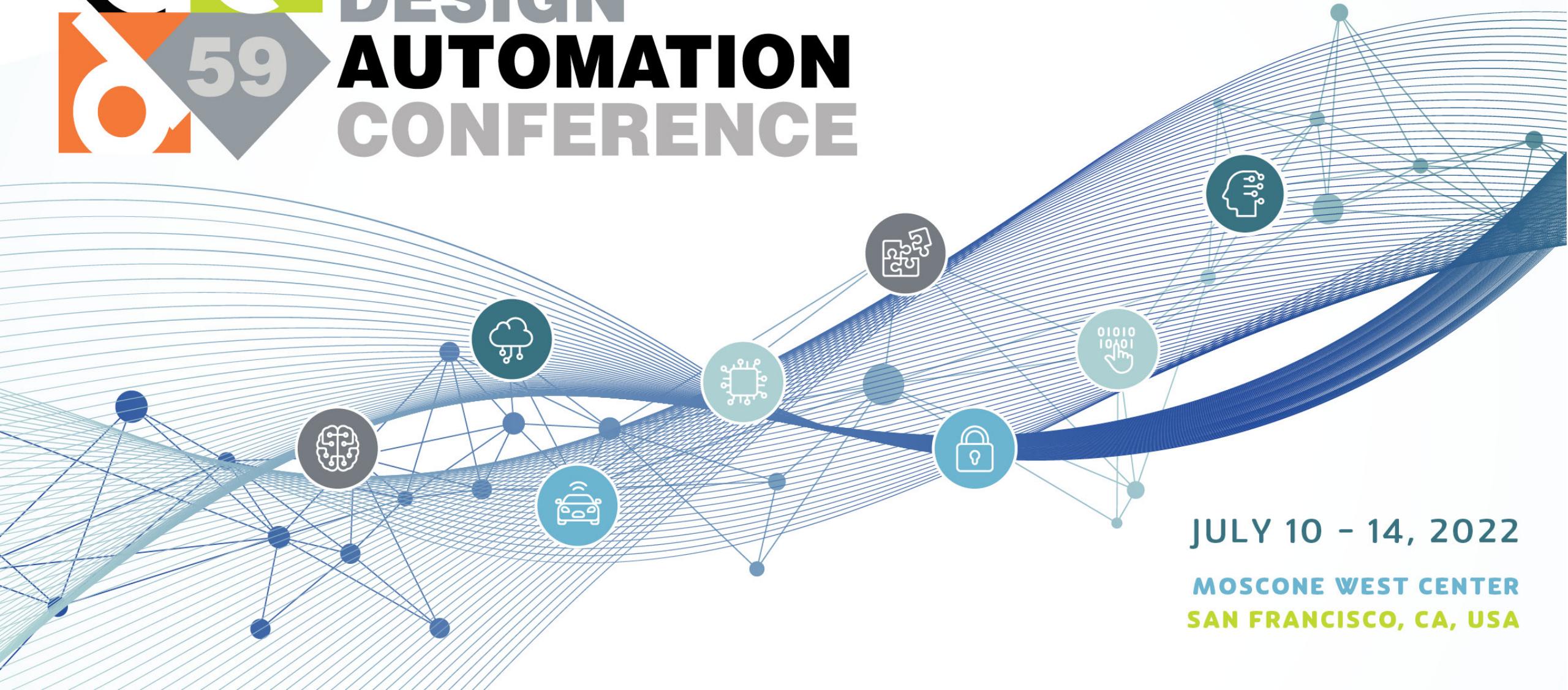




# DESIGN AUTOMATION CONFERENCE



JULY 10 - 14, 2022

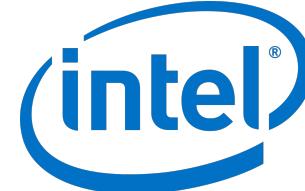
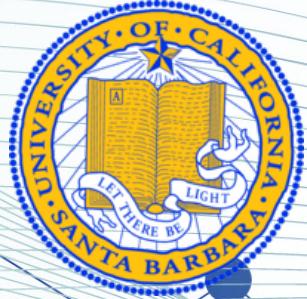
MOSCONE WEST CENTER  
SAN FRANCISCO, CA, USA



# Enabling Deeper Quantum Compiler Optimization at High Level

Yufei Ding, Gushu Li, Anbang Wu, Yuan Xie

07/11/2022



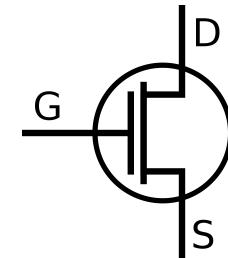
# The Quantum Revolution

1<sup>st</sup> Quantum  
Revolution

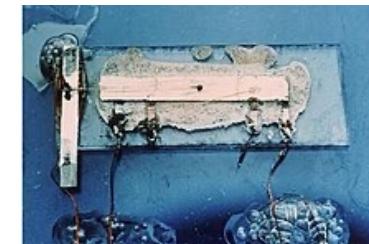
1900  
Quantum  
Mechanics



1947  
Transistor



1956  
MOSFET



1958  
Integrated  
Circuit

Classical  
Computer

1980

2<sup>nd</sup> Quantum Revolution: the power of  
quantum is not fully exploited

# The Quantum Revolution

2<sup>nd</sup> Quantum  
Revolution

1980  
Quantum  
Computer



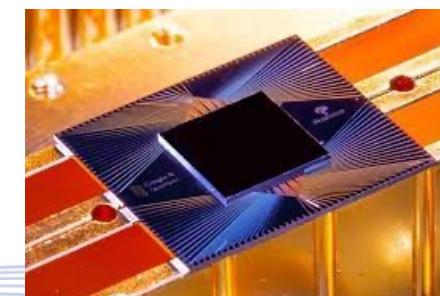
1994  
Cryptography

2019  
Supremacy

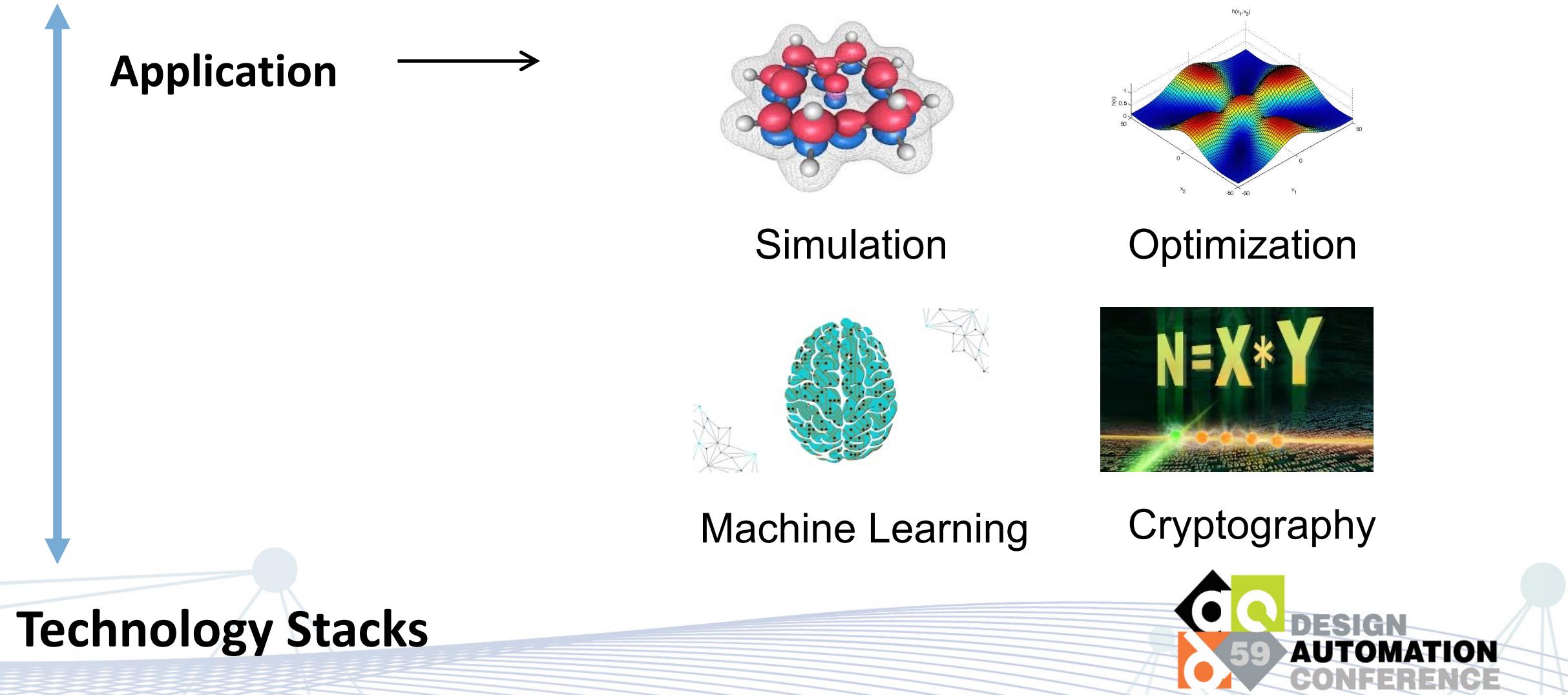
Practical  
Quantum  
Computing



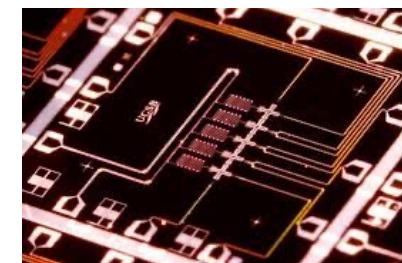
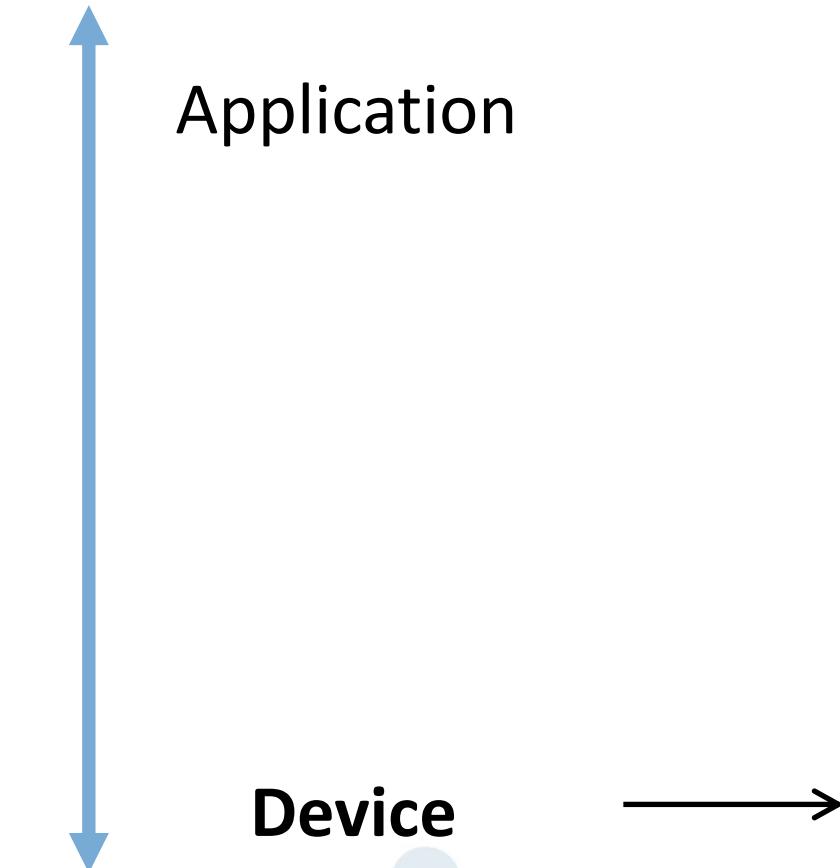
Now



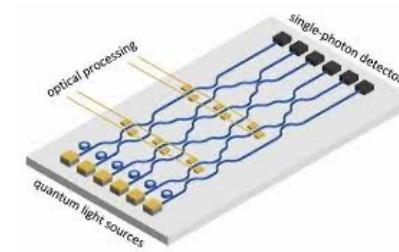
# Quantum Computing System Stacks



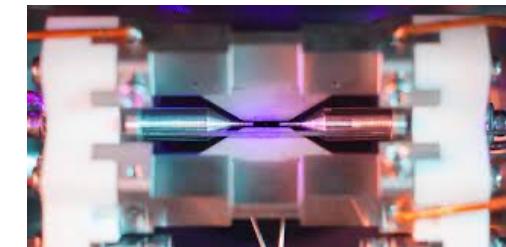
# Quantum Computing System Stacks



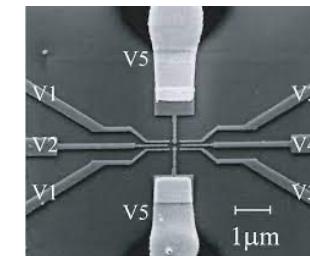
Superconducting



Photonics



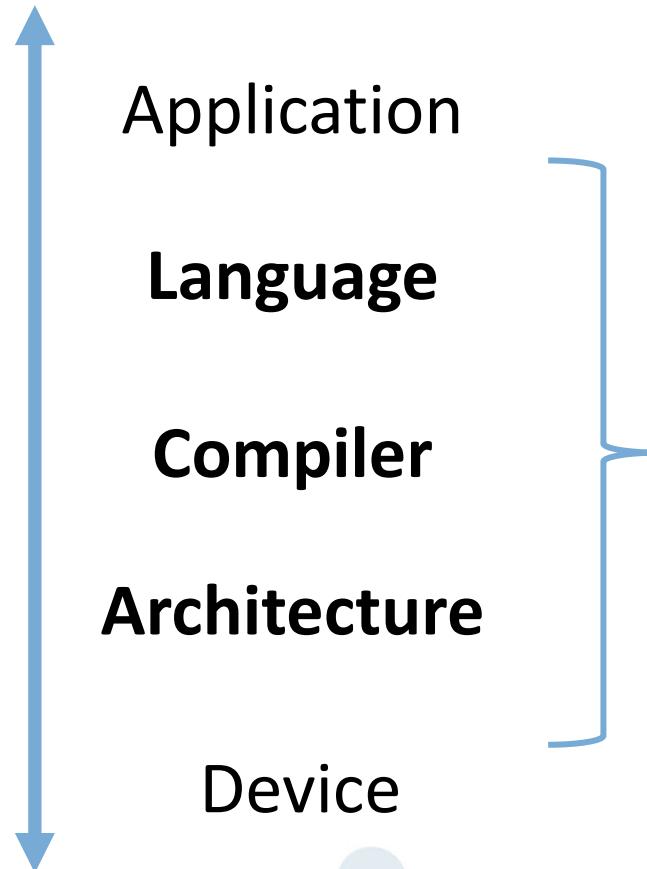
Ion Trap



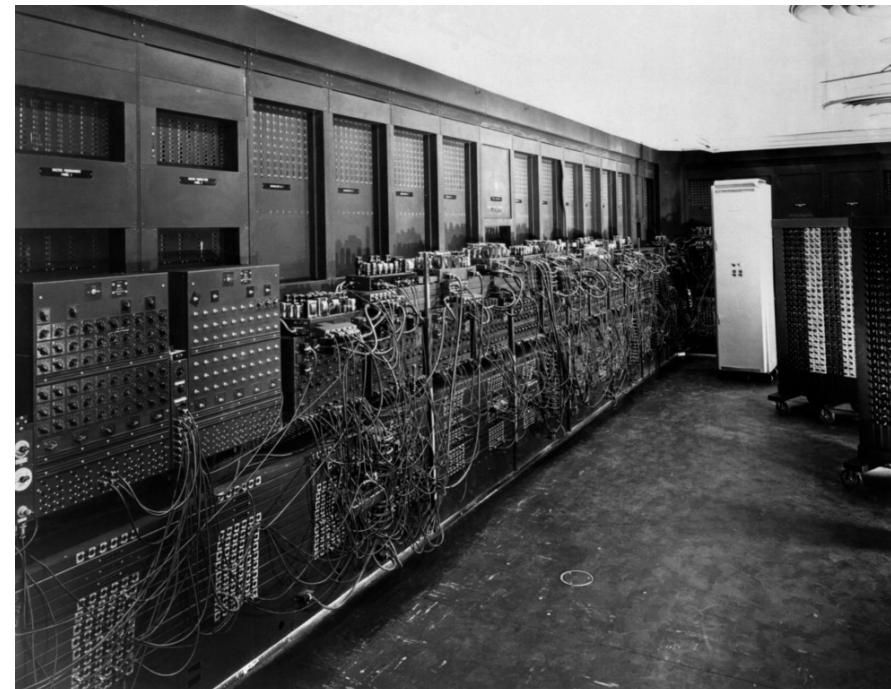
Quantum Dot

Technology Stacks

# Quantum Computing System Stacks



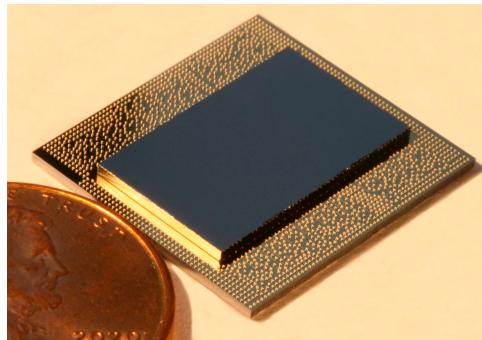
ENIAC, the first electronic general purpose digital computer, 1945



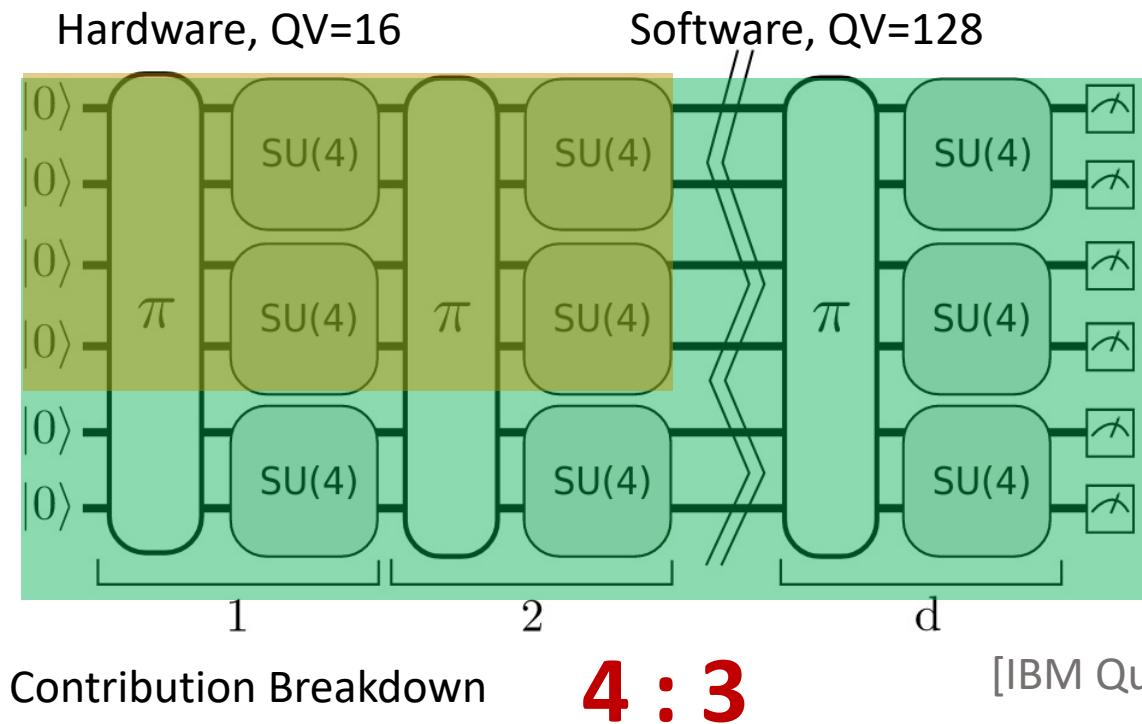
Technology Stacks

# Hardware vs Software

- IBM benchmarking results



IBM Q Montreal



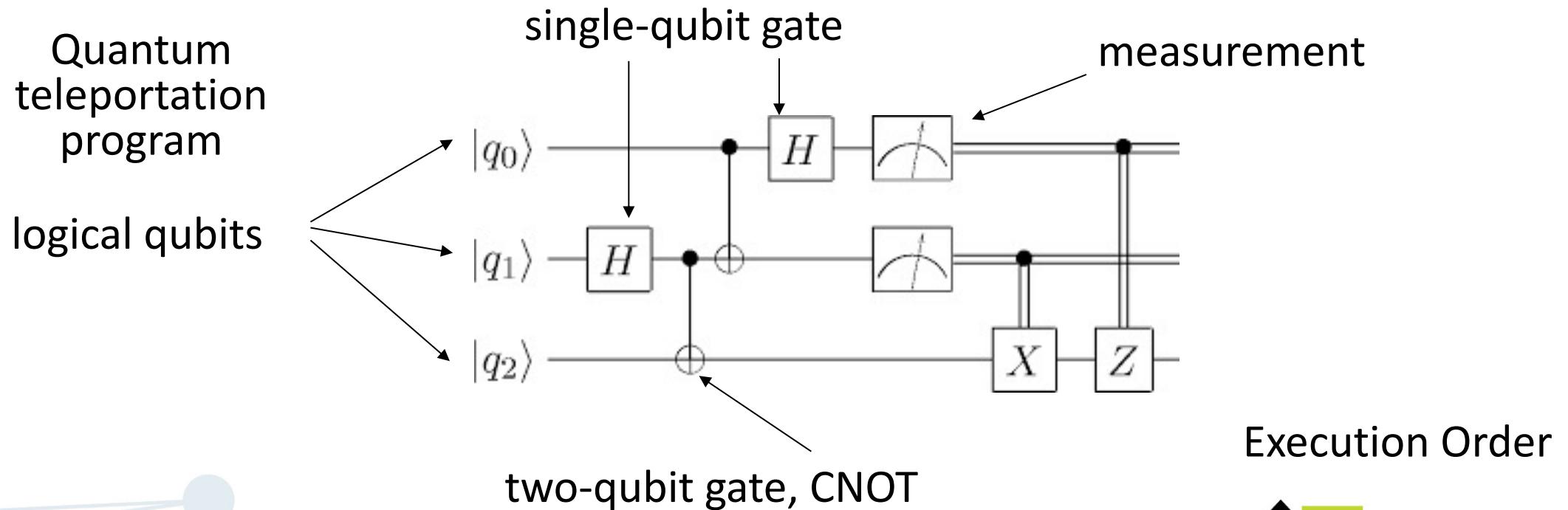
Quantum Volume (QV): the size of Hilbert space that a quantum processor can explore reliably

Quantum system research extends the computation capability!

And both software and hardware are important

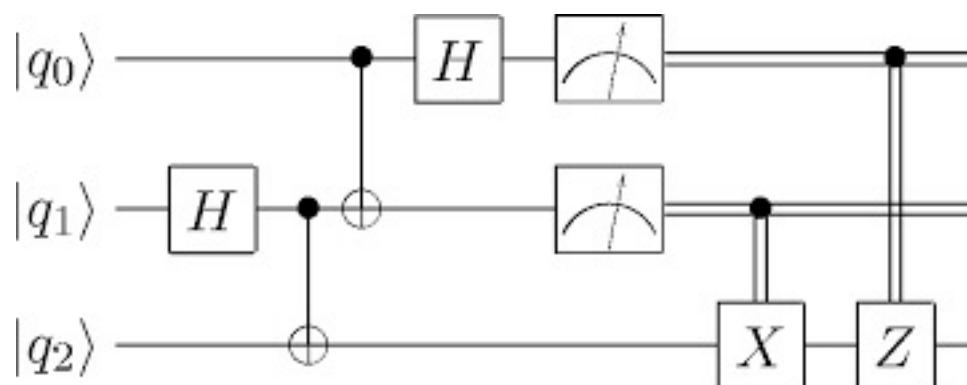
# Background

- Quantum software – quantum circuit



# Background

- Quantum software – program state



State vector:

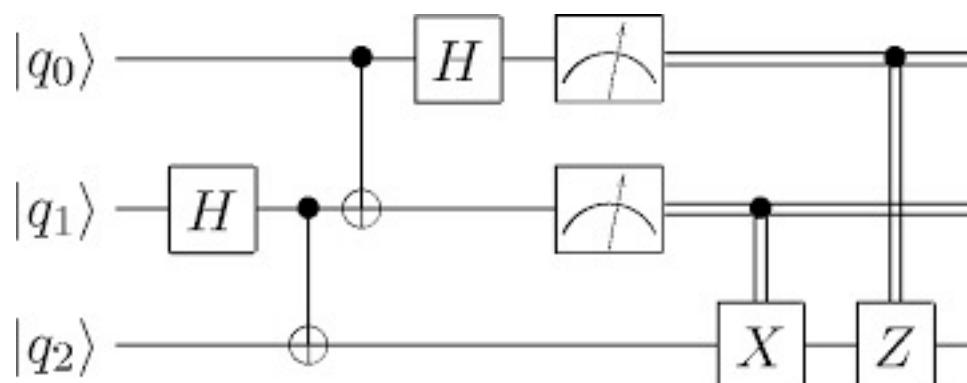
1-qubit,  $|\psi\rangle = a_0|0\rangle + a_1|1\rangle$ ,  $[a_0, a_1]$

3-qubit,  $|\psi\rangle = a_{000}|000\rangle + a_{001}|001\rangle + \dots + a_{111}|111\rangle$   
 $[a_{000}, \dots, a_{111}]$

n-qubit, state vector  $[a_0, \dots, a_{2^n-1}]$  of size  $2^n$

# Background

- Quantum software – program semantics



Gate matrices:

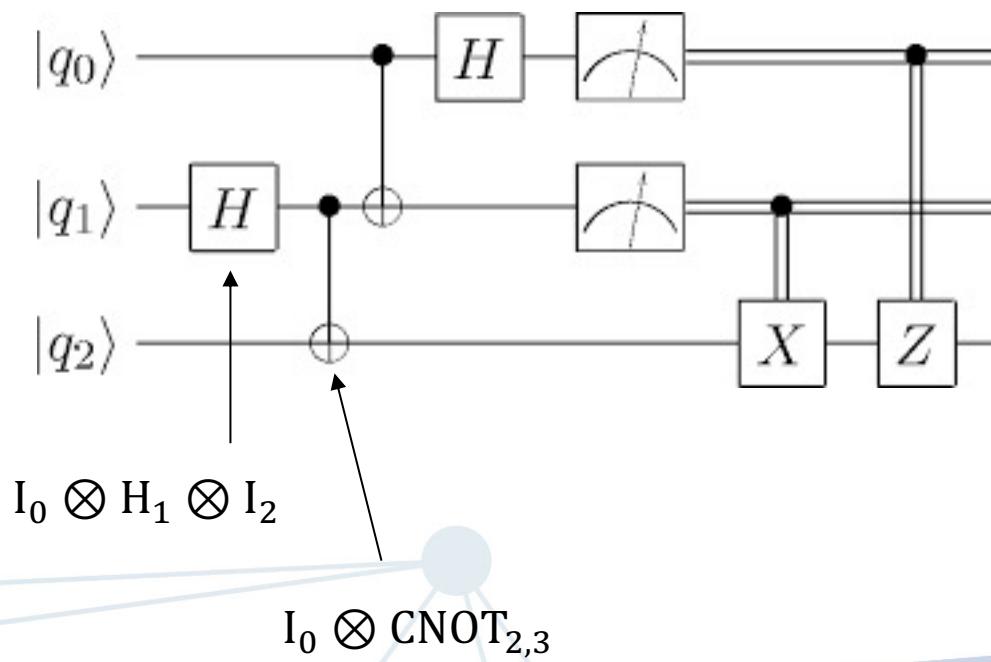
$$1\text{-qubit, } H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$2\text{-qubit, CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

n-qubit, matrices of size  $2^n$  by  $2^n$

# Background

- Quantum software – program semantics



Gate matrices:

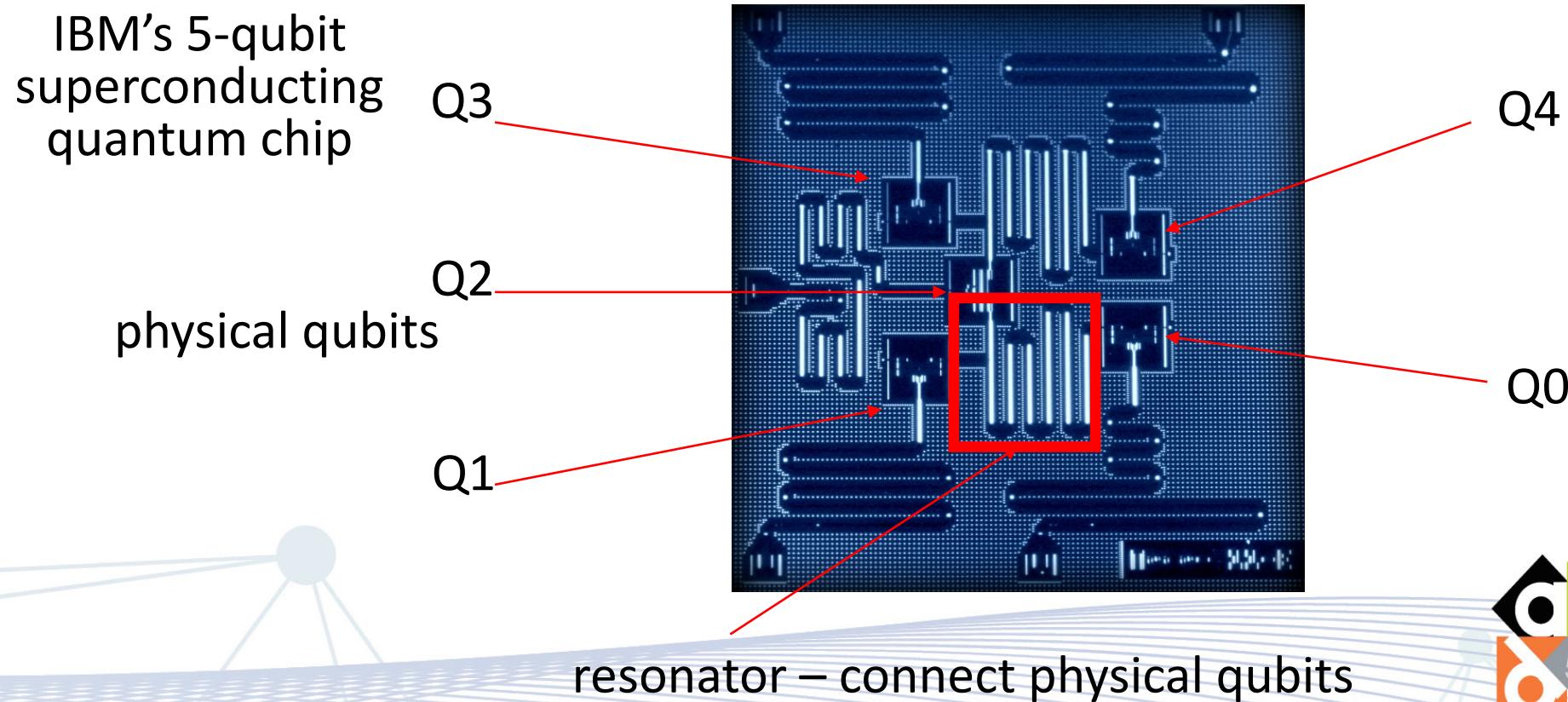
$$1\text{-qubit, } H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

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n-qubit, matrices of size  $2^n$  by  $2^n$

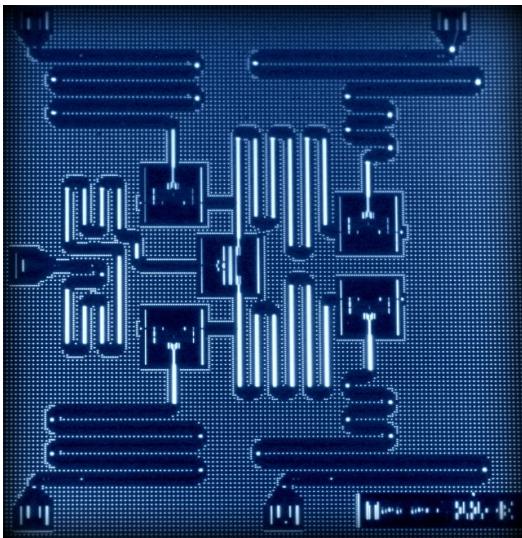
# Background

- Quantum hardware – the superconducting architecture

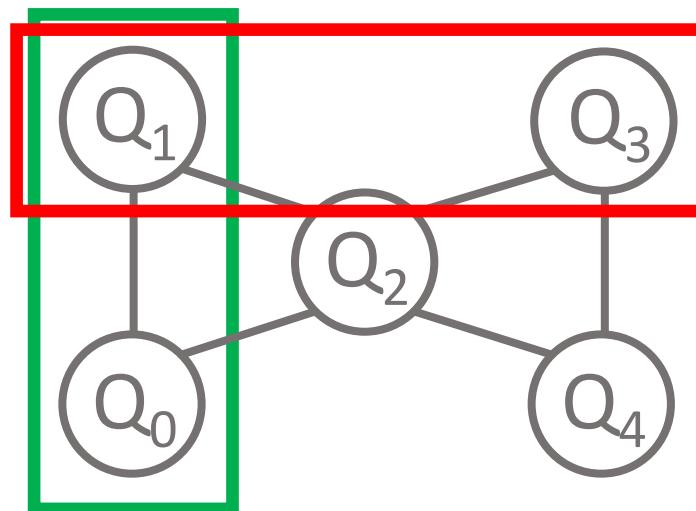


# Background

- Quantum hardware – the superconducting architecture



IBM's 5-qubit  
superconducting  
quantum chip



Coupling graph – limited  
qubit connection

CNOT gates only  
allowed on the  
connected edges

CNOT Q0, Q1

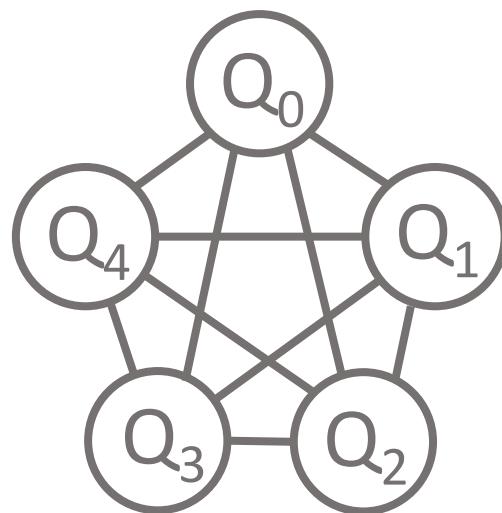


CNOT Q1, Q3

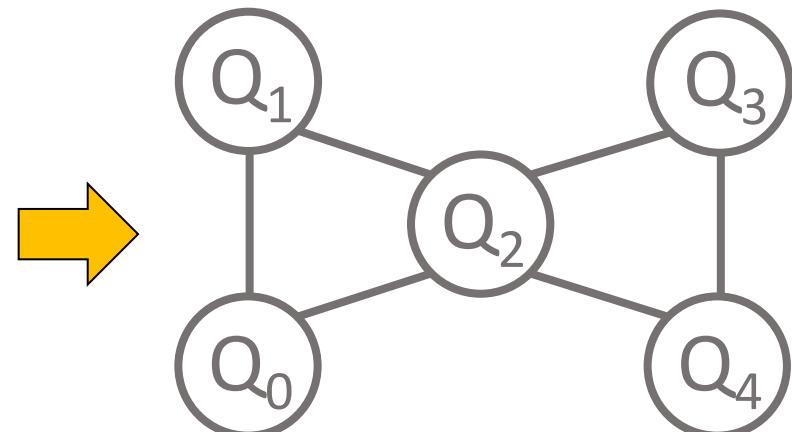


# Mismatch

- When we write a quantum program, we may not know the underlying architecture



Ideal device –  
complete graph

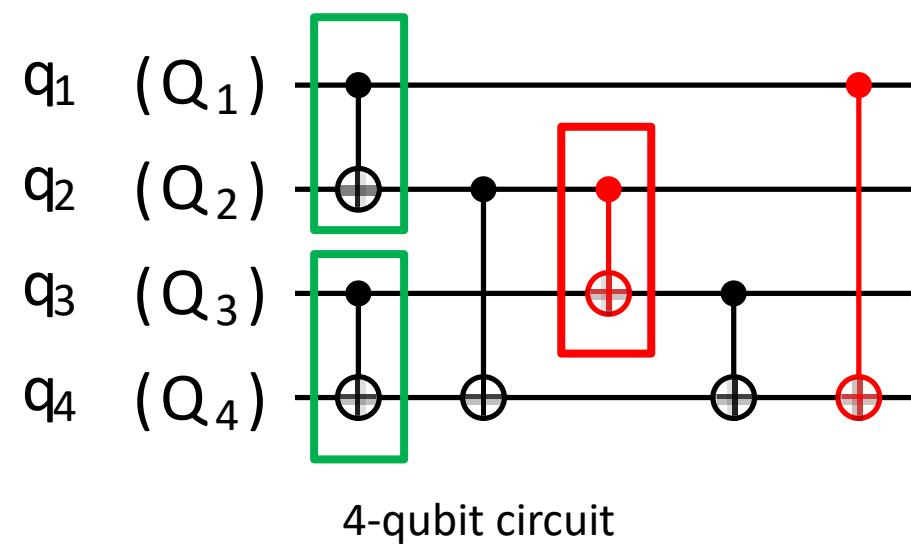
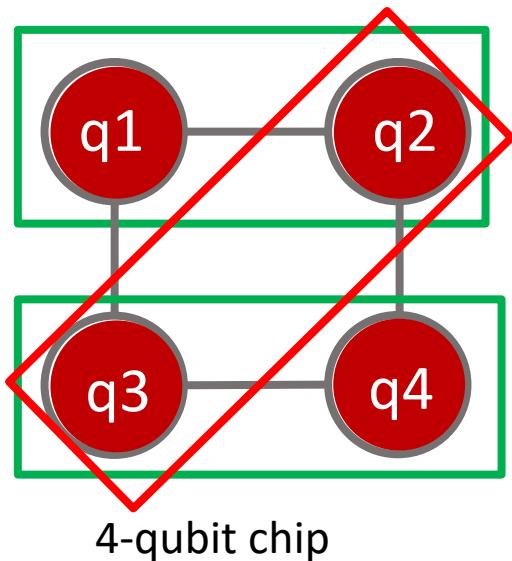


Coupling graph – limited  
qubit connection

Some gates may  
not be executable

# Qubit Mapping

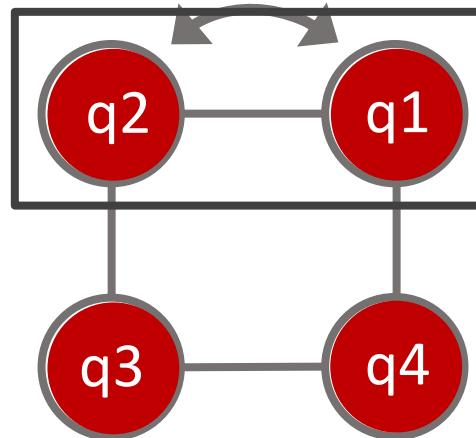
- An Example



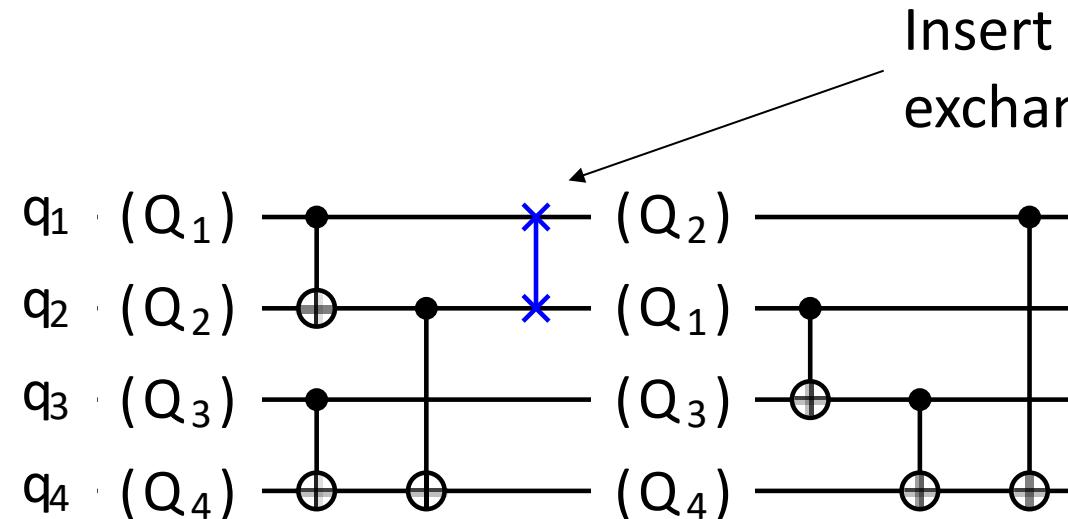
Baths of NOTs  
are executable

# Qubit Mapping

- An Example



4-qubit chip



Insert additional gate SWAP:  
exchange the mapping

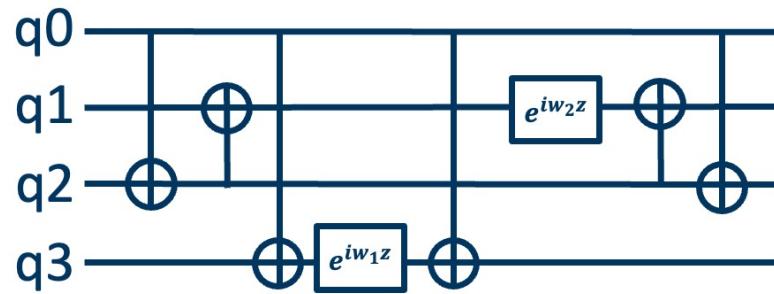
$$\begin{array}{c} q_1 \\ q_2 \end{array} = \begin{array}{c} \text{*} \\ \text{*} \end{array} = \begin{array}{ccccc} & & & & \\ \bullet & \otimes & \bullet & \otimes & \bullet \\ & \otimes & \bullet & \otimes & \bullet \end{array}$$

1 SWAP = 3 CNOT

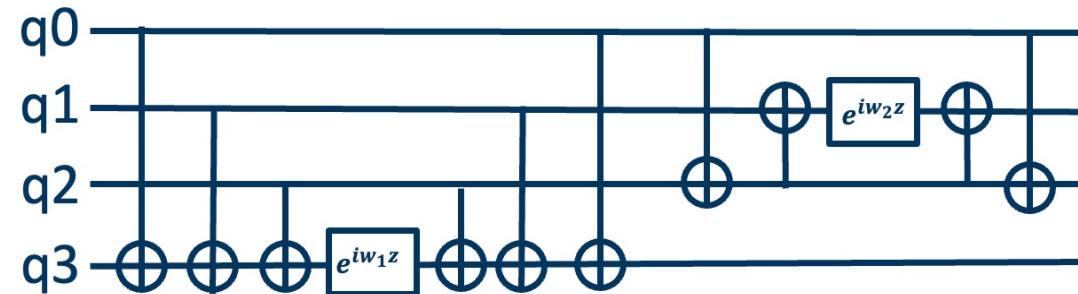
Each additional SWAP leads to more noise

# Quantum Compiler Optimization

- Find some circuit identities
- Select the best one according to some metrics (e.g., # gate, # depth) and constraints (e.g., sparse connection on hardware)



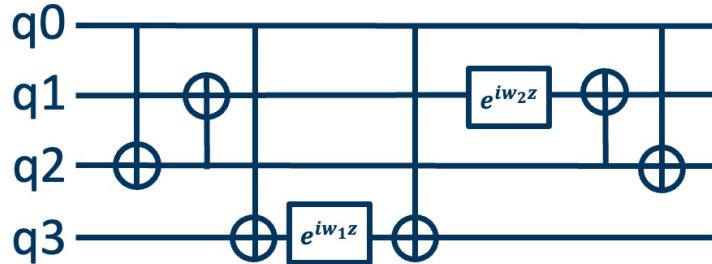
Gate: 6 + 2; Depth: 6



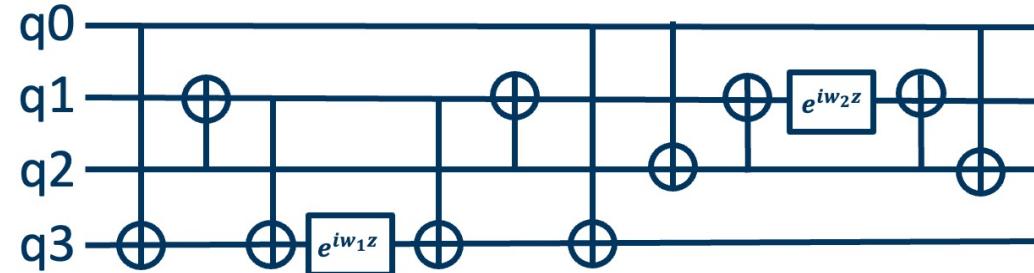
Gate: 10 + 2; Depth: 12

# Challenge

- How can we find **large-scale** quantum circuit identities efficiently?



?=



- ❖ Calculate their **matrix representations** and check the **equivalence**?
    - \* Scalability issue: Matrix size of  $2^n \times 2^n$  and/or huge combinatorial search space.
  - ❖ Limit our **compiler optimization scope**: peephole optimization, local swap insertions in qubit mapping, etc.
    - \* Missing large-scope optimizations.



# Paulihedral: A Generalized Block-Wise Compiler Optimization Framework for Quantum Simulation Kernels

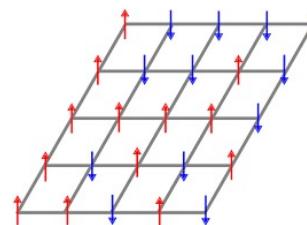
Gushu Li, Anbang Wu, Yunong Shi, Ali Javadi-Abhari, Yufei Ding,  
Yuan Xie

ASPLOS 2022

# Opportunities at High-Level

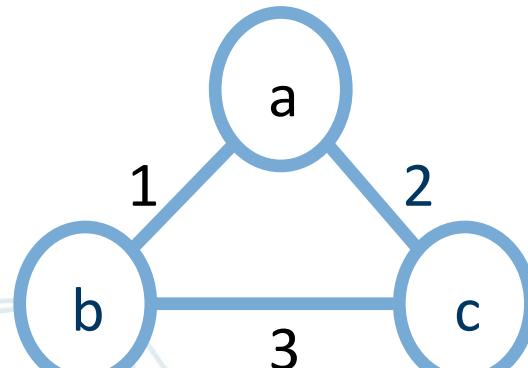
- More **abstract compact** form? Yes
- **Simulation** is widely used in quantum algorithm design

Simulation



$$H = -\sum J_{ij} Z_i Z_j - \mu \sum h_j Z_j$$

Graph Cut



$$H = \frac{1}{2}(Z_a Z_b - I) + \frac{2}{2}(Z_a Z_c - I) + \frac{3}{2}(Z_b Z_c - I)$$

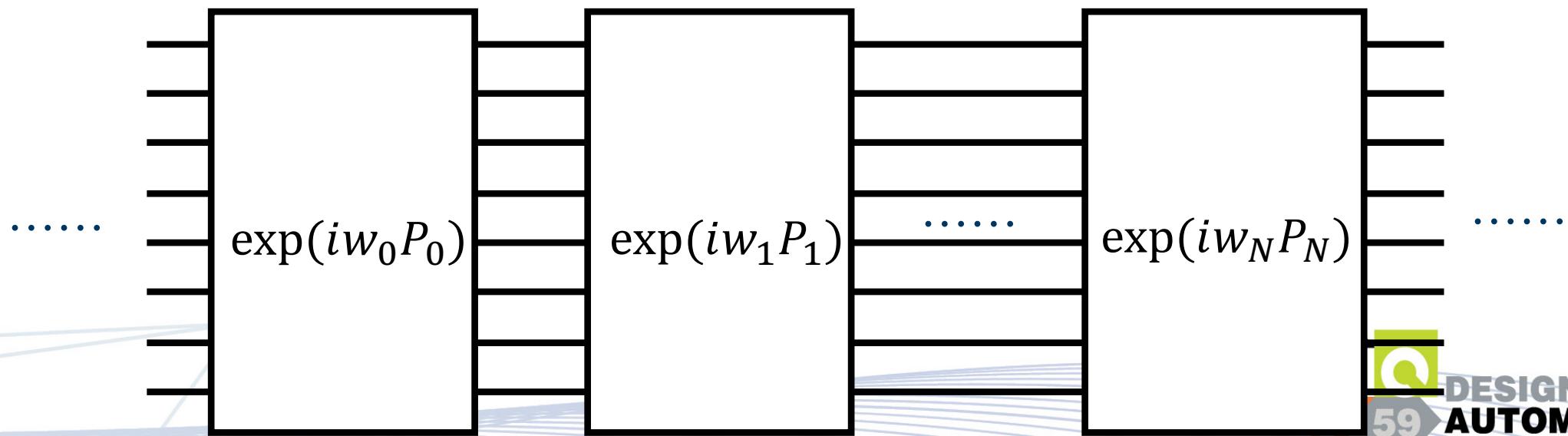
And many more

# Quantum Simulation Kernel

- A widely-used subroutine

$$\exp(iHt)$$

$H = \sum_i w_i P_i$ ,  $P_i$  is a Pauli string,  $w_i \in \mathbb{R}$  is weight



# High-Level IR: Pauli IR

(P, w)

Basic unit of our IR: a pair of Pauli string P and a real number w.

- Pauli string P is just **Kronecker product** of 1-qubit Pauli matrices (I, X, Y, Z).

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

A size-n P can concisely express a  $2^n \times 2^n$  matrix

- Examples of 4-qubit Pauli string:  $X_3Y_2Z_1I_0, Z_3Z_2Z_1Z_0, X_3Y_2Y_1X_0$
- Active qubits** for a Pauli string: qubits with a non-I Pauli matrix.

# High-Level IR: Pauli IR

(P, w)

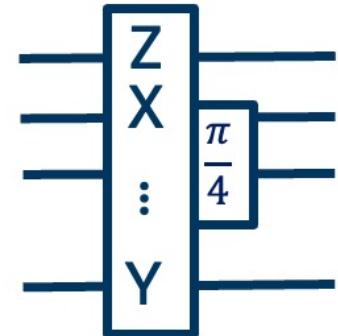
denotes a **2<sup>n</sup> x 2<sup>n</sup>** unitary gate  $\exp(iwP)$ .

Example:  $(ZX\dots Y, \frac{\pi}{4})$

Pauli IR (kernel)

$\exp(i \frac{\pi}{4} ZX\dots Y)$

Exponential form

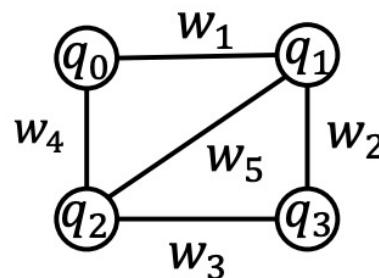


Circuit form

Universal in terms of unitary gates

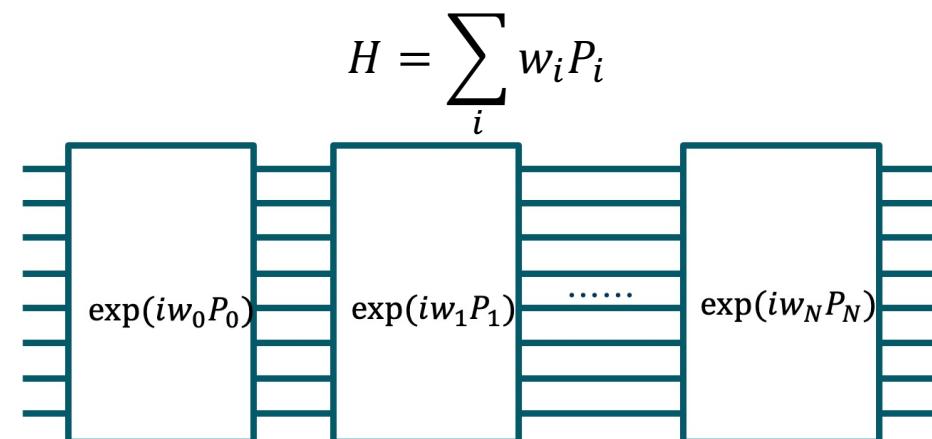
# Compile to Pauli IR

- **Great news:** It is already there, at least for many quantum algorithm design.
- (e.g., QAOA, VQE, and many other quantum simulation algorithms like UCCSD).



$\exp(iw_1I_3I_2Z_1Z_0\gamma)$   
 $\exp(iw_2Z_3I_2Z_1I_0\gamma)$   
 $\exp(iw_3Z_3Z_2I_1I_0\gamma)$   
 $\exp(iw_4I_3Z_2I_1Z_0\gamma)$   
 $\exp(iw_5I_3Z_2Z_1I_0\gamma)$

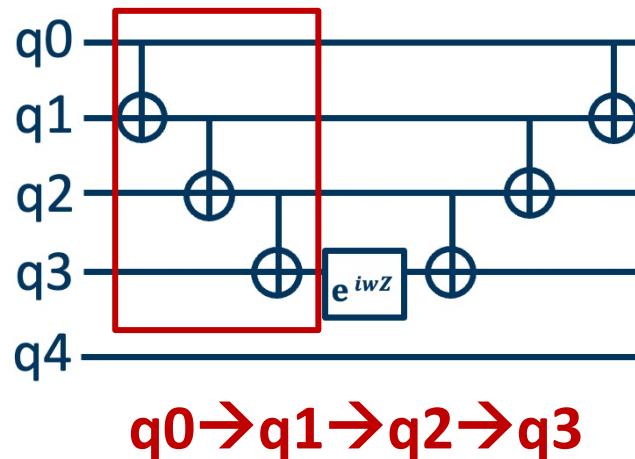
QAOA Ansatz on graph Max-Cut



# From Pauli IR to Gates

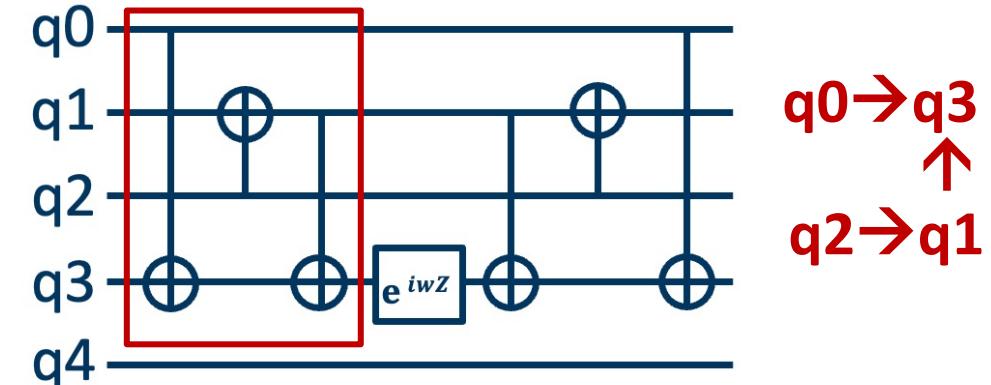
- **Very flexible compilation/synthesis**

$$\exp(iwI_4Z_3Z_2Z_1Z_0)$$



❖ if **z-basis parity on q3, q2, q1, q0 = 0**, apply a global phase  $\exp(iw)$ ; otherwise, apply  $\exp(-iw)$ .

CNOT tree  
for parity check



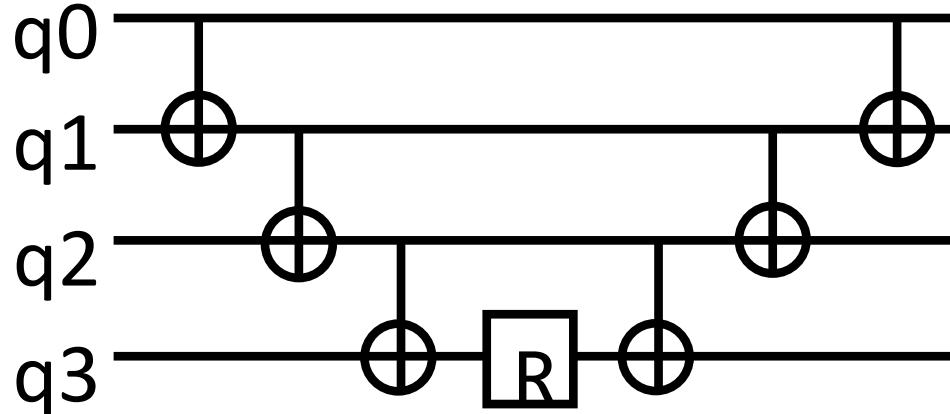
It thus could generate many great **circuit identities** with the same  $(P, w)$ .

# Example: Qubit Mapping

- How can Paulihederal change the mapping/SWAP insertion?

# Conventional Compilation

- Find SWAP in gate sequence



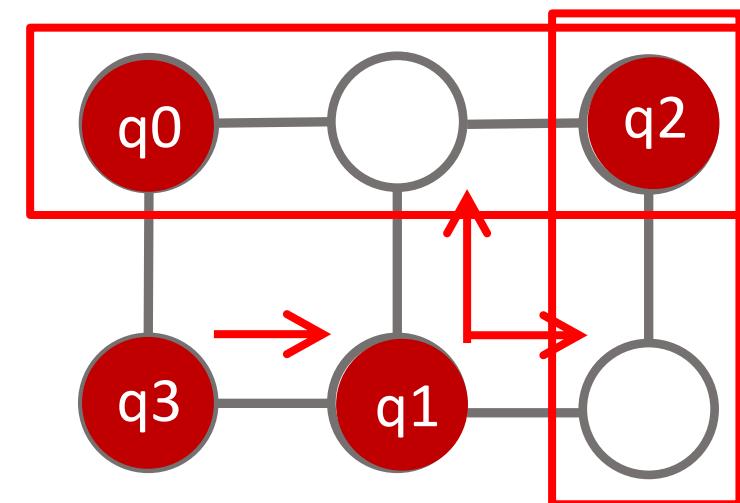
CNOT q0, q1

CNOT q1, q2

CNOT q2, q3

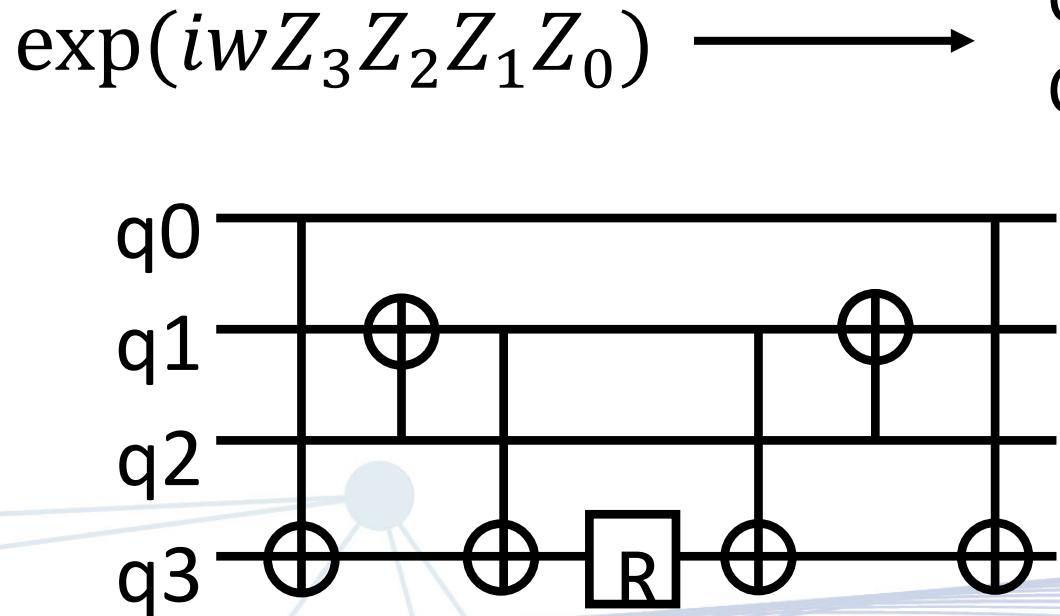
$$\exp(iwZ_3Z_2Z_1Z_0)$$

SWAP q1, ...  
CNOT q0, q1  
CNOT q1, q2  
SWAP q3, ...  
SWAP q3, ...  
CNOT q2, q3



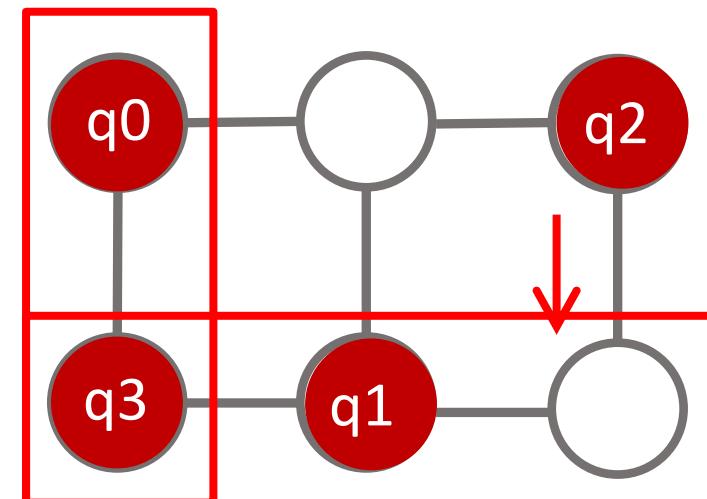
# Paulihedral Compilation

- Leverage high-level information



$$\exp(iwZ_3Z_2Z_1Z_0)$$

SWAP q2, ...  
CNOT q0, q3  
CNOT q2, q1  
CNOT q1, q3

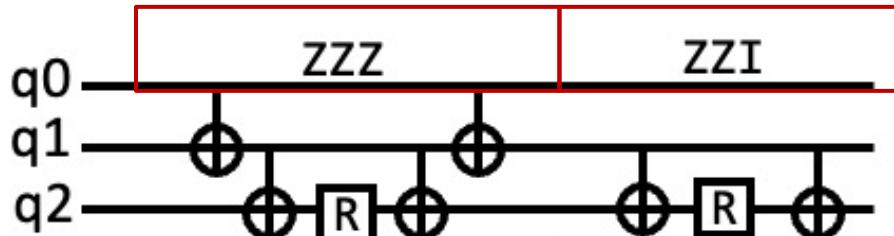


Find a tree embedding, then  
generate the CNOT tree

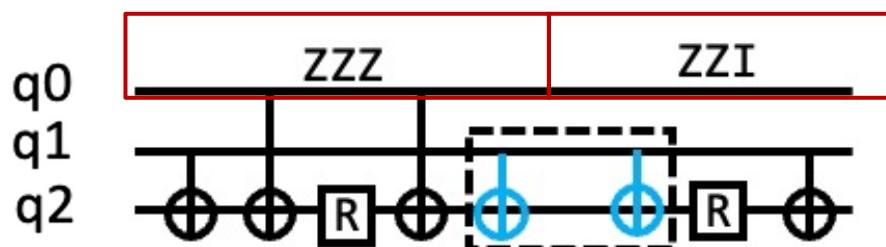
# More Gate Cancellation

- Find **global gate cancellation** among Pauli strings

- Circuit synthesis for  $\exp(iZ_2Z_1I_0) \cdot \exp(iZ_2Z_1Z_0)$



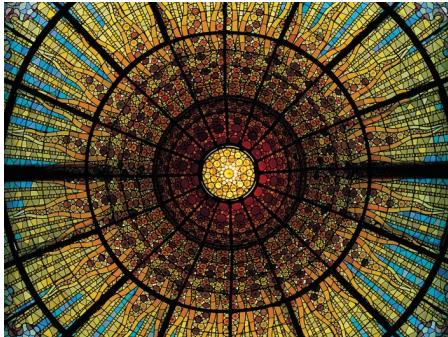
- ❖ Naïve synthesis for each separate kernel
- ❖ No gate cancellation.



- ❖ **Common subtree-centric gate synthesis** for large-scope gate cancellation.
- ❖ CNOTs cancel out with each other.

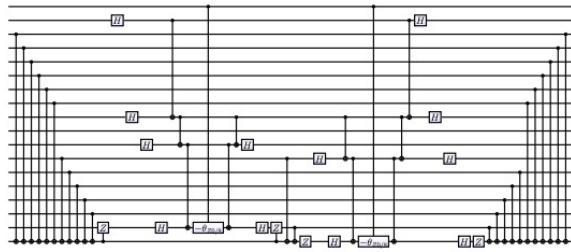
More active qubits overlapping between nearby Pauli IR kernels → more gate cancellation.

# Moreover



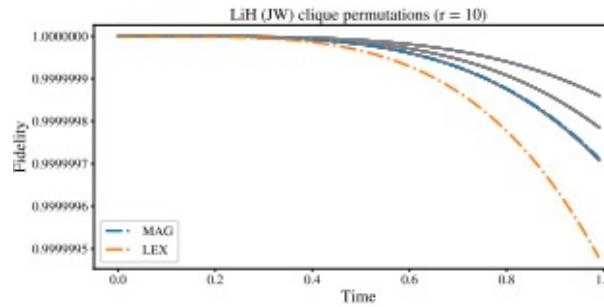
[Livio, 2012]

symmetry preserving

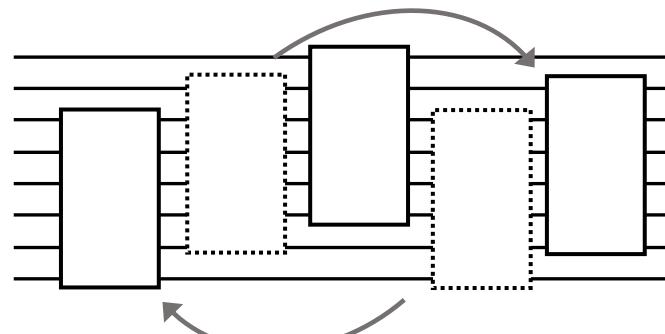


[Hastings et al. 2015]  
more gate cancellation

Please see paper for details



[Gui et al. 2019]  
error mitigation

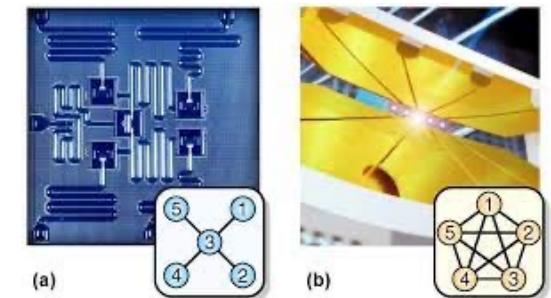


large-scope scheduling

$$(a_2^\dagger a_0 - a_0^\dagger a_2) = \frac{i}{2}(X_2 Z_1 Y_0 - Y_2 Z_1 X_0)$$
$$(a_3^\dagger a_1 - a_1^\dagger a_3) = \frac{i}{2}(X_3 Z_2 Y_1 - Y_3 Z_2 X_1)$$
$$(a_3^\dagger a_2^\dagger a_1 a_0 - a_0^\dagger a_1^\dagger a_2 a_3) =$$
$$\frac{i}{8}(X_3 Y_2 X_1 X_0 + X_3 X_2 X_1 X_0 + Y_3 Y_2 Y_1 X_0 + Y_3 Y_2 X_1 Y_0 - X_3 X_2 Y_1 X_0 - X_3 X_2 X_1 Y_0 - Y_3 X_2 Y_1 Y_0 - X_3 Y_2 Y_1 Y_0).$$

[McArdle et al. 2020]

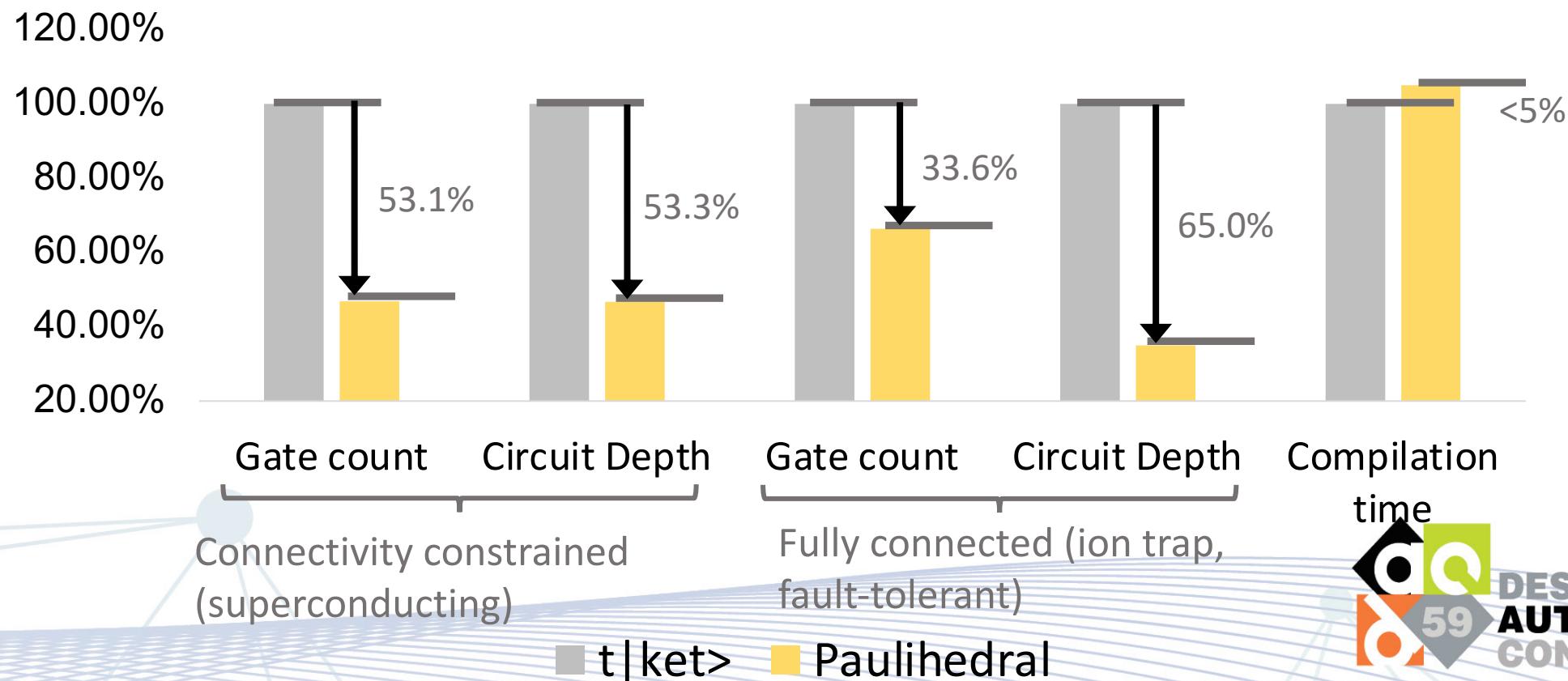
parameter sharing



[Linke et al. 2017]  
different backends

# Evaluation

- Benchmarks: molecule/Ising/Heisenberg/random Hamiltonian, UCCSD/QAOA graph ansatz





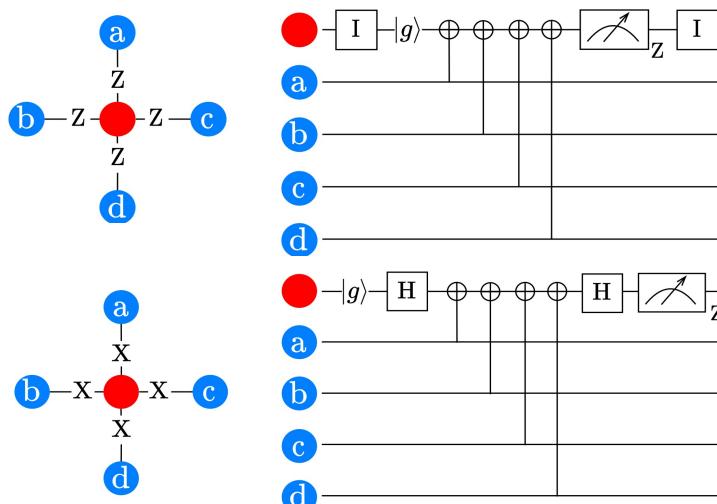
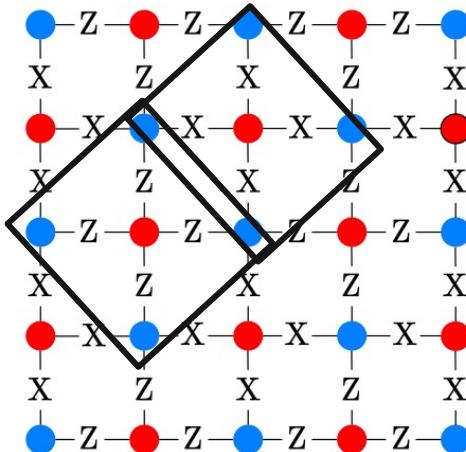
# A Synthesis Framework for Stitching Surface Code with Superconducting Quantum Devices

Anbang Wu, Gushu Li, Hezi Zhang, Gian Giacomo Guerreschi,  
Yufei Ding, Yuan Xie

ISCA 2022

# QEC Program: Surface Code

- Surface code: one of the best QEC in terms of error correction capabilities (up to about 1% error).



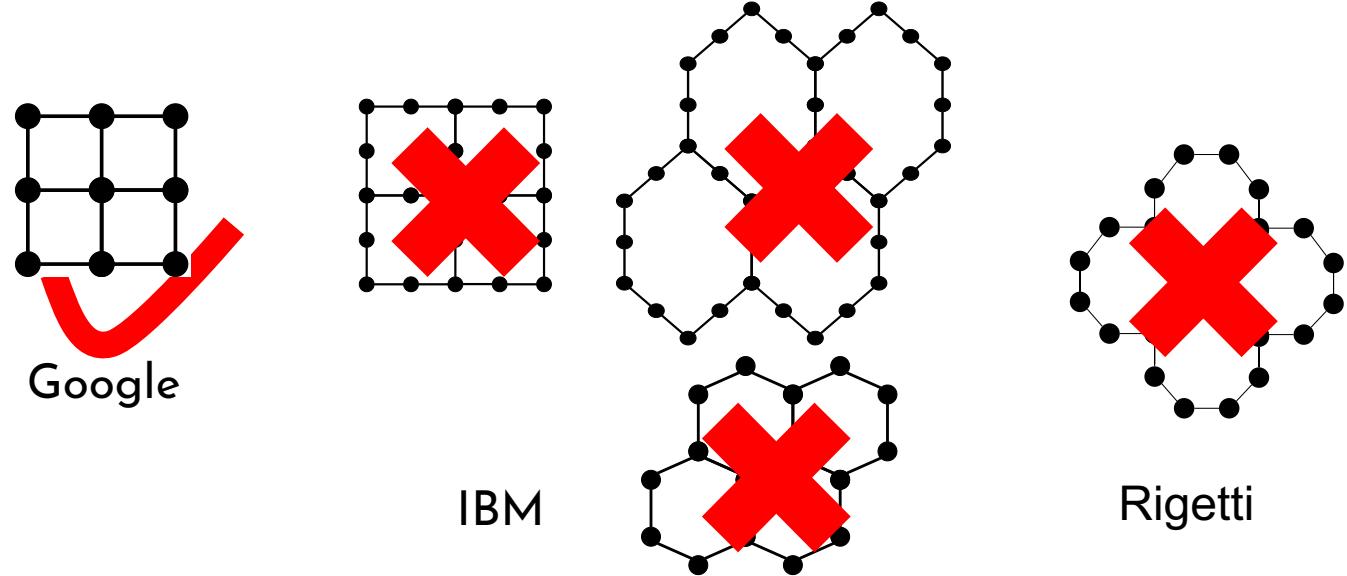
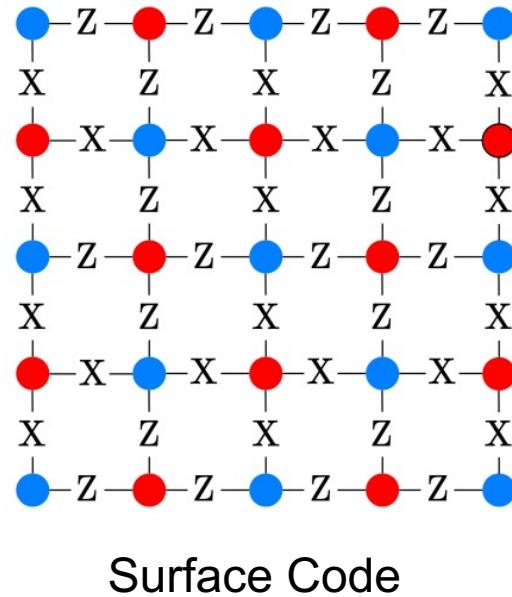
Circuits can be perfectly mapped to the hardware (coupling graph) on the left side.

- 2-D lattice qubit structure: **long-range entanglement** to protect the logic qubit from local noises.

**Data qubits** (blue): encode the correct subspace for logical operations.

**Syndrome qubit** (red): ensure data qubits are working collaboratively by checking their X, Z parity.

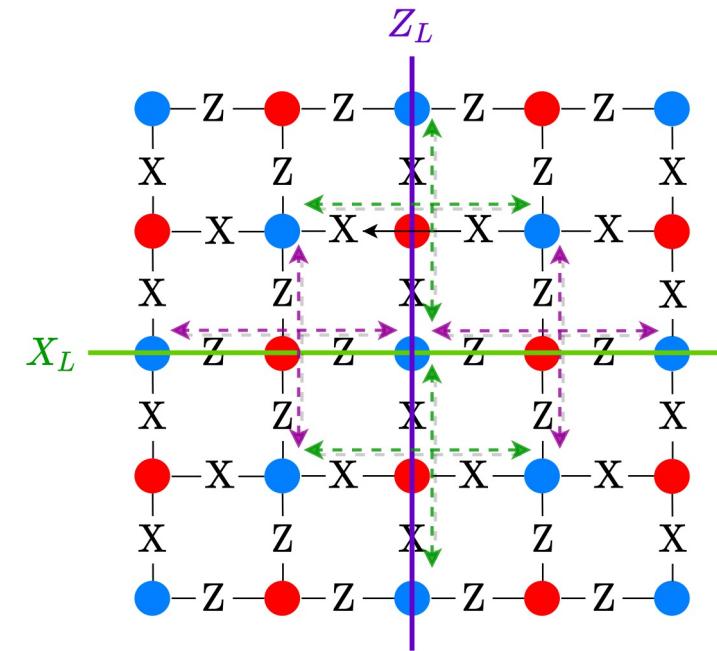
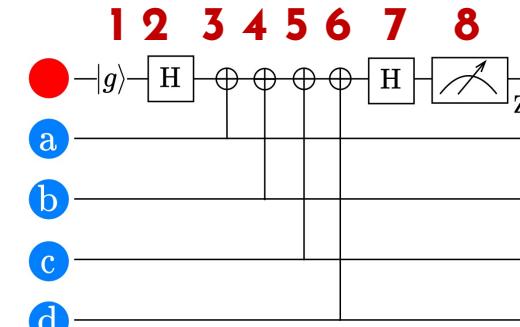
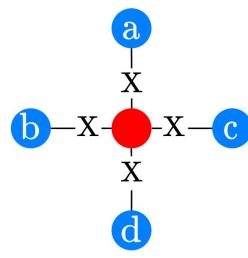
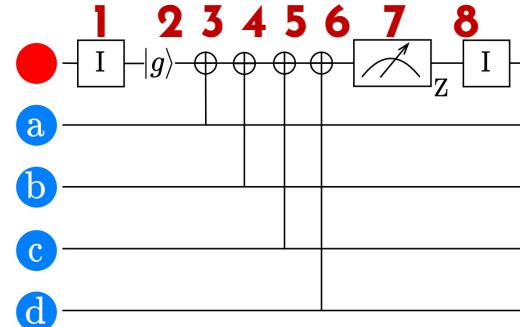
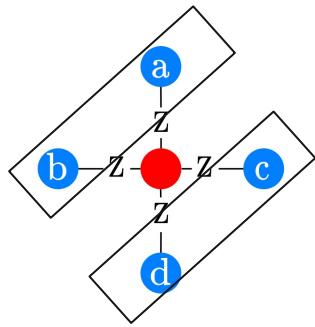
# Mismatch between Surface Code and Sparse Architectures



Some recent study designed new QEC codes tailored for these **sparse** architecture.

- ❖ Can the “mismatch” be mitigated by compiler optimization?

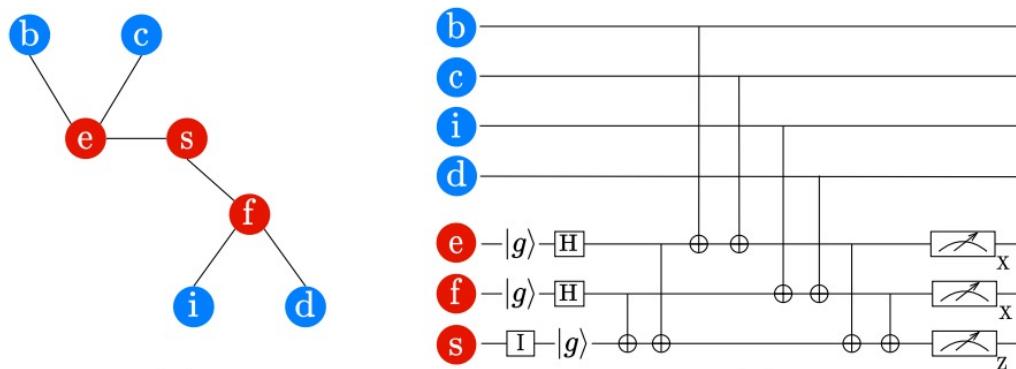
# What is Special about QEC program?



- 1) **4-degree qubit**: each syndrome qubit (red) measures the **parity** of 4 data qubits (blue).
- 2) **Fixed data qubit Layout**: moving data qubits would invalidate those high-level logical operations.
- 3) **Stabilizer measurement scheduling**: zigzag (instead of clockwise) gives maximum parallelism.

# Our QEC Compiler

- How to resolve the mismatch problem from a compiler's perspective? If so, to what extent?



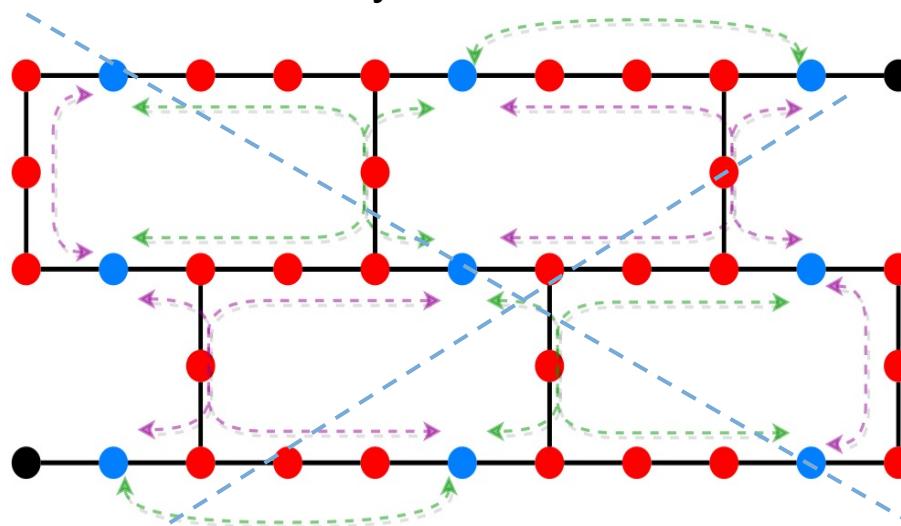
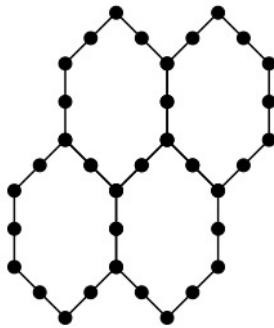
**Bridge tree** to encode a “**logic syndrome qubit**, and use it to replace a 4-degree node.  
[L. Lao et. al. PRA2020]

- ❖ **Three key submodules** to resolve the surface code mismatch problem:
  1. How are data qubits allocated?
  2. How to find “small and local “ bridge trees?
  3. How to schedule stabilizer measurement circuits?

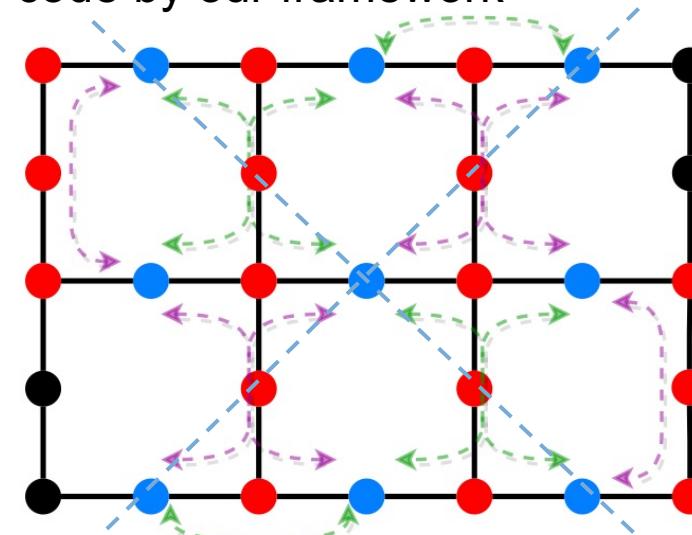
# Our QEC Compiler

- We build the **first automated framework** for compiling surface code to sparse quantum devices with a **modularized** design.

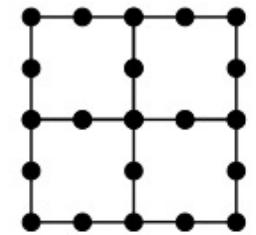
➤ The synthesized distance-3 surface code by our framework



(a) Heavy Hexagon



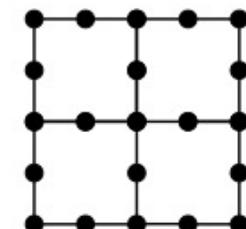
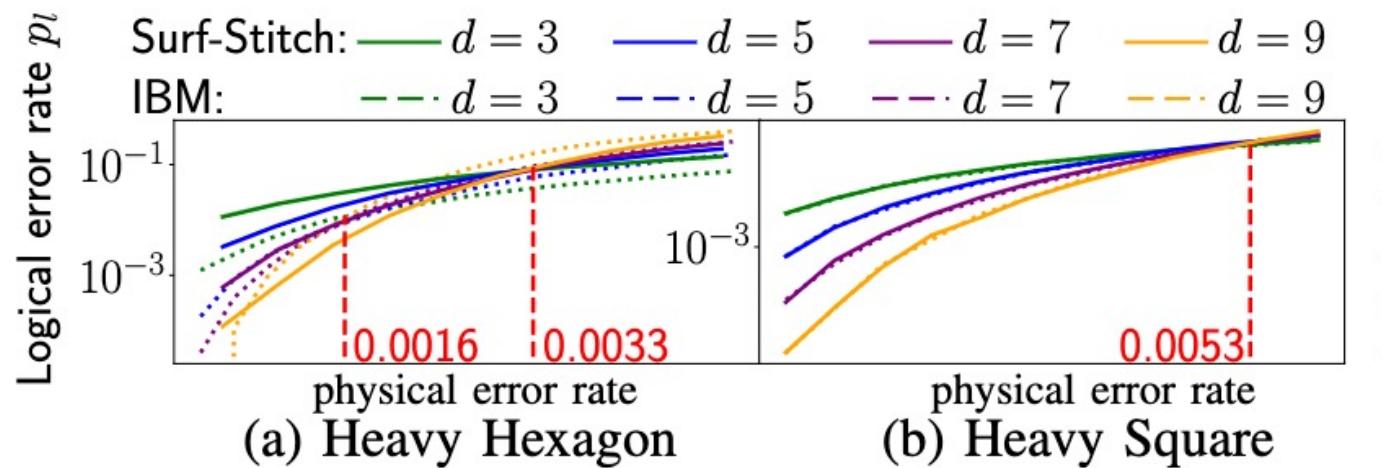
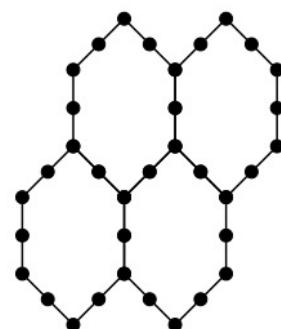
(b) Heavy Square



We could work with different architectures + missing links/qubits.

# Performance of Our QEC Compiler

- The *error threshold* of the compiled surface code is comparable or even better than IBM's **manually designed** QEC codes tailored for the sparse architectures.



More results in our paper.

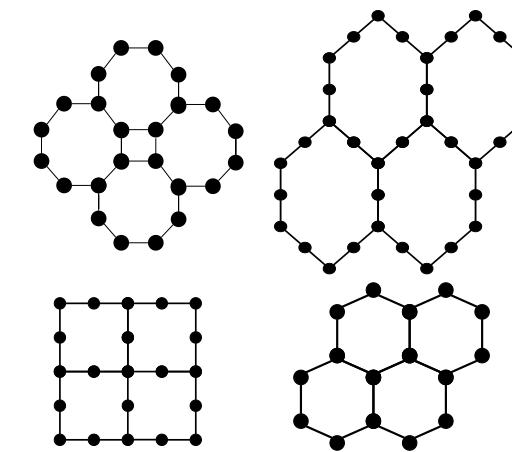
# Our Vision for Future QEC Development

- General QEC design and mapping could be formulated as a compiler problem.

Different stabilizer codes,  
subsystem codes,  
and their hybrid concatenation,  
etc.  
...

Automatic Mapping  
and Optimization  
Framework

Metrics:  
Error Threshold?  
Qubit Utilization?  
Fabrication Yield?  
....



Hardware

A compiler framework to automate the designs of **hardware-aware** QEC codes +  
its associated error decoder!!!

# Q & A

- Thank you!