Tutorial on QuantumFlow+VACSEN: A Visualization System for Quantum Neural Networks on Noisy Quantum Devices

Session 3: QuantumFlow Co-Design Framework

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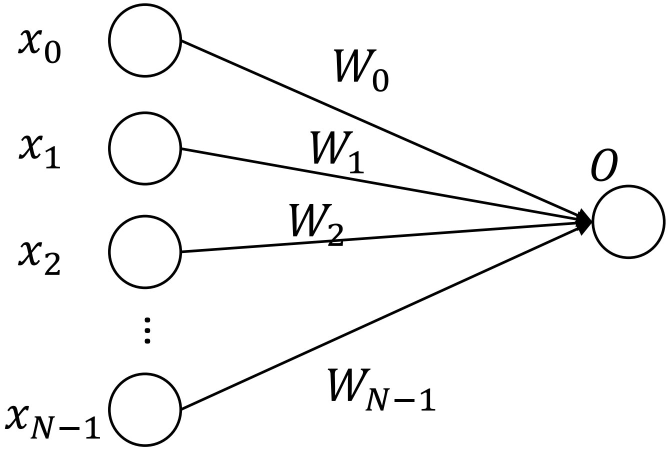
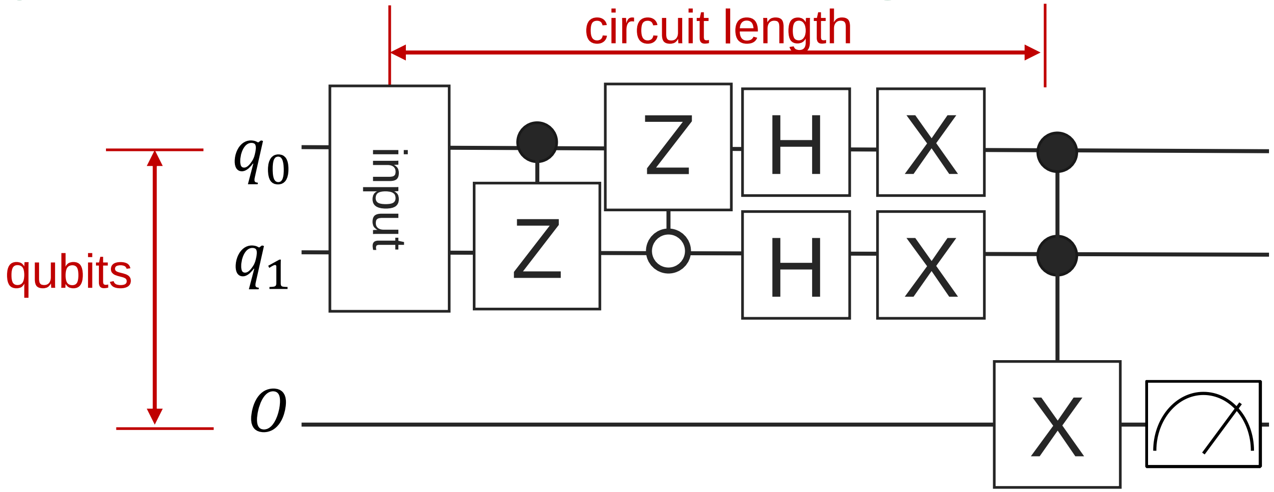
What's the complexity? Quantum Advantage?





- **Classical computer with 1 MAC**

  Time: $O(N)$

  Space (Comp. Res.): $O(1)$

  $\boldsymbol{Time \times Space: O(N)}$

- **Classical computer with N MAC**

  Time: $O(1)$

  Space (Comp. Res.): $O(N)$

  $\boldsymbol{Time \times Space: O(N)}$

- **Time-Space Complexity in Quantum computer**

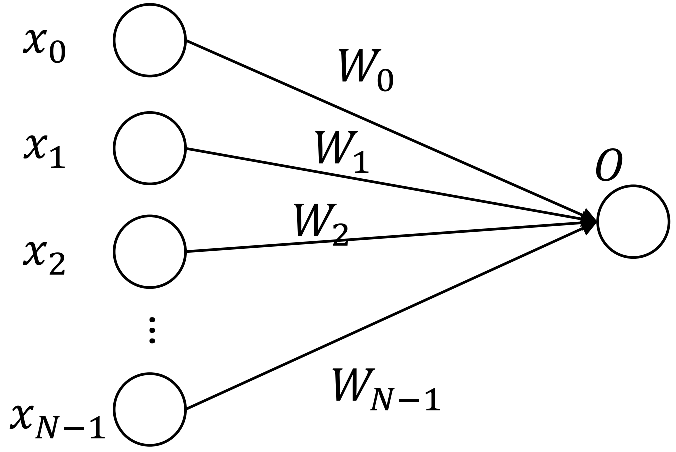
  Time: Circuit Length

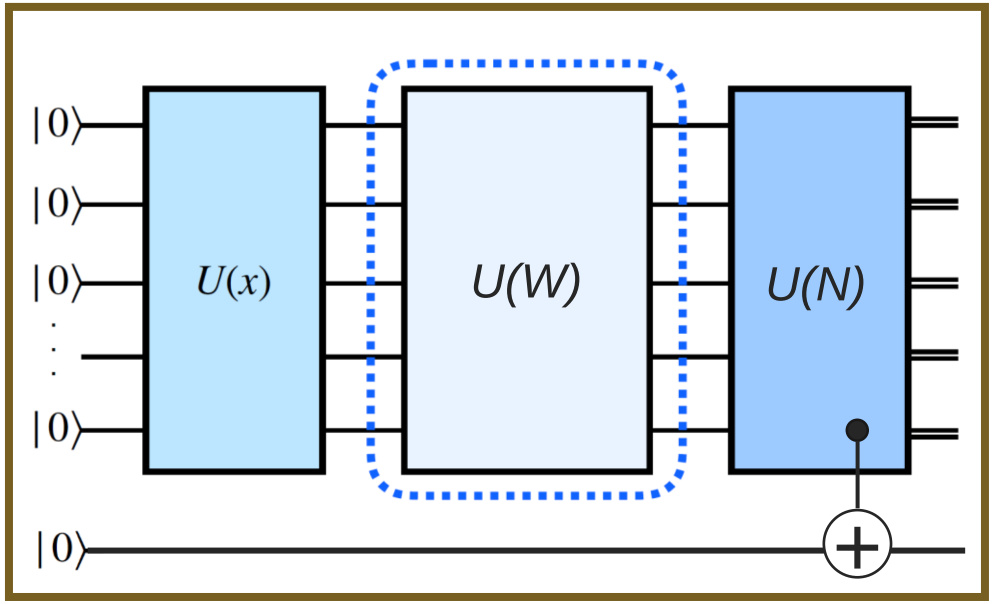
  Space (Comp. Res.): Qubits

  $\boldsymbol{Time \times Space\ (T-S): Qubits \times Circuit\ Length}$

- **Given that $T-S$ complexity on classical computer is $\boldsymbol{O(N)}$, Quantum Advantage is achieved if $T-S$ complexity on Quantum can be $\boldsymbol{O(ploylogN)}$ or lower. ------- Exponential Speedup!**

What's the Goals?



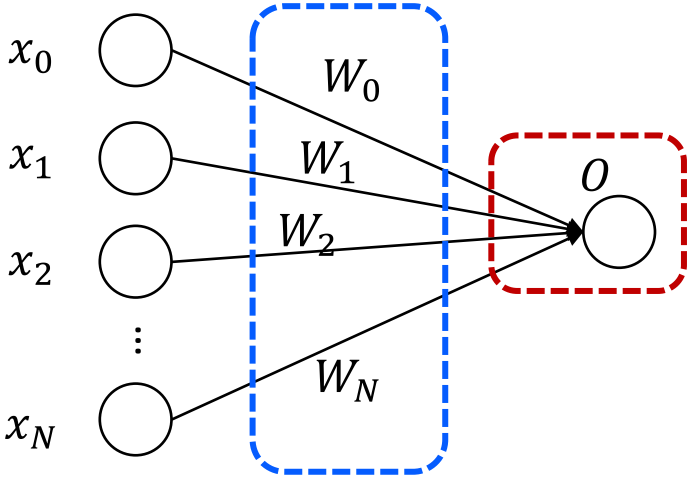


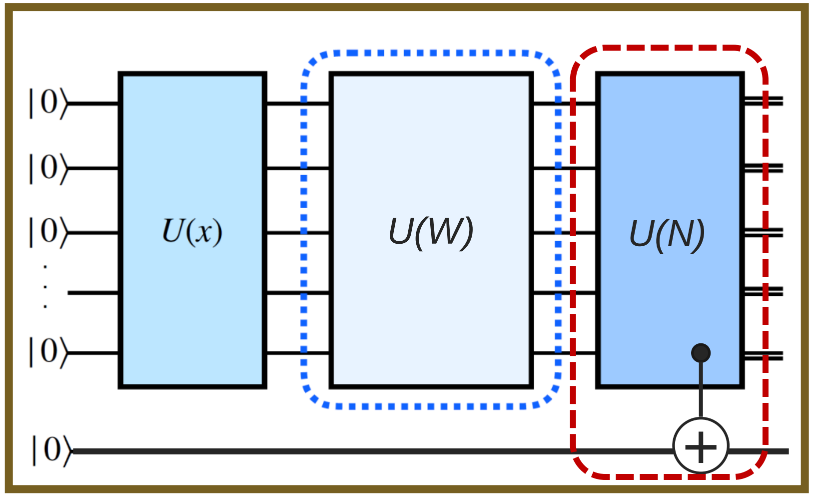
Input features	Number of Qubits	Number of Gates
U(x)	$O(log\text{N})$	$\boldsymbol{O}(?)$
U(W)	$O(log\text{N})$	$\boldsymbol{O}(?)$
U(N)	$O(1)$	$O(logN)$

n: input data number
Potential Quantum Advantage

What's the Goals?

Goal 1: Correctly Implement!





Goal 2: Efficiently Implement!

$$O = \delta\left(\sum_{i \in [0,N)} x_i \times W_i\right)$$

where δ is a quadratic function

Classical Computing:

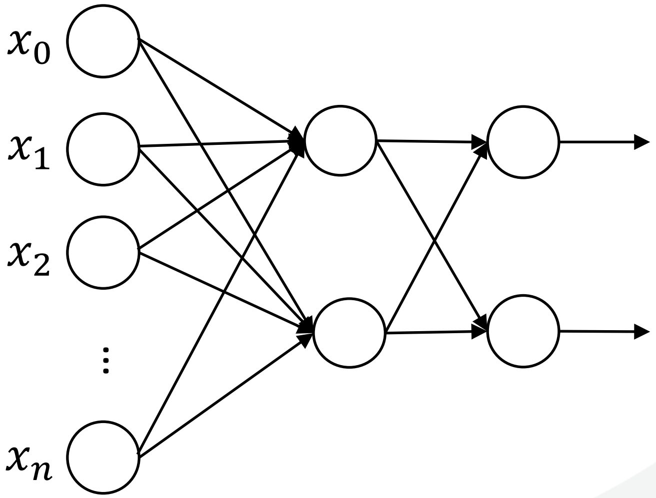
Complexity of $O(N)$

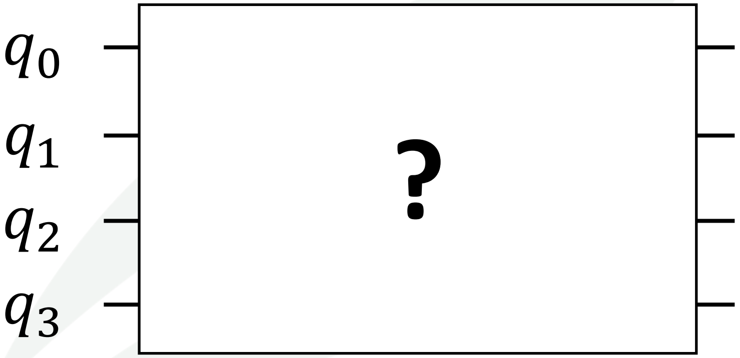
Quantum Computing:

Can we reduce complexity to

$O(ploylogN)$, say $O(log^2 n)$?

Goal 3: Scale-Up!





Agenda – Session 3: QuantumFlow

- **General Framework for Quantum-Based Neural Network Accelerator**
  - Data Preparation and Encoding
  - *Colab Hands-On (1): From Classical Data to Quantum Data*
  - Quantum Circuit Design
  - *Colab Hands-On (2): A Quantum Neuron*
- **Co-Design toward Quantum Advantage**
  - Challenges?
  - Feedforward Neural Network
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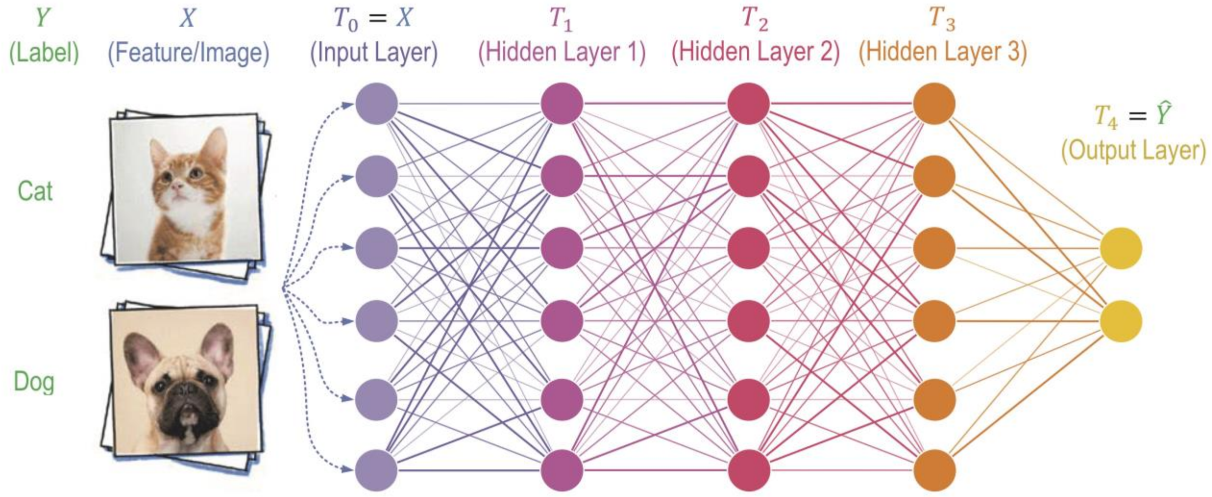
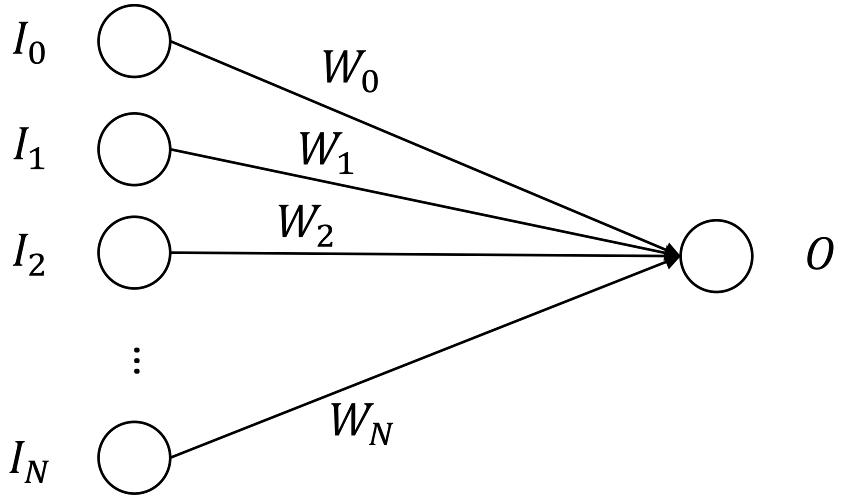
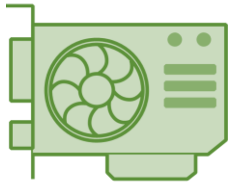
  - *Colab Hands-On (4): QuantumFlow*
  - Results

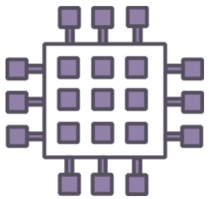
Neural Network Accelerator Design on Classical Hardware



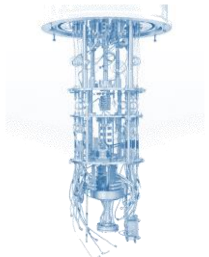




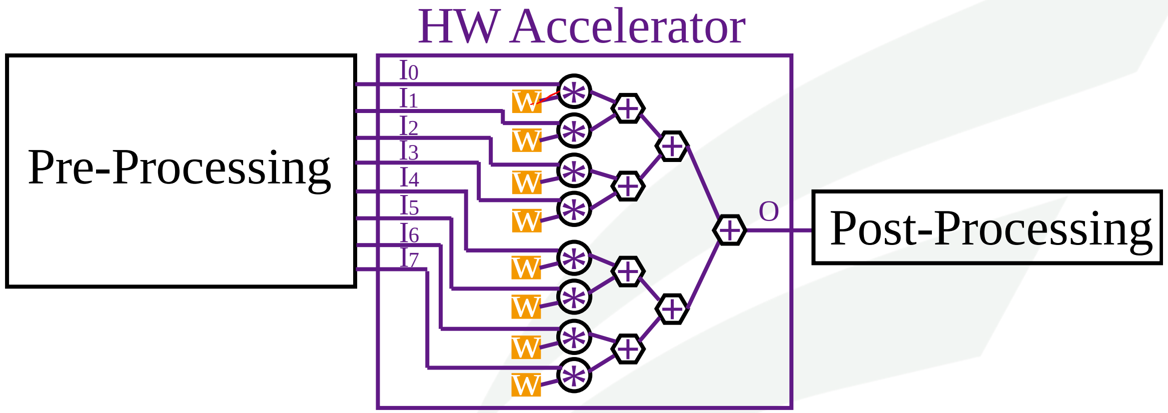
CPU/GPU Accelerator



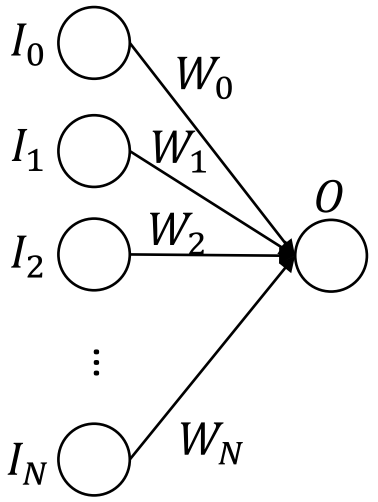
FPGA/ASIC Accelerator

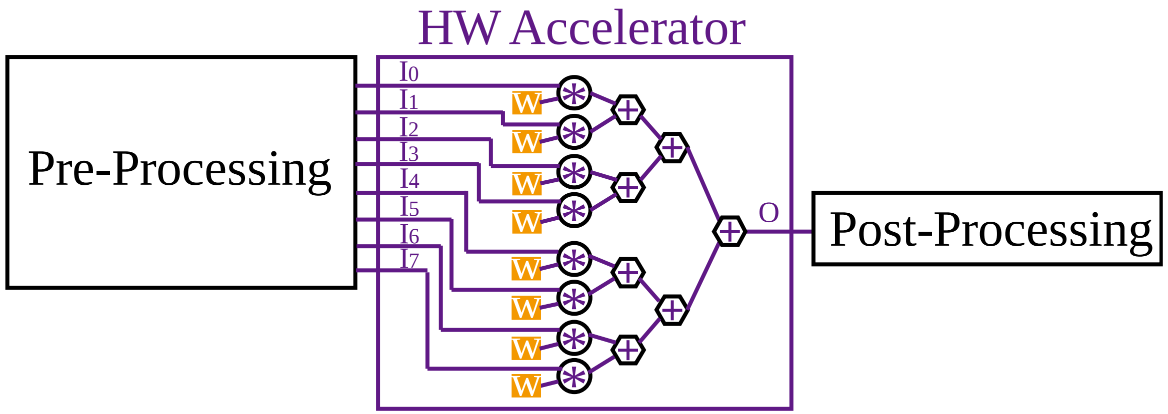


Quantum Accelerator

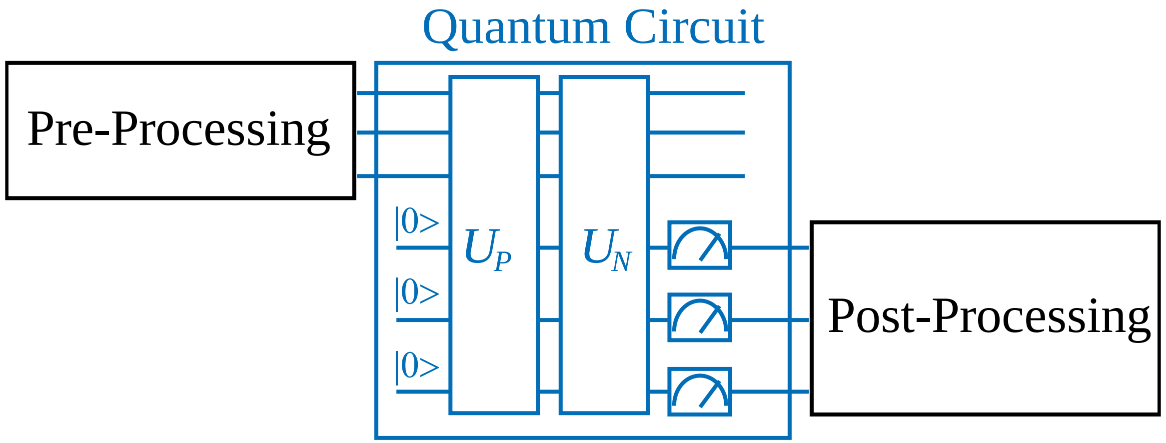


Neural Network Accelerator Design from Classical to Quantum Computing





1. Data Pre-Processing (*PreP*)
2. HW Acceleration
3. Data Post-Processing (*PostP*)

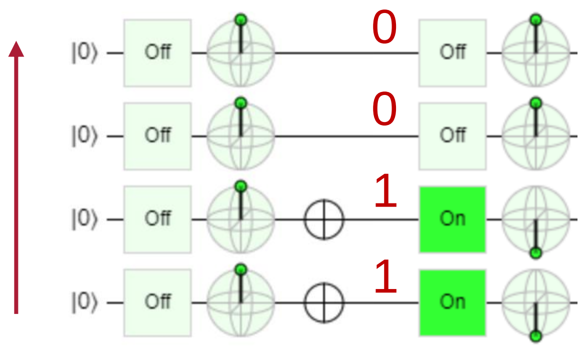


1. Data Pre-Processing (*PreP*)
2. HW/Quantum Acceleration
   - (2.1) U_p Quantum-State-Preparation
   - (2.2) U_N Quantum Neural Computation
   - (2.3) M Measurement
3. Data Post-Processing (PostP)

$$PreP + U_P + U_N + M + PostP$$

What Data Can Be Encoded to Quantum Computers, and how?

- **Can we encode an arbitrary number into quantum computer? Is it efficient?**
  - **Yes / No**



Yes, we can!

Number: 12

Number: 1.5

Number: 0.75

No, because it uses too many qubits!

This encoding is similar to classical bits, where each qubit is regarded as a binary number!

1-to-N mapping! (Boolean Function)

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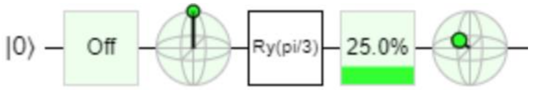

Yes, we can!

0.25 in one qubit

No, (1) data needs in the range of [0,1]!
(2) same complexity *O(1)* as classical

1-to-1 mapping! (Angle Encoding)

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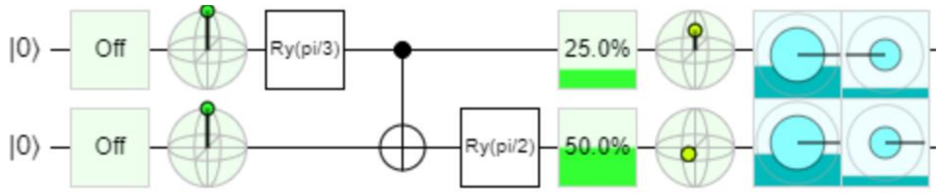

Yes, 4 # in 2 qubits

$$\begin{bmatrix}+0.61 & -0.35\\ +0.61 & +0.35\end{bmatrix}$$

No, (1) sum of the square of data need to be 1
(2) may have high cost to encode data

N-to-logN mapping! (Amplitude Encoding)

Encoding: 1-to-N *v.s.* 1-to-1 *v.s.* N-to-*log*N

Data Encoding	# of Qubit (C v.s. Q)	Data Limitation	Encoding Complexity
1-to-N	O(N) v.s. O(N^2)	**Almost No!**	**Low**
1-to-1	O(N) v.s. O(N)	[0,+1]	**Low**
N-to-logN	**O(N) v.s. O(*log*N)**	[-1,+1] and $\sum x^2 = 1$	High

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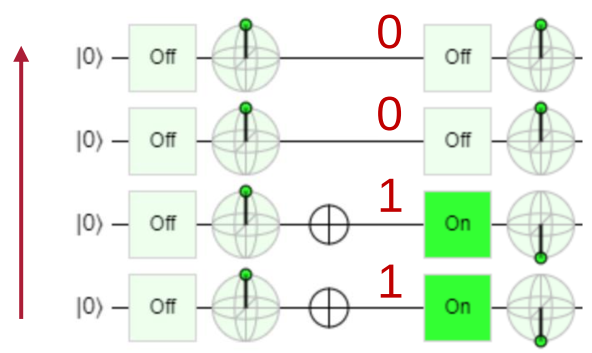

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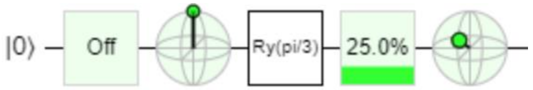

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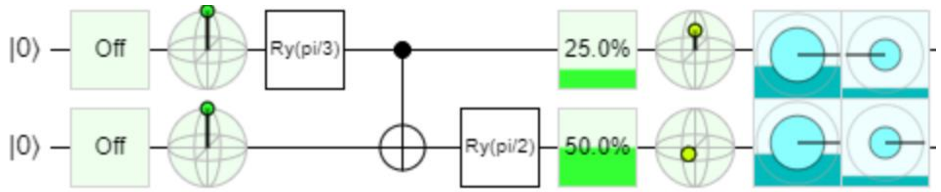

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n-to-logn mapping! (Amplitude Encoding)

Hands-On: QuantumFlow

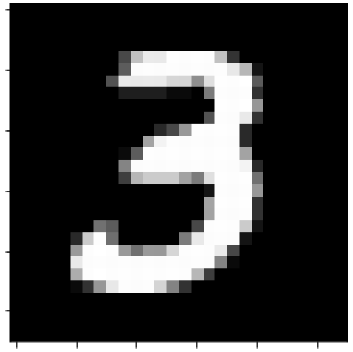
A Co-Design Framework of Neural Networks and Quantum Circuits Towards Quantum Advantage

Published at Nature Communications 2021

Presenter: Zhepeng Wang

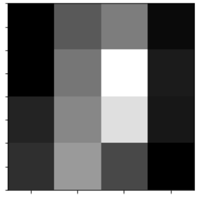
PreP + U_P + U_N + *M* + *PostP* : Data Pre-Processing

- **Given:** (1) 28×28 image, (2) the number of qubits to encode data (say Q=4 qubits in the example)
- **Do:** (1) downsampling from 28×28 to $2^Q = 16 = 4 \times 4$; (2) converting data to be the state vector in a unitary matrix
- **Output:** A unitary matrix, $M_{16\times16}$



Step 1: Downsampling

From 28×28 to 4×4



$$\begin{bmatrix} 0.0039 & 0.2118 & 0.2941 & 0.0275 \\ 0.0039 & 0.2784 & 0.5961 & 0.0667 \\ 0.0863 & 0.3176 & 0.5216 & 0.0588 \\ 0.1137 & 0.3608 & 0.1725 & 0.0039 \end{bmatrix}$$

$$\begin{bmatrix} 0.0039 & 0.2118 & 0.2941 & 0.0275 \\ 0.0039 & 0.2784 & 0.5961 & 0.0667 \\ 0.0863 & 0.3176 & 0.5216 & 0.0588 \\ 0.1137 & 0.3608 & 0.1725 & 0.0039 \end{bmatrix}$$

Step 2: Formulate Unitary Matrix

Applying SVD method

(See Listing 1 in ASP-DAC SS Paper)

Unitary matrix: $M_{16\times16}$

[SS] W. Jiang, et al. [When Machine Learning Meets Quantum Computers: A Case Study](), ASP-DAC'21

PreP + U_P + U_N + *M* + *PostP* --- Data Encoding / Quantum State Preparation

- **Given:** The unitary matrix provided by *PreP*, $M_{16\times16}$
- **Do:** Quantum-State-Preparation, encoding data to qubits
- **Verification:** Check the amplitude of states are consistent with the data in the unitary matrix, $M_{16\times16}$

Let's use a 2-qubit system as an example to encode a matrix $M_{4\times4}$

$$\begin{bmatrix} 0.3 & 0.5 \\ 0.7 & 0.9 \end{bmatrix} \xrightarrow{PreP} \underbrace{\begin{bmatrix} \mathbf{0.2343} & X & X & X \\ \mathbf{0.3904} & X & X & X \\ \mathbf{0.5466} & X & X & X \\ \mathbf{0.7028} & X & X & X \end{bmatrix}}_{\text{data\_matrix}} \xrightarrow{U_P}$$

State Transition:

data_matrix $\quad |00\rangle$

IBM Qiskit Implementation:

inp = QuantumRegister(4, "in_qubit")
circ = QuantumCircuit(inp)
iniG = **UnitaryGate(data_matrix, label="input")**
circ.append(iniG, inp[0:4])

Hands-On Tutorial (1)

PreP + U_P

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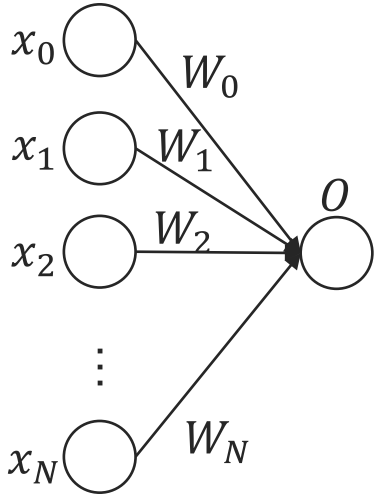
PreP + U_P + U_N + *M* + *PostP* --- Neural Computation



- **Given:** (1) A circuit with encoded input data $\boldsymbol{x}$; (2) the trained binary weights $\boldsymbol{w}$ for one neural computation, which will be associated to each data.
- **Do:** Place quantum gates on the qubits, such that it performs $\frac{(\boldsymbol{x} * \boldsymbol{w})^2}{\|\boldsymbol{x}\|}$.
- **Verification:** Whether the output data of quantum circuit and the output computed using torch on classical computer are the same.

Target: $O = \left[\frac{\sum_i (x_i \times w_i)}{\sqrt{\|x\|}}\right]^2$

- **Assumption 1:** Parameters/weights (W_0 --- W_N) are binary weight, either +1 or -1
- **Assumption 2:** The weight $W_0 = +1$, otherwise we can use $-\boldsymbol{w}$ (quadratic func.)

Step 1: $m_i = x_i \times w_i$

Step 2: $n = \left[\frac{\sum_i (m_i)}{\sqrt{\|x\|}}\right]$

Step 3: $O = n^2$

Quantum Neuron Design: Step 1

Step 1: $m_i = x_i \times w_i$

EX: 4 input data on 2 qubits

$$\boldsymbol{x} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad \boldsymbol{w} = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \end{bmatrix} \qquad \begin{matrix} w_0 = 1 \\ w_1 = 1 \\ w_2 = 1 \\ w_3 = -1 \end{matrix}$$

$$w_3 = -1 \Rightarrow \boxed{m_3 = -1 \times a_3 = -a_3}$$

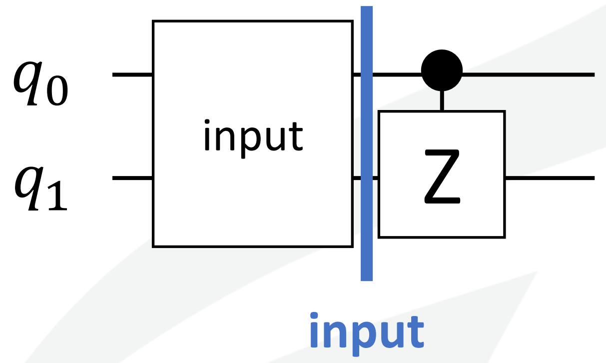
Output = **U** × **Input**

a_0	$\vert 00\rangle$
a_1	$\vert 01\rangle$
a_2	$\vert 10\rangle$
$m_3 = -a_3$	$\vert 11\rangle$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \times$$

a_0	$\vert 00\rangle$
a_1	$\vert 01\rangle$
a_2	$\vert 10\rangle$
a_3	$\vert 11\rangle$

Quantum Circuit



PreP + U_P + U_N + *M* + *PostP* --- Neural Computation: Step 1

Step 1: $m_i = x_i \times w_i$

EX: 4 input data on 2 qubits

$$\boldsymbol{w} = \begin{bmatrix} +1 \\ +1 \\ +1 \\ -1 \end{bmatrix} \text{or} \begin{bmatrix} +1 \\ +1 \\ -1 \\ -1 \end{bmatrix} \text{or} \begin{bmatrix} +1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \text{or} \begin{bmatrix} +1 \\ +1 \\ -1 \\ +1 \end{bmatrix} \text{or} \begin{bmatrix} +1 \\ -1 \\ -1 \\ +1 \end{bmatrix} \text{or} \begin{bmatrix} +1 \\ -1 \\ +1 \\ +1 \end{bmatrix}$$

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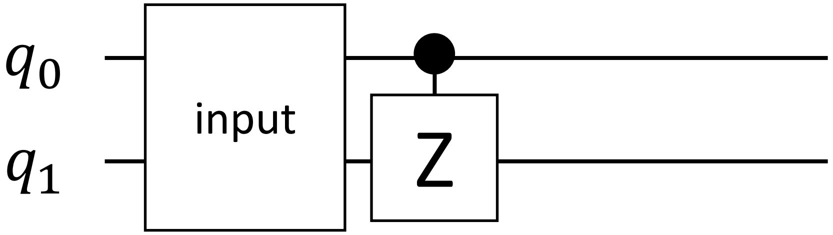
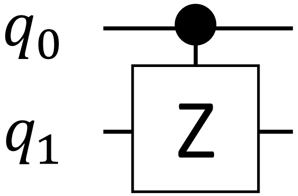
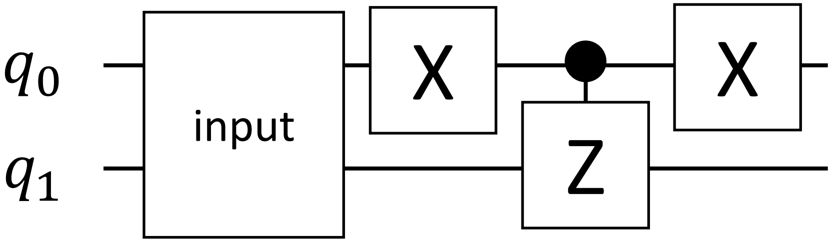
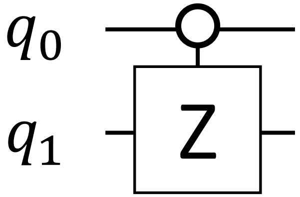




Flip the sign of $|11\rangle$

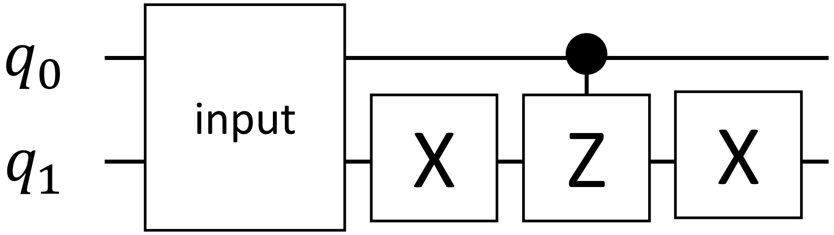
$$\boldsymbol{w} = \begin{bmatrix} +1 \\ -1 \\ +1 \\ +1 \end{bmatrix}$$

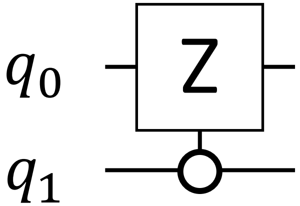




Flip the sign of $|01\rangle$

$$\boldsymbol{w} = \begin{bmatrix} +1 \\ +1 \\ -1 \\ +1 \end{bmatrix}$$





Flip the sign of $|10\rangle$

Quantum Neuron Design: Step 2

Step 2: $n = \left[\frac{\sum_i (m_i)}{\sqrt{\|x\|}} \right]$

EX: 4 input data on 2 qubits

Output = **U** × **Input**

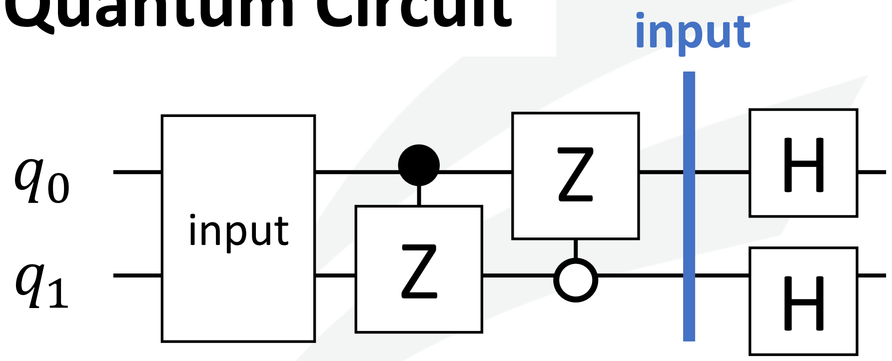
$\sum_i (m_i) / \sqrt{\|x\|}$	$\|00\rangle$
Do not care 1	$\|01\rangle$
Do not care 2	$\|10\rangle$
Do not care 3	$\|11\rangle$

$$= \frac{1}{\sqrt{\|x\|}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \times$$

m_0	$\|00\rangle$
m_1	$\|01\rangle$
m_2	$\|10\rangle$
m_3	$\|11\rangle$

note: $\|x\| = 2^N$

Quantum Circuit



Quantum Neuron Design: Step 3

Step 3: $O = n^2$

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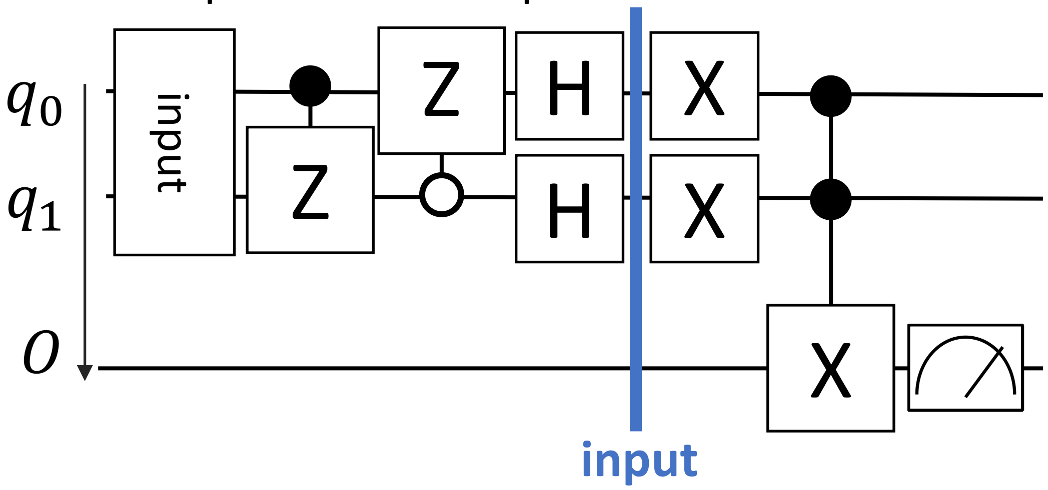

Input

$\sum_i (m_i) / \sqrt{\|x\|}$	$\|000\rangle$
0	$\|001\rangle$
Do not care 1	$\|010\rangle$
0	$\|011\rangle$
Do not care 2	$\|100\rangle$
0	$\|101\rangle$
Do not care 3	$\|110\rangle$
0	$\|111\rangle$

$X^{\otimes 2}$

Do not care 3	$\|000\rangle$
0	$\|001\rangle$
Do not care 2	$\|010\rangle$
0	$\|011\rangle$
Do not care 1	$\|100\rangle$
0	$\|101\rangle$
$\sum_i (m_i) / \sqrt{\|x\|}$	$\|110\rangle$
0	$\|111\rangle$

CCX

Do not care	$\|000\rangle$
0	$\|001\rangle$
Do not care	$\|010\rangle$
0	$\|011\rangle$
Do not care	$\|100\rangle$
0	$\|101\rangle$
0	$\|110\rangle$
$\sum_i (m_i) / \sqrt{\|x\|}$	$\|111\rangle$

Output

$$P\{O = |1\rangle\} = P\{|001\rangle\} + P\{|011\rangle\} + P\{|101\rangle\} + P\{|111\rangle\} = \left[\frac{\sum_i (m_i)}{\sqrt{\|x\|}}\right]^2$$

Hands-On Tutorial (2)

PreP $+ U_P + U_N$

Agenda – Session 3: QuantumFlow

- **General Framework for Quantum-Based Neural Network Accelerator**
  - Data Preparation and Encoding
  - *Colab Hands-On (1): From Classical Data to Quantum Data*
  - Quantum Circuit Design
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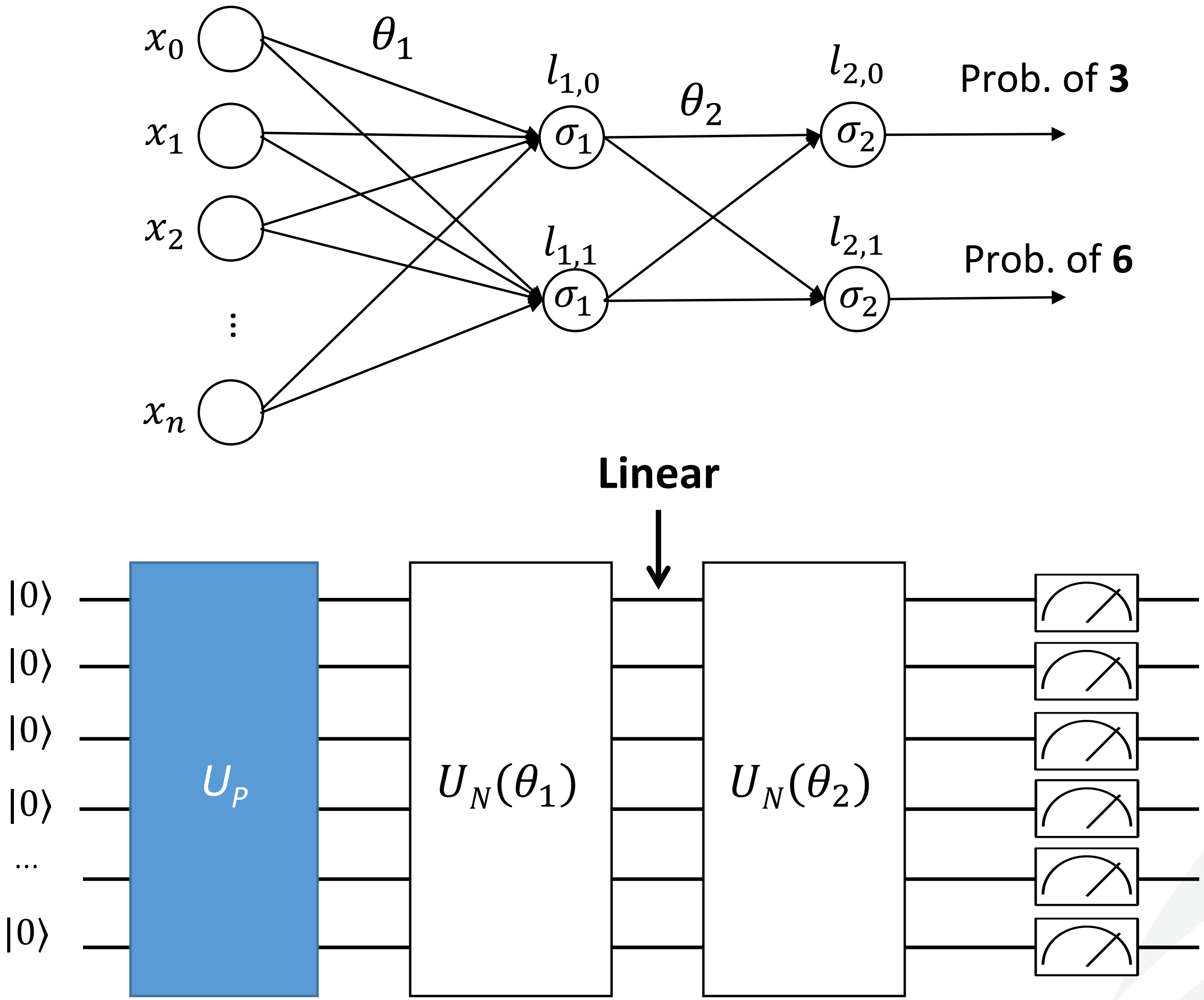
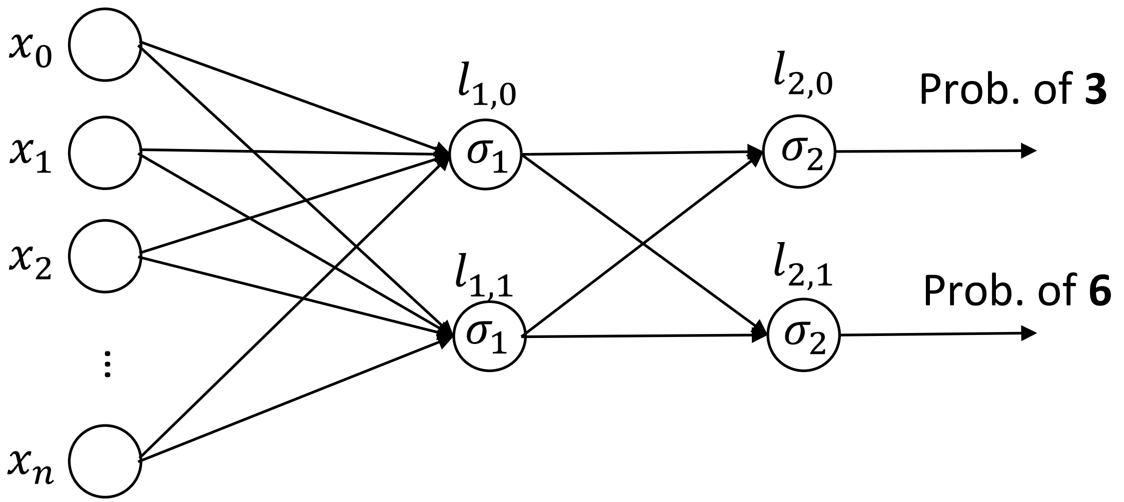
Challenge 1: Non-linearity is Needed, But Difficult in Quantum Circuit



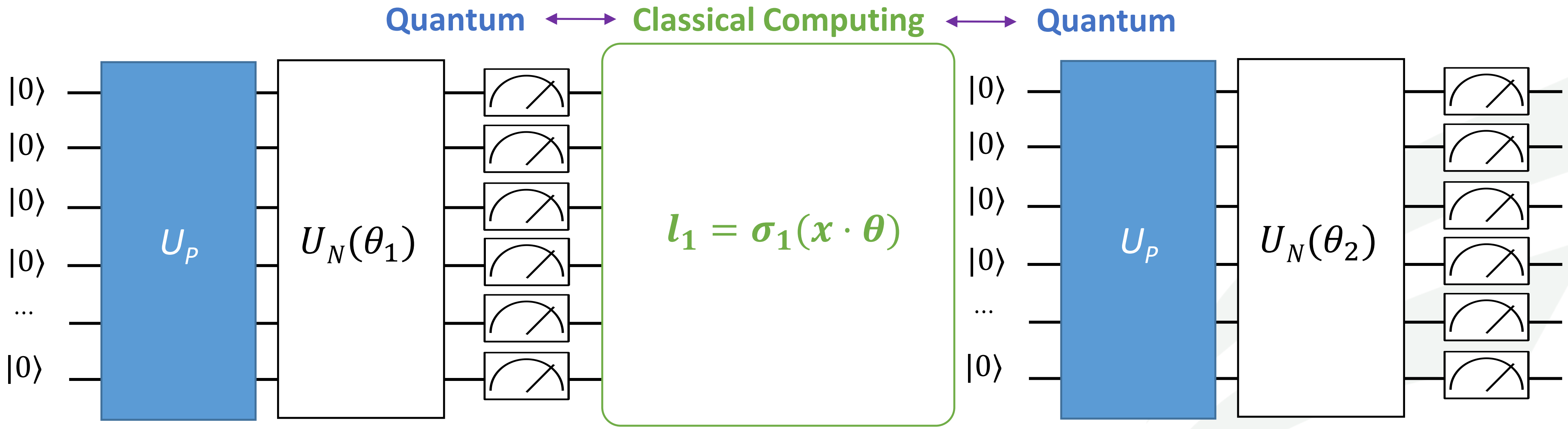
Challenge 2: Quantum-Classical Interface is Expensive



Ref [1]

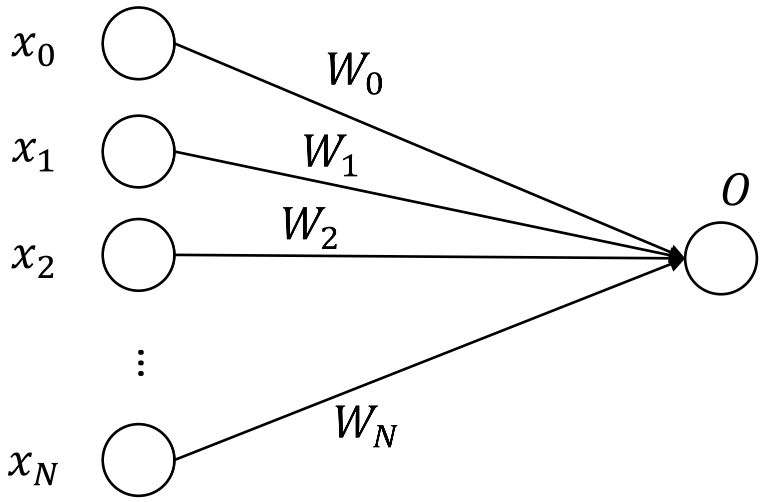
Table 2 Complexity of each step in hybrid quantum-classical computing for deep neural network with U-LYR.

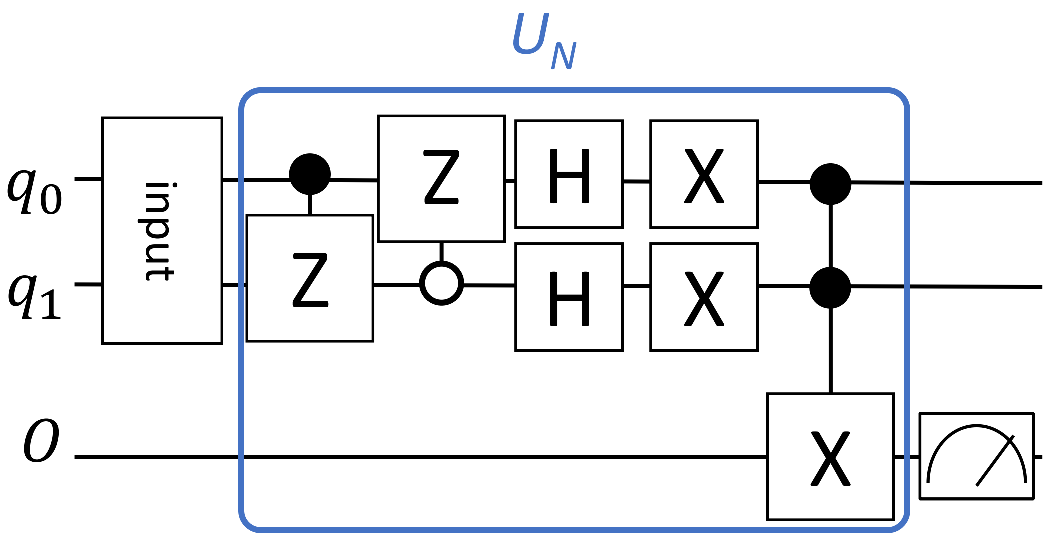
Complexity	State-preparation	Computation	Measurement
Depth (T)	$O(d \cdot \sqrt{n})$	$O(d \cdot \log^2 n)$	$O(d)$
Qubits (S)	$O(n)$	$O(n \cdot \log n)$	$O(n \cdot \log n)$
Cost (TS)	$O(d \cdot n^{\frac{3}{2}})$	$O(d \cdot n \cdot \log^3 n)$	$O(d \cdot n \cdot \log n)$
Total (TS)	$O(d \cdot n^{\frac{3}{2}})$ dominate		



[1] W. Jiang, et al. A Co-Design Framework of Neural Networks and Quantum Circuits Towards Quantum Advantage, Nature Communications

Challenge 3: High Complexity in the Previous Design





Cost Complexity

Classical Computing		
	No Parallelism	Full Parallelism
Time (T)	O(N)	O(1)
Space (S)	O(1)	O(N)
Cost (TS)	O(N)	O(N)

Quantum Computing		
	Previous Design	Optimization
Circuit Depth (T)	O(N)	???
Qubits (S)	O($\log N$)	O(N)
Cost (TS)	O($N \cdot \log N$)	**target** O(ploylog N)

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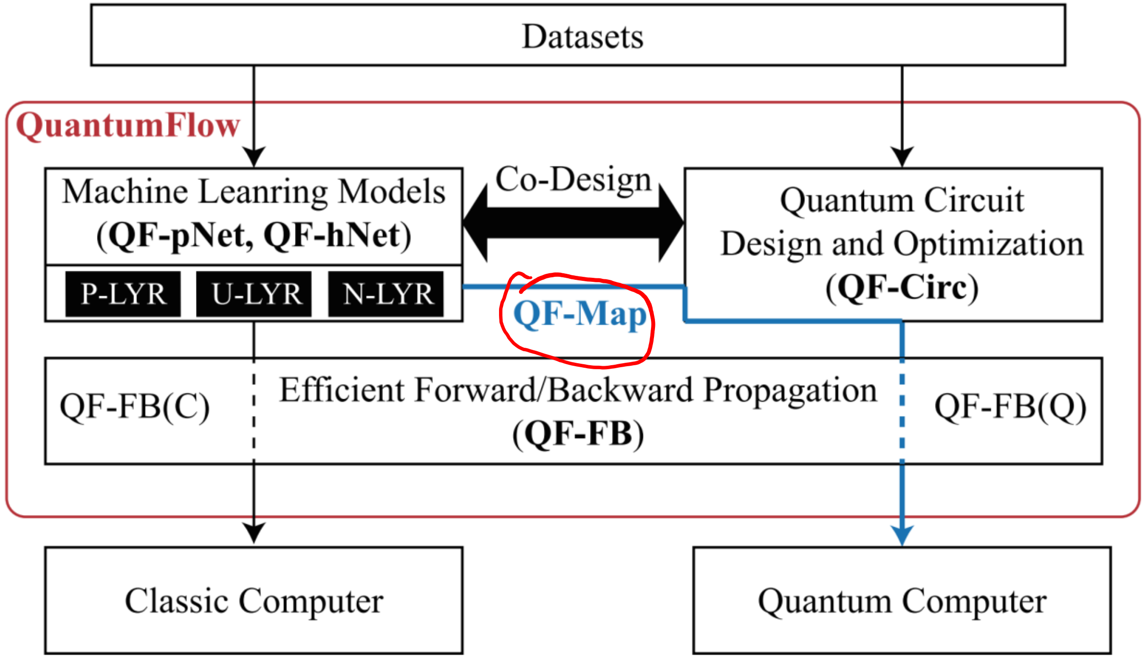
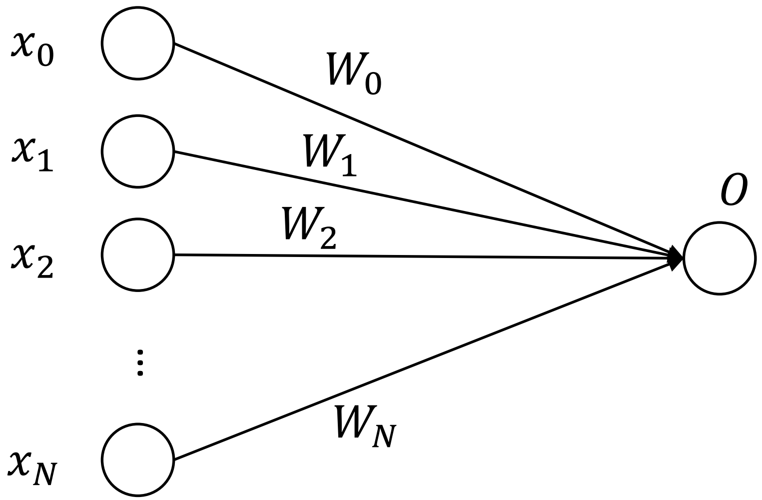
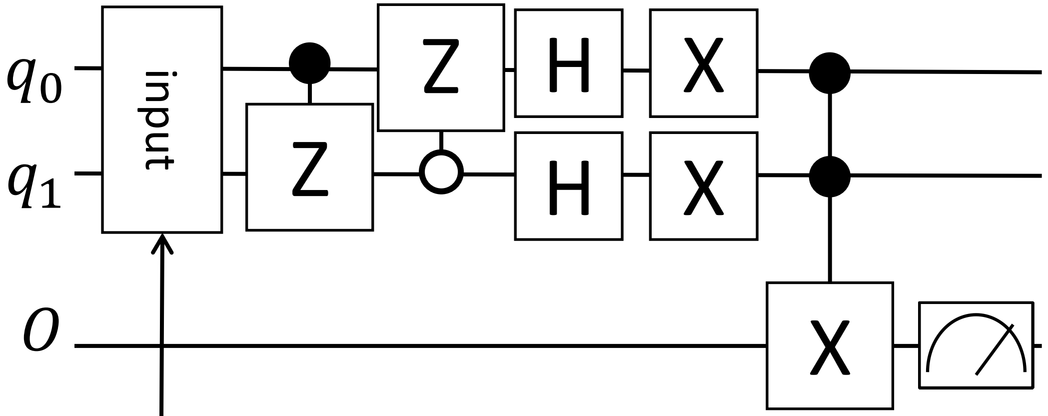
Co-Design Framework



Design Direction 1: NN → Quantum Circuit



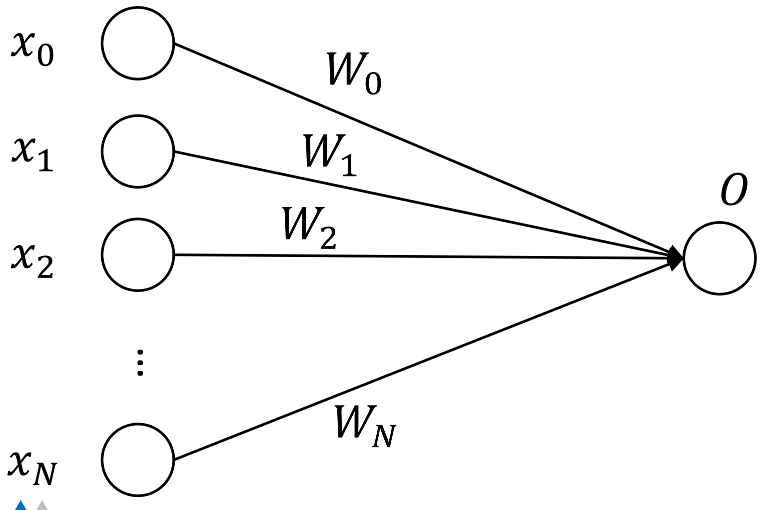


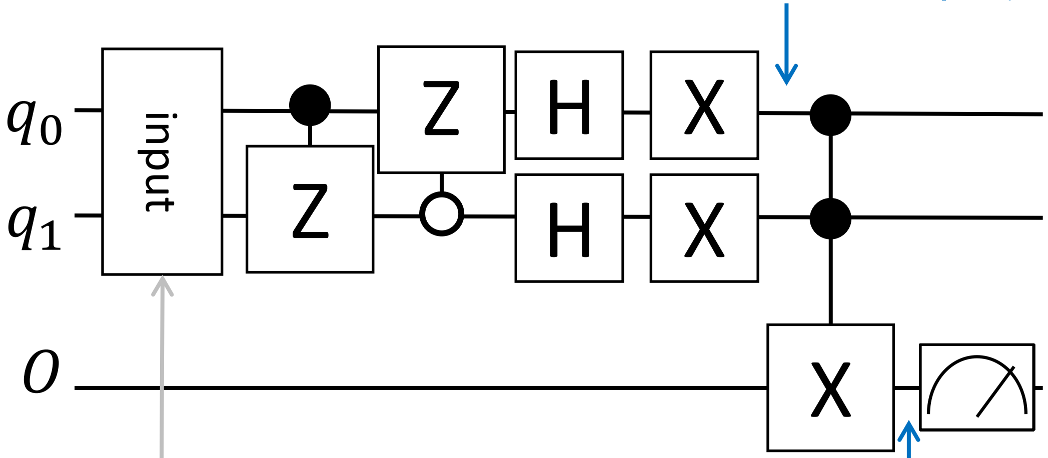
Converting to Unitary Matrix

Digital Numbers → Amplitude Encoding

N numbers are encoded to $\log N$ qubits

Design Direction 2: Quantum Circuit → NN





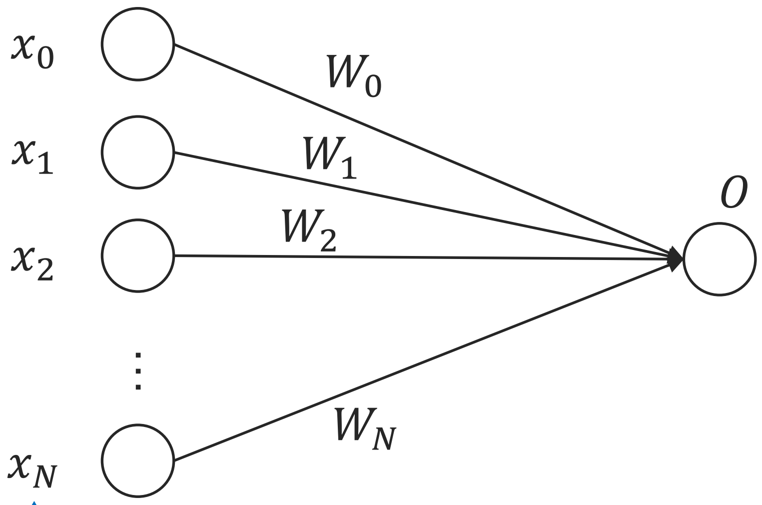
Digital Numbers → Amplitude Encoding

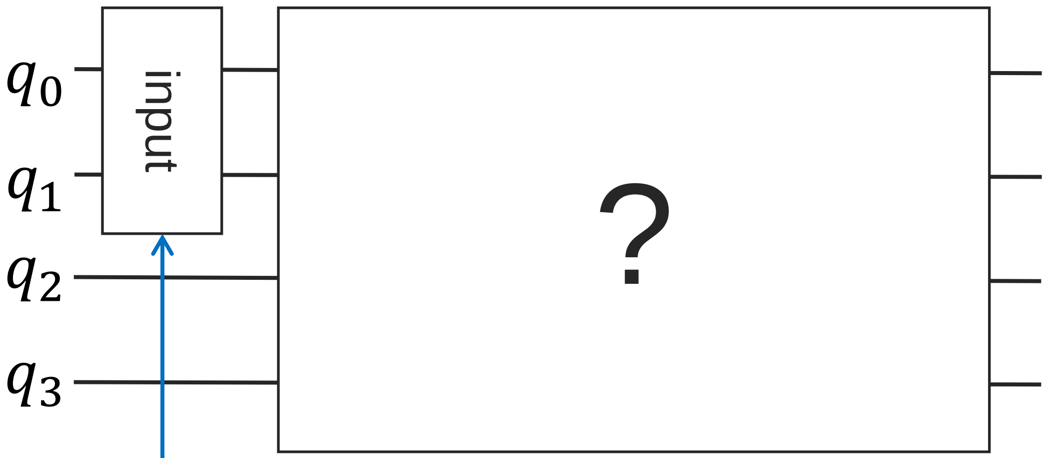
Converting to Unitary Matrix

N numbers are encoded to $\log N$ qubits

Random Variable ← Probability of $O = |1\rangle$ is $\boldsymbol{n}^2$

Design Direction 3: NN → Quantum Circuit





Random Variable

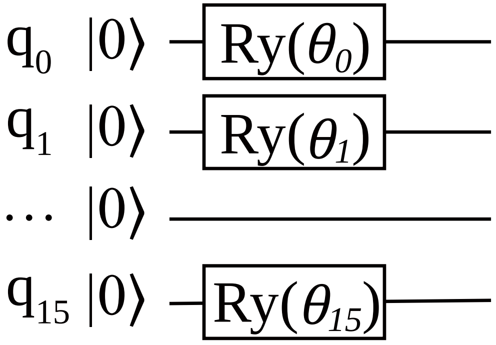
Rotation Gate, say Ry

Angle Encoding

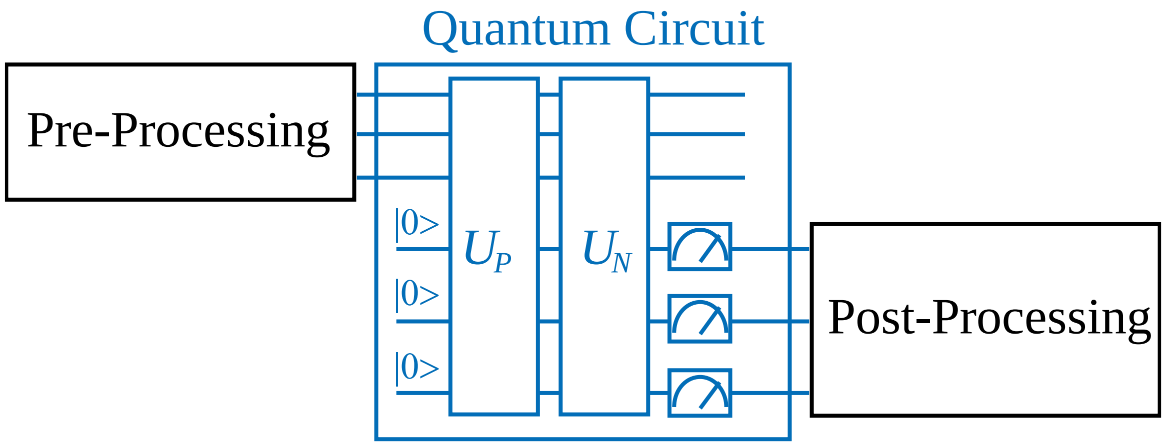
N **numbers are encoded to** ***N*** **qubits**

0.0	0.9	0.0	0.0
0.0	0.1	0.0	0.0
0.0	1.0	0.5	0.5
0.0	0.0	0.0	0.0

r.v.	-1	+1
x_0	0.0	1.0
x_1	0.9	0.1
…	…	
x_{15}	0.0	1.0



Apply Our Framework to Address Challenges 1 & 2 (non-linear & Q-C comm.)



(1) Data Pre-Processing (*PreP*)

(2) HW/Quantum Acceleration

(2.1) rvU_p Quantum-State-Preparation

(2.2) rvU_N Quantum Neural Computation

(2.3) M Measurement

(3) Data Post-Processing (*PostP*)

rvU_P --- Data Encoding / Quantum State Preparation

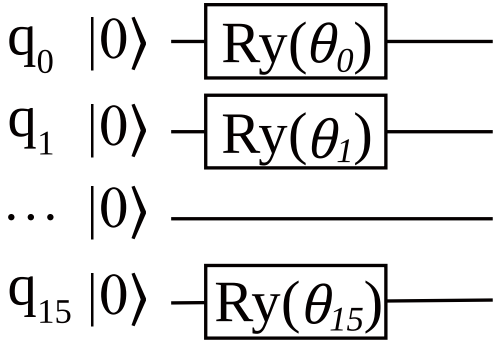
- **Given:** A vector of input data, ranging from [0,1] (do scaling in *PreP* if range out of [0,1])
- **Do:** Applying rotation gate to encode each data to one qubits
- **Output:** A quantum circuit, where the probability of each qubit to be $|1\rangle$ is the same as the corresponding input data

0.0	0.9	0.0	0.0
0.0	0.1	0.0	0.0
0.0	1.0	0.5	0.5
0.0	0.0	0.0	0.0

→

r.v.	-1	+1
x_0	0.0	1.0
x_1	0.9	0.1
…	…	
x_{15}	0.0	1.0

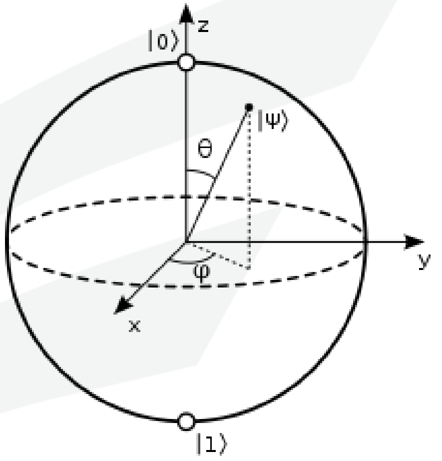
→



Determination of θ_i:

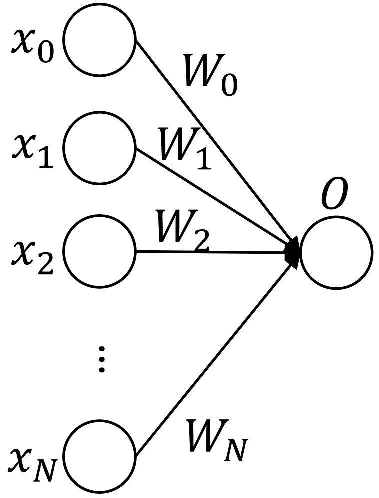
$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + (\cos\phi + i \cdot \sin\phi) \cdot \sin\frac{\theta}{2}|1\rangle$$

$$\theta_i = 2 \times \arcsin(\sqrt{x_i})$$



[1] W. Jiang, et al. A Co-Design Framework of Neural Networks and Quantum Circuits Towards Quantum Advantage, Nature Communications

rvU_N --- Neural Computation



- **Given:** (1) A circuit with encoded input data $\boldsymbol{x}$; (2) the trained binary weights $\boldsymbol{w}$ for one neural computation, which will be associated to each data.
- **Do:** Place quantum gates on qubits, such that it performs $\frac{(\boldsymbol{x}*\boldsymbol{w})^2}{||\boldsymbol{x}||^2}$, where $\boldsymbol{x}$ are random variables

Target: $O = \left[\frac{\sum_i (x_i \times w_i)}{||\boldsymbol{x}||}\right]^2$

- **Assumption 1:** Parameters/weights (W_0 --- W_N) are binary weight, either +1 or -1
- **Assumption 2:** The weight $W_0 = +1$, otherwise we can use $-\boldsymbol{w}$ (quadratic func.)

Step 1: $m_i = x_i \times w_i$

Step 2: $n = \left[\frac{\sum_i (m_i)}{||\boldsymbol{x}||}\right]$

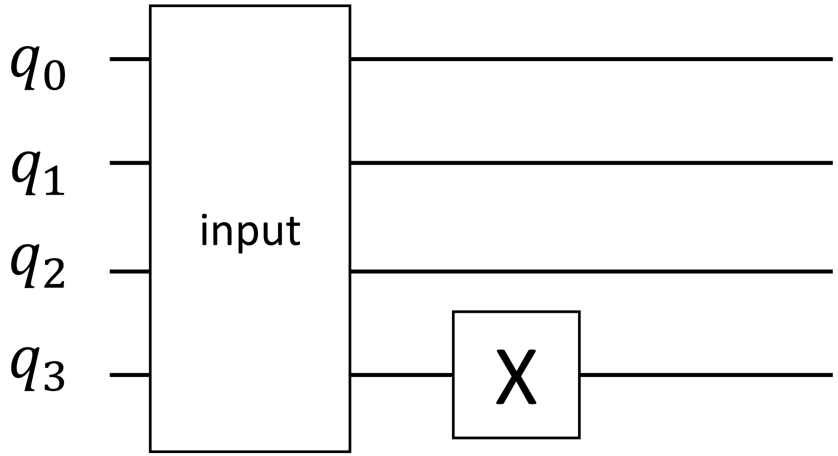
Step 3: $O = n^2$

rvU_N --- Neural Computation: Step 1

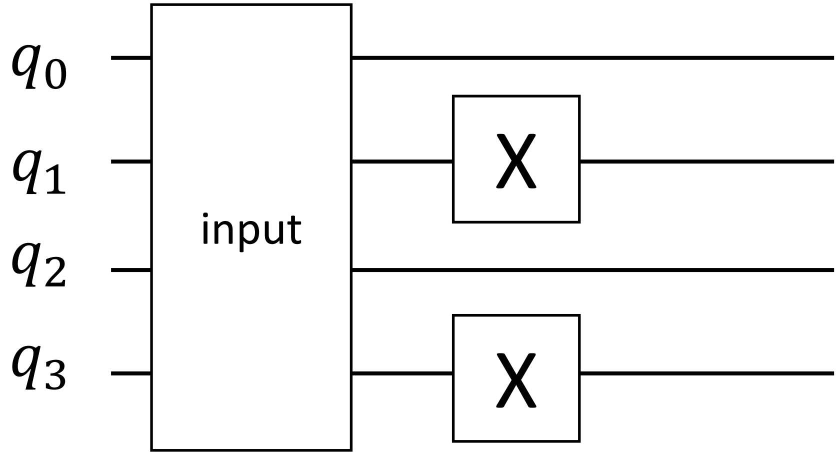
Step 1: $m_i = x_i \times w_i$

EX: 4 input data on 4 qubits

$$\boldsymbol{w} = \begin{bmatrix} +1 \\ +1 \\ +1 \\ -1 \end{bmatrix}$$



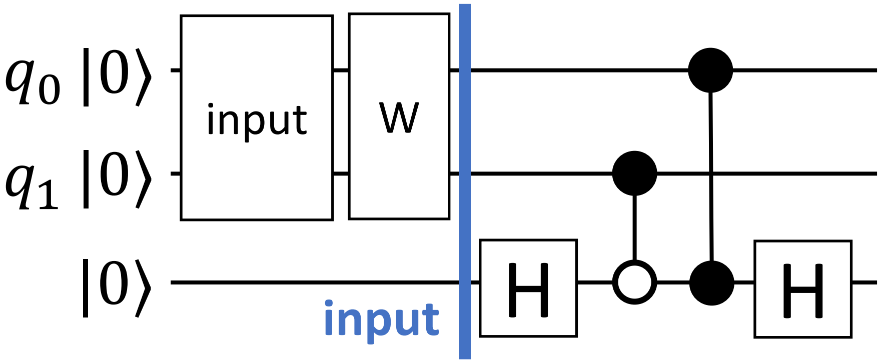
$$\boldsymbol{w} = \begin{bmatrix} +1 \\ -1 \\ +1 \\ -1 \end{bmatrix}$$



rvU_N --- Neural Computation: Step 2

Step 2: $n = \left[\frac{\sum_i (m_i)}{\|\boldsymbol{x}\|}\right]$

EX: 2 input data on 2 qubits



r.v.	-1 ($\|1\rangle$)	+1 ($\|0\rangle$)
m_0	p_0	q_0
m_1	p_1	q_1

r.v.	-1	0	+1
n	p_0p_1	$p_0q_1 + p_1q_0$	q_0q_1

Input

$\sqrt{q_0q_1}$	$\|000\rangle$
0	$\|001\rangle$
$\sqrt{q_0p_1}$	$\|010\rangle$
0	$\|011\rangle$
$\sqrt{p_0q_1}$	$\|100\rangle$
0	$\|101\rangle$
$\sqrt{p_0p_1}$	$\|110\rangle$
0	$\|111\rangle$

IIH+CZs

$\sqrt{q_0q_1}/\sqrt{2}$	$\|000\rangle$
$\sqrt{q_0q_1}/\sqrt{2}$	$\|001\rangle$
$-\sqrt{q_0p_1}/\sqrt{2}$	$\|010\rangle$
$\sqrt{q_0p_1}/\sqrt{2}$	$\|011\rangle$
$\sqrt{p_0q_1}/\sqrt{2}$	$\|100\rangle$
$-\sqrt{p_0q_1}/\sqrt{2}$	$\|101\rangle$
$-\sqrt{p_0p_1}/\sqrt{2}$	$\|110\rangle$
$-\sqrt{p_0p_1}/\sqrt{2}$	$\|111\rangle$

IIH

$\sqrt{q_0q_1}$	$\|000\rangle$
---	$\|100\rangle$
0	$\|001\rangle$
---	$\|101\rangle$
0	$\|010\rangle$
---	$\|110\rangle$
$-\sqrt{p_0p_1}$	$\|011\rangle$
---	$\|111\rangle$

rvU_N --- Neural Computation: Step 3

Classical:

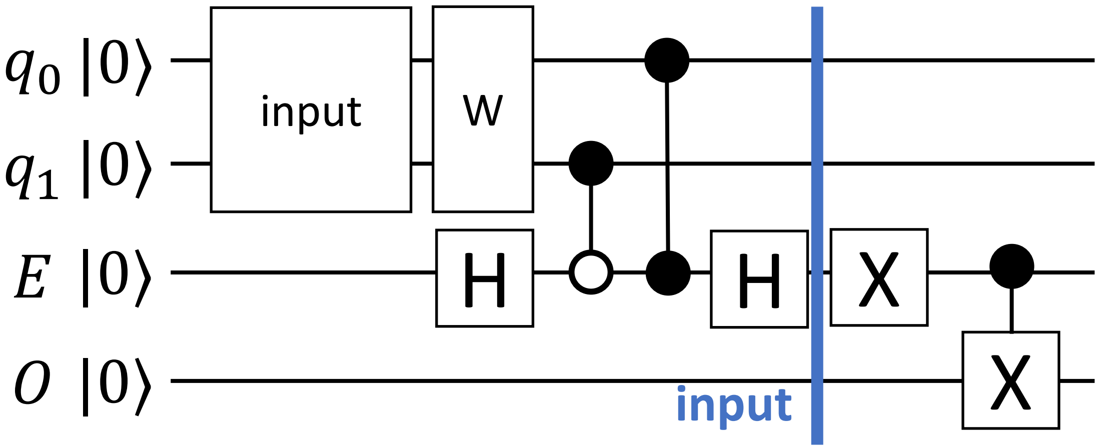
$E(O) = E(n^2)$
$= 0 \times (p_0q_1 + p_1q_0) + 1 \times (q_0q_1 + p_0p_1)$

Step 3: $O = n^2$

r.v.	-1	0	+1
n	p_0p_1	$p_0q_1 + p_1q_0$	q_0q_1

r.v.	0	+1
n^2	$p_0q_1 + p_1q_0$	$q_0q_1 + p_0p_1$

EX: 2 input data on 2 qubits



Input

$\sqrt{q_0q_1}$	$\|000\rangle$
---	$\|001\rangle$
0	$\|010\rangle$
---	$\|011\rangle$
0	$\|100\rangle$
---	$\|101\rangle$
$-\sqrt{p_0p_1}$	$\|110\rangle$
---	$\|111\rangle$

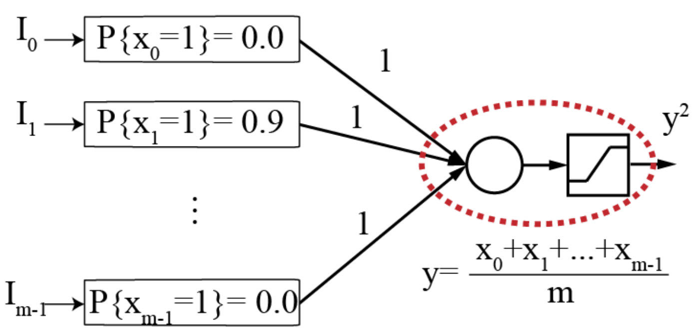
IIX

---	$\|000\rangle$
$\sqrt{q_0q_1}$	$\|001\rangle$
---	$\|010\rangle$
0	$\|011\rangle$
---	$\|100\rangle$
0	$\|101\rangle$
---	$\|110\rangle$
$-\sqrt{p_0p_1}$	$\|111\rangle$

Quantum:

$P(E = |1\rangle)$
$= \sqrt{q_1q_0}^2 + (-\sqrt{p_1p_0})^2$
$= q_0q_1 + p_0p_1$

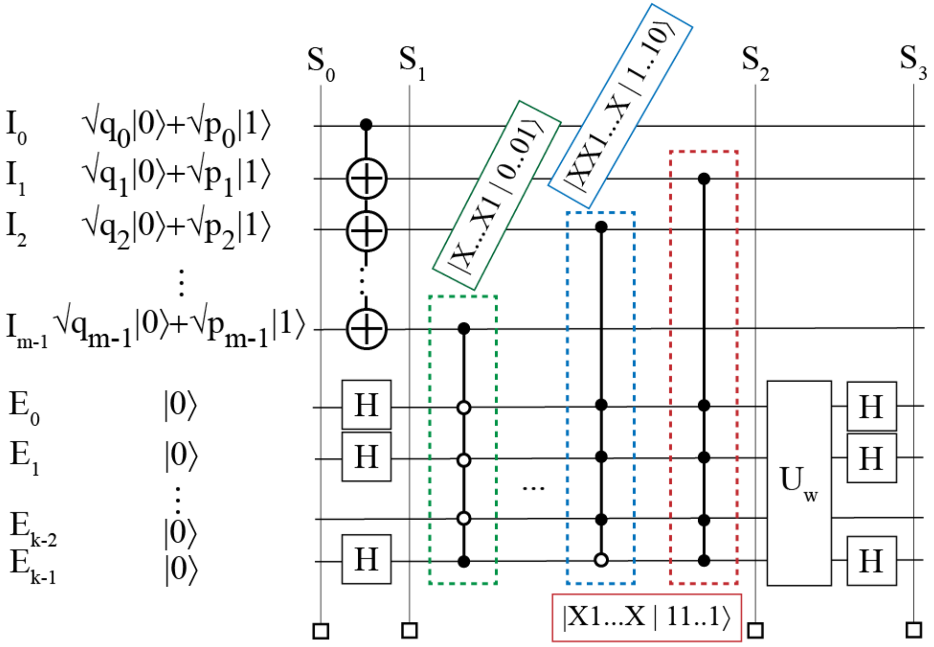
rvU_N --- Neural Computation



	$\|1\rangle$	$\|0\rangle$
	-1	+1
x_0	p_0	q_0
x_1	p_1	q_1
⋮		
x_{m-1}	p_{m-1}	q_{m-1}

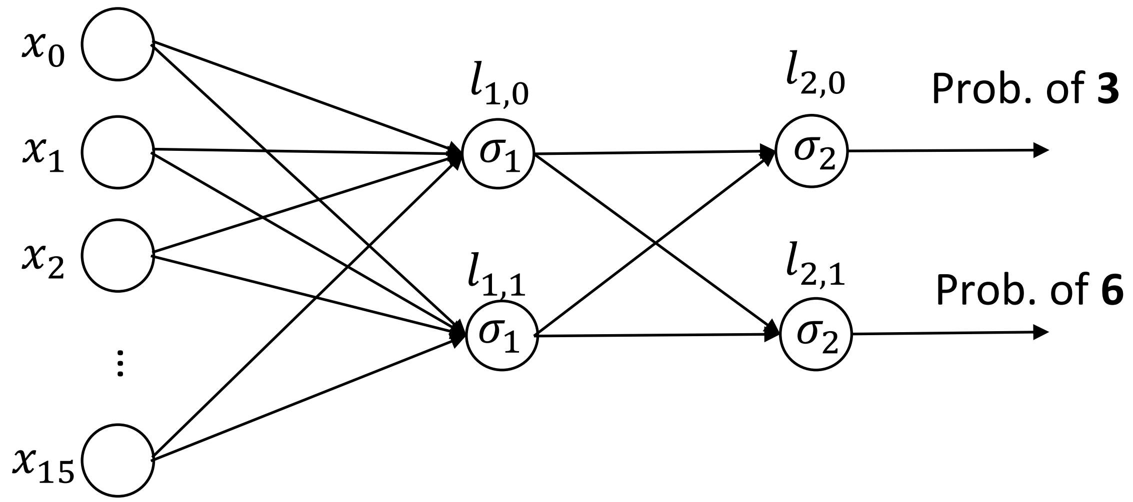
y	-1	$\frac{-m+2}{m}$	… 0 …	$\frac{m-2}{m}$	1
	$\prod p_i$	$p_{m-1}\ldots p_1q_0$ + $p_{m-1}\ldots q_1p_0$ + ⋮ + $q_{m-1}\ldots p_1p_0$	…	$q_{m-1}\ldots q_1p_0$ + $q_{m-1}\ldots p_1q_0$ + ⋮ + $p_{m-1}\ldots q_1q_0$	$\prod q_i$

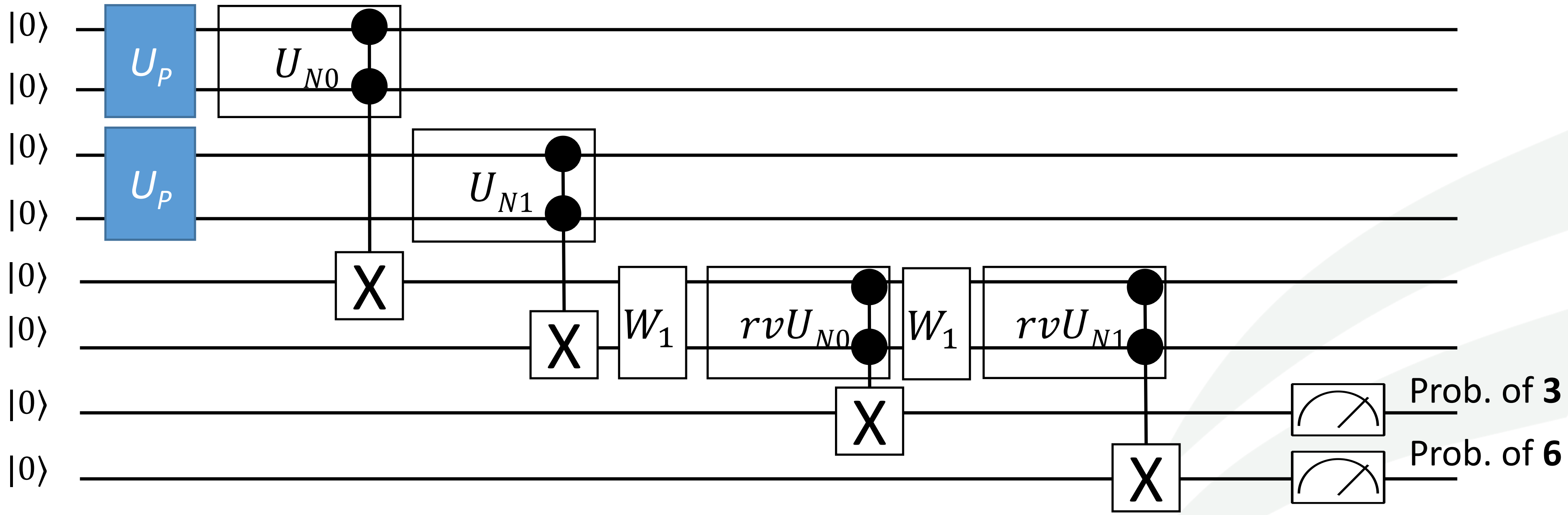
y^2	0 …	$(\frac{m-2}{m})^2$	1
		$p_{m-1}\ldots p_1q_0$ + $p_{m-1}\ldots q_1p_0$ + ⋮ + $q_{m-1}\ldots p_1p_0$ + $q_{m-1}\ldots q_1p_0$ + $q_{m-1}\ldots p_1q_0$ + ⋮ + $p_{m-1}\ldots q_1q_0$	$\prod q_i$ + $\prod p_i$



m-k Encoder States	Amplitude S_0	S_1	S_2	S_3
$\|00\ldots0\rangle\otimes\|0..0\rangle$ $\|00\ldots0\rangle\otimes\|0..1\rangle$ … $\|00\ldots0\rangle\otimes\|1..1\rangle$	$\sqrt{q_{m-1}q_{m-2}\ldots q_0}$ 0 … 0	$\frac{1}{2^{k/2}}$ $\sqrt{q_{m-1}q_{m-2}\ldots q_0}$ $\sqrt{q_{m-1}q_{m-2}\ldots q_0}$ … $\sqrt{q_{m-1}q_{k-1}\ldots q_0}$	$\frac{1}{2^{k/2}}$ $\sqrt{q_{m-1}q_{m-2}\ldots q_0}$ $\sqrt{q_{m-1}q_{m-2}\ldots q_0}$ … $\sqrt{q_{m-1}q_{m-2}\ldots q_0}$	$\sqrt{q_{m-1}q_{m-2}\ldots q_0}$ xxxxxxxxxx … xxxxxxxxxx
$\|00\ldots1\rangle\otimes\|0..0\rangle$ $\|00\ldots1\rangle\otimes\|0..1\rangle$ … $\|00\ldots1\rangle\otimes\|1..1\rangle$	$\sqrt{q_{m-1}q_{m-2}\ldots p_0}$ 0 … 0	$\frac{1}{2^{k/2}}$ $\sqrt{q_{m-1}q_{m-2}\ldots p_0}$ $\sqrt{q_{m-1}q_{m-2}\ldots p_0}$ … $\sqrt{q_{m-1}q_{m-2}\ldots p_0}$	$\frac{1}{2^{k/2}}$ $\sqrt{q_{m-1}q_{m-2}\ldots p_0}$ $-\sqrt{q_{m-1}q_{m-2}\ldots p_0}$ … $\sqrt{q_{m-1}q_{m-2}\ldots p_0}$	(m-2)/m $\sqrt{q_{m-1}q_{m-2}\ldots p_0}$ xxxxxxxxxx … xxxxxxxxxx
…	…	…	…	…
$\|11\ldots1\rangle\otimes\|0..0\rangle$ $\|11\ldots1\rangle\otimes\|0..1\rangle$ … $\|11\ldots1\rangle\otimes\|1..1\rangle$	$\sqrt{p_{m-1}p_{m-2}\ldots p_0}$ 0 … 0	$\frac{1}{2^{k/2}}$ $\sqrt{p_{m-1}q_{m-2}\ldots q_0}$ $\sqrt{p_{m-1}q_{m-2}\ldots q_0}$ … $\sqrt{p_{m-1}q_{m-2}\ldots q_0}$	$\frac{1}{2^{k/2}}$ $\sqrt{p_{m-1}q_{m-2}\ldots q_0}$ $-\sqrt{p_{m-1}q_{m-2}\ldots q_0}$ … $-\sqrt{p_{m-1}q_{m-2}\ldots q_0}$	(2-m)/m $\sqrt{q_{m-1}q_{m-2}\ldots p_0}$ xxxxxxxxxx … xxxxxxxxxx

Implementing Feedforward Net w/ Non-Linearity, w/o Measurement!





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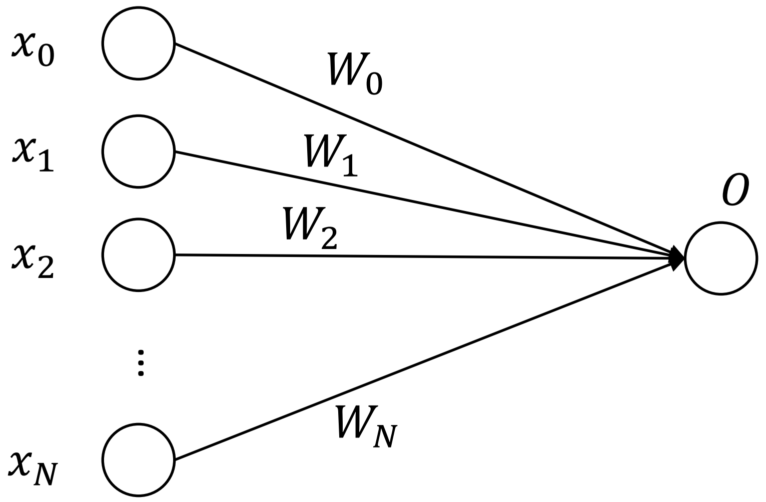
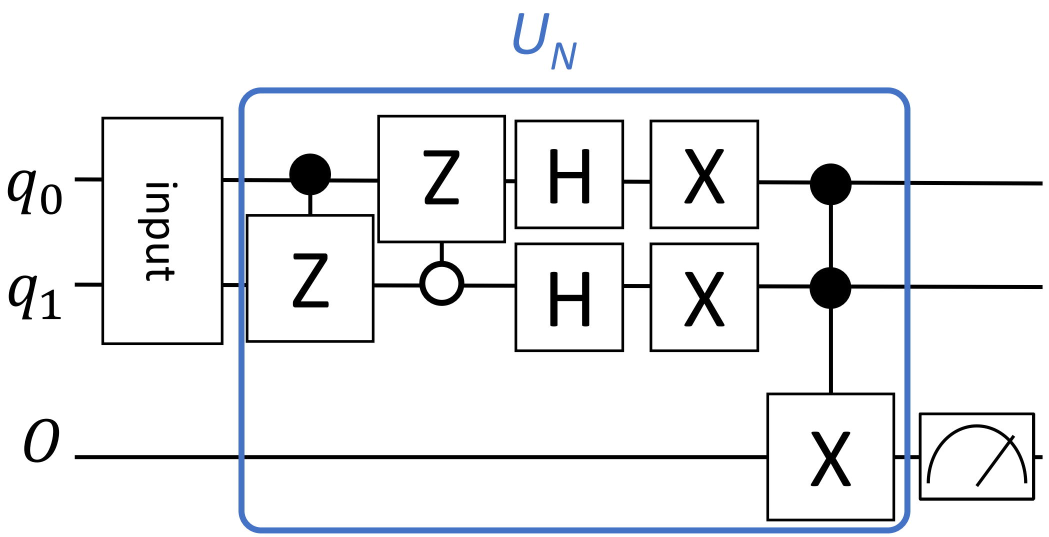




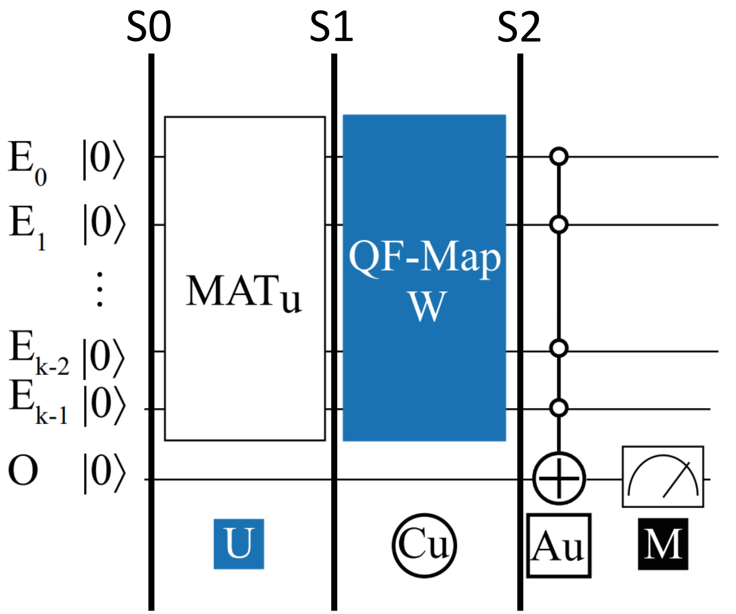
Cost Complexity

Classical Computing		
	No Parallelism	Full Parallelism
Time (T)	O(N)	O(1)
Space (S)	O(1)	O(N)
Cost (TS)	O(N)	O(N)

Quantum Computing		
	Previous Design	Optimization
Circuit Depth (T)	O(N)	???
Qubits (S)	O($\log N$)	O($\cancel{N}$) log N
Cost (TS)	O($N \cdot \log N$)	**target** O(ploylog N)

QuantumFlow: Taking NN Property to Design QC

$$[0, 0.9, 0, 0, 0, 0.1, 0, 0, 1.0, 0.5, 0.5, 0, 0, 0, 0]^T \xrightarrow{U} [0, 0.59, 0, 0, 0, 0.07, 0, 0, 0.66, 0.33, 0.33, 0, 0, 0, 0]^T$$



S0 -> S1:

$$(v_o; v_{x1}; v_{x2}; \dots; v_{xn}) \times \begin{pmatrix} 1 \\ 0 \\ \dots \\ 0 \end{pmatrix} = (v_0)$$

$$S1 = [0, 0.59, 0, 0, 0, 0.07, 0, 0, 0.66, 0.33, 0.33, 0, 0, 0, 0]^T$$

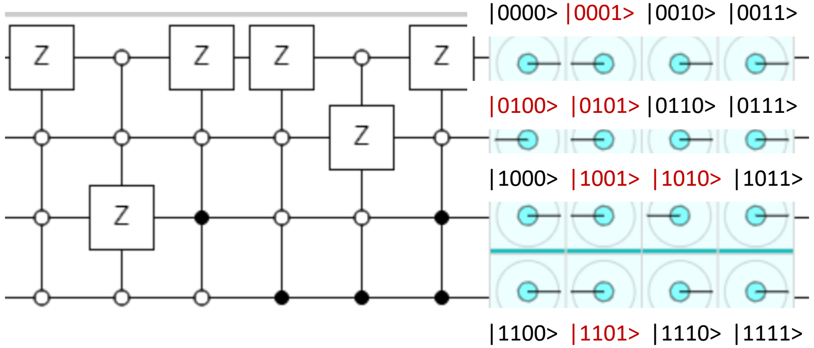
S1 -> S2:

$$W = [+1, -1, +1, +1, -1, -1, +1, +1, +1, -1, -1, +1, +1, -1, +1, +1]^T$$

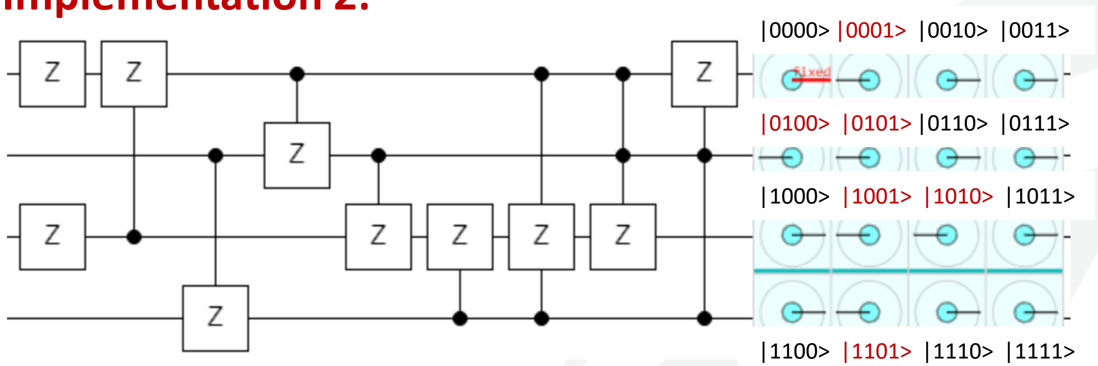
|0000> |0001> |0010> |0011> |0100> |0101> |0110> |0111> |1000> |1001> |1010> |1011> |1100> |1101> |1110> |1111>

$$S2 = [0, -0.59, 0, 0, -0, -0.07, 0, 0, 0, -0.66, -0.33, 0.33, 0, -0, 0, 0]^T$$

Implementation 1 (example in Quirk):

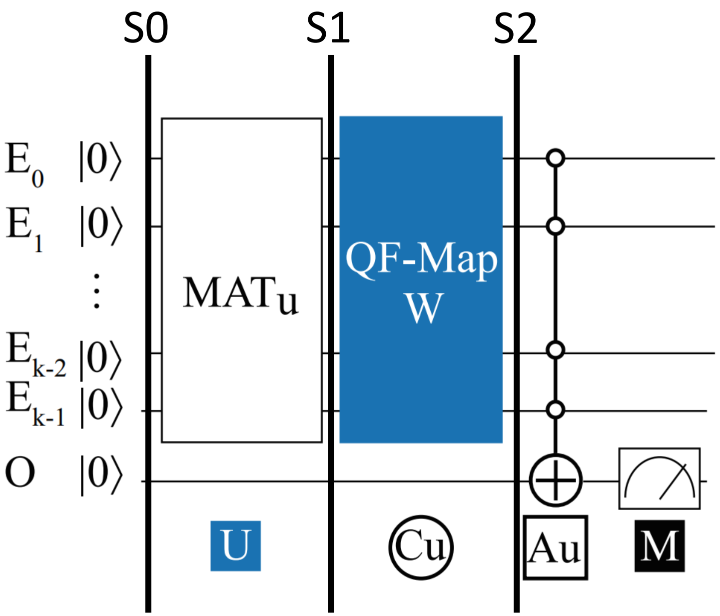


Implementation 2:



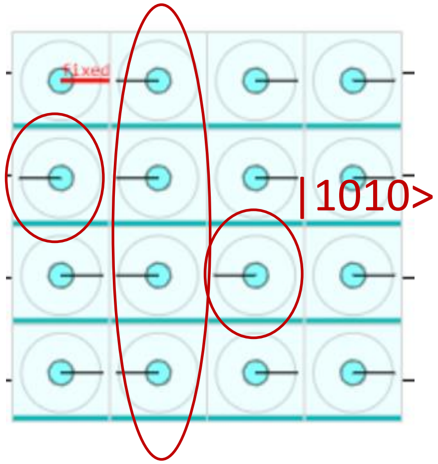
[ref] Tacchino, F., et al., 2019. An artificial neuron implemented on an actual quantum processor. *npj Quantum Information*, 5(1), pp.1-8.

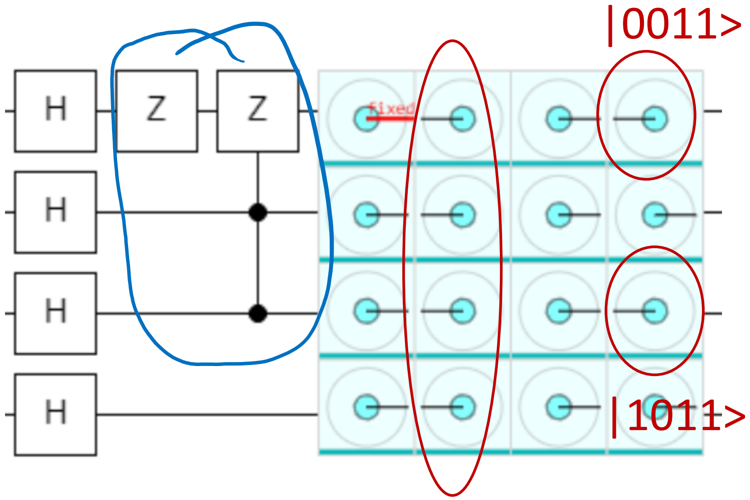
QuantumFlow: Taking NN Property to Design QC



Property from NN

- The **weight order** is not necessary to be fixed, which can be adjusted if the order of inputs are adjusted accordingly
- **Benefit:** No need to require the positions of sign flip are exactly the same with the weights; instead, only need the number of signs are the same.





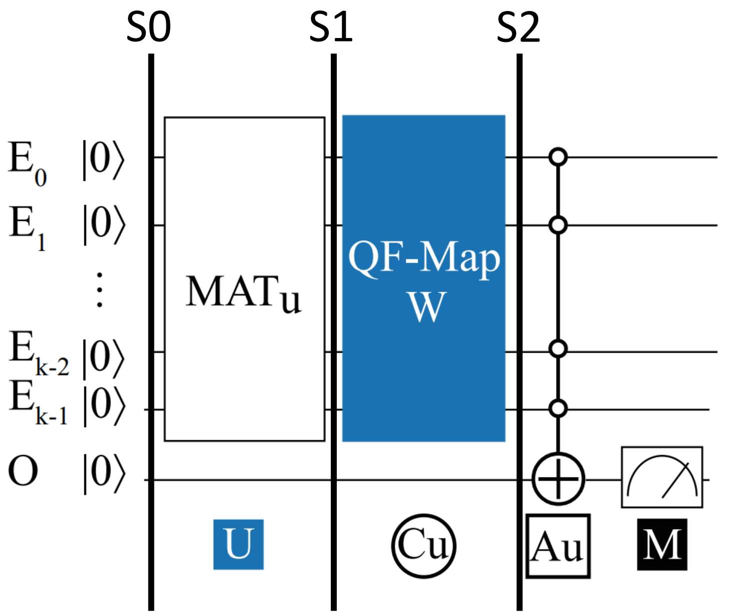
$$S1 = [0, 0.59, 0, \mathbf{0}, \mathbf{0}, 0.07, 0, 0, 0.66, \mathbf{0.33}, \mathbf{0.33}, 0, 0, 0, 0]^T$$

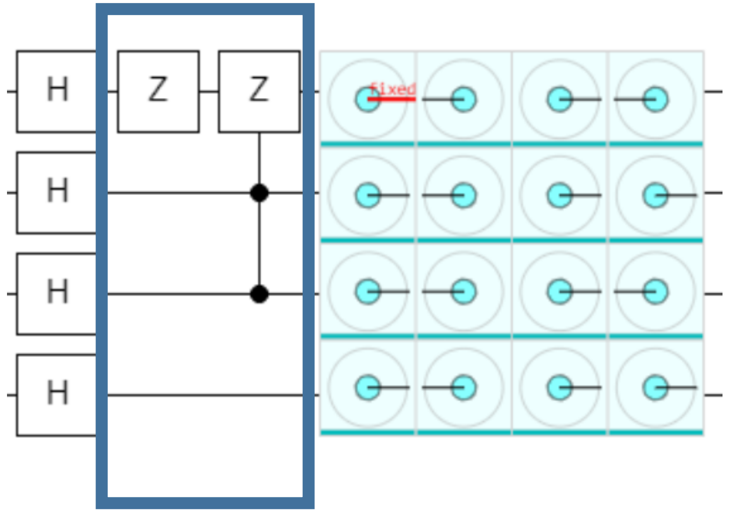
ori + - - +

fin - + + -

$$S1' = [0, 0.59, 0, \mathbf{0.33}, \mathbf{0.33}, 0.07, 0, 0, 0.66, \mathbf{0}, \mathbf{0}, 0, 0, 0, 0]^T$$

QuantumFlow: Taking NN Property to Design QC





Algorithm 4: QF-Map: weight mapping algorithm

Input: (1) An integer $R \in (0, 2^{k-1}]$; (2) number of qbits k;
Output: A set of applied gate G

```
void recursive(G,R,k){
    if (R < 2^{k-2}){
        recursive(G,R,k − 1); // Case 1 in the third step
    }
    else if (R == 2^{k-1}){
        G.append(PG_{2^{k-1}}); // Case 2 in the third step
        return;
    }else{
        G.append(PG_{2^{k-1}});
        recursive(G,2^{k-1} − R,k − 1); // Case 3 in the third step
    }
}
// Entry of weight mapping algorithm
set main(R,k){
    Initialize empty set G;
    recursive(G,R,k);
    return G
}
```

Used gates and Costs

Gates	Cost
Z	1
CZ	1
C^2Z	3
C^3Z	5
C^4Z	6
…	…
C^kZ	2k-1

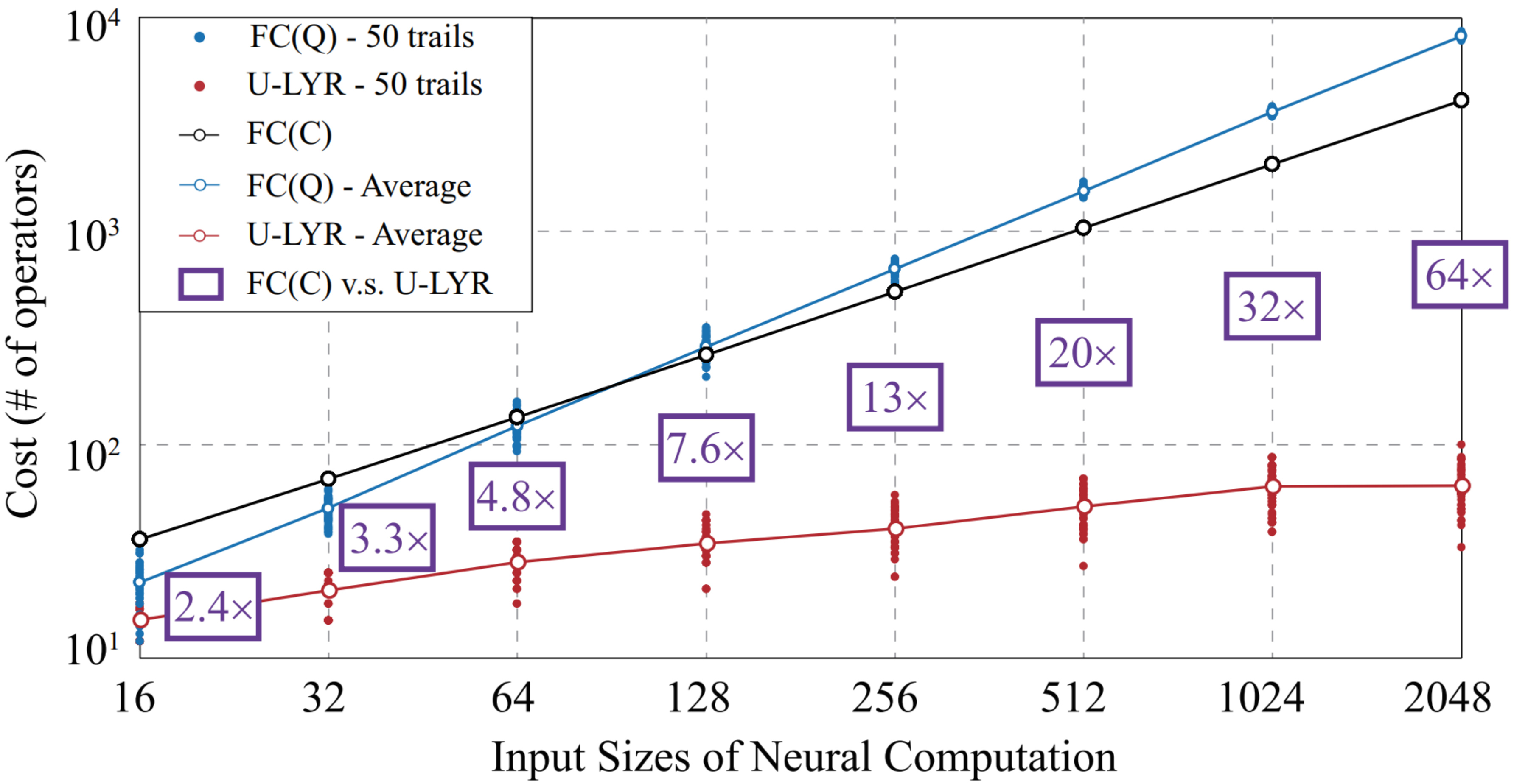
Worst case: all gates

$O(k^2)$

Agenda – Session 3: QuantumFlow

- **General Framework for Quantum-Based Neural Network Accelerator**
  - Data Preparation and Encoding
  - *Colab Hands-On (1): From Classical Data to Quantum Data*
  - Quantum Circuit Design
  - *Colab Hands-On (2): A Quantum Neuron*
- **Co-Design toward Quantum Advantage**
  - Challenges?
  - Feedforward Neural Network
  - *Colab Hands-On (3): End-to-End Neural Network on MNIST*
  - Optimization for Quantum Neuron
  - *Colab Hands-On (4): QuantumFlow*
  - Results

QuantumFlow Results



[ref] Tacchino, F., et al., 2019. An artificial neuron implemented on an actual quantum processor. *npj Quantum Information*, *5*(1), pp.1-8.

QuantumFlow Achieves Over 10X Cost Reduction

Dataset	Structure			MLP(C)			FFNN(Q)				QF-hNet(Q)			
	In	L1	L2	L1	L2	Tot.	L1	L2	Tot.	**Red.**	L1	L2	Tot.	**Red.**
{1,5}	16	4	2	132	18	150	80	38	118	**1.27**×	74	38	112	**1.34**×
{3,6}	16	4	2				96	38	134	**1.12**×	58	38	96	**1.56**×
{3,8}	16	4	2				76	34	110	**1.36**×	58	34	92	**1.63**×
{3,9}	16	4	2				98	42	140	**1.07**×	68	42	110	**1.36**×
{0,3,6}	16	8	3	264	51	315	173	175	348	**0.91**×	106	175	281	**1.12**×
{1,3,6}	16	8	3				209	161	370	**0.85**×	139	161	300	**1.05**×
{0,3,6,9}	64	16	4	2064	132	2196	1893	572	2465	**0.89**×	434	572	1006	**2.18**×
{0,1,3,6,9}	64	16	5	2064	165	2229	1809	645	2454	**0.91**×	437	645	1082	**2.06**×
{0,1,2,3,4}	64	16	5				1677	669	2346	**0.95**×	445	669	1114	**2.00**×
{0,1,3,6,9}*	256	8	5	4104	85	4189	5030	251	5281	**0.79**×	135	251	386	**10.85**×

*: Model with 16×16 resolution input for dataset {0,1,3,6,9} to test scalability, whose accuracy is 94.09%, which is higher than 8×8 input with accuracy of 92.62%.

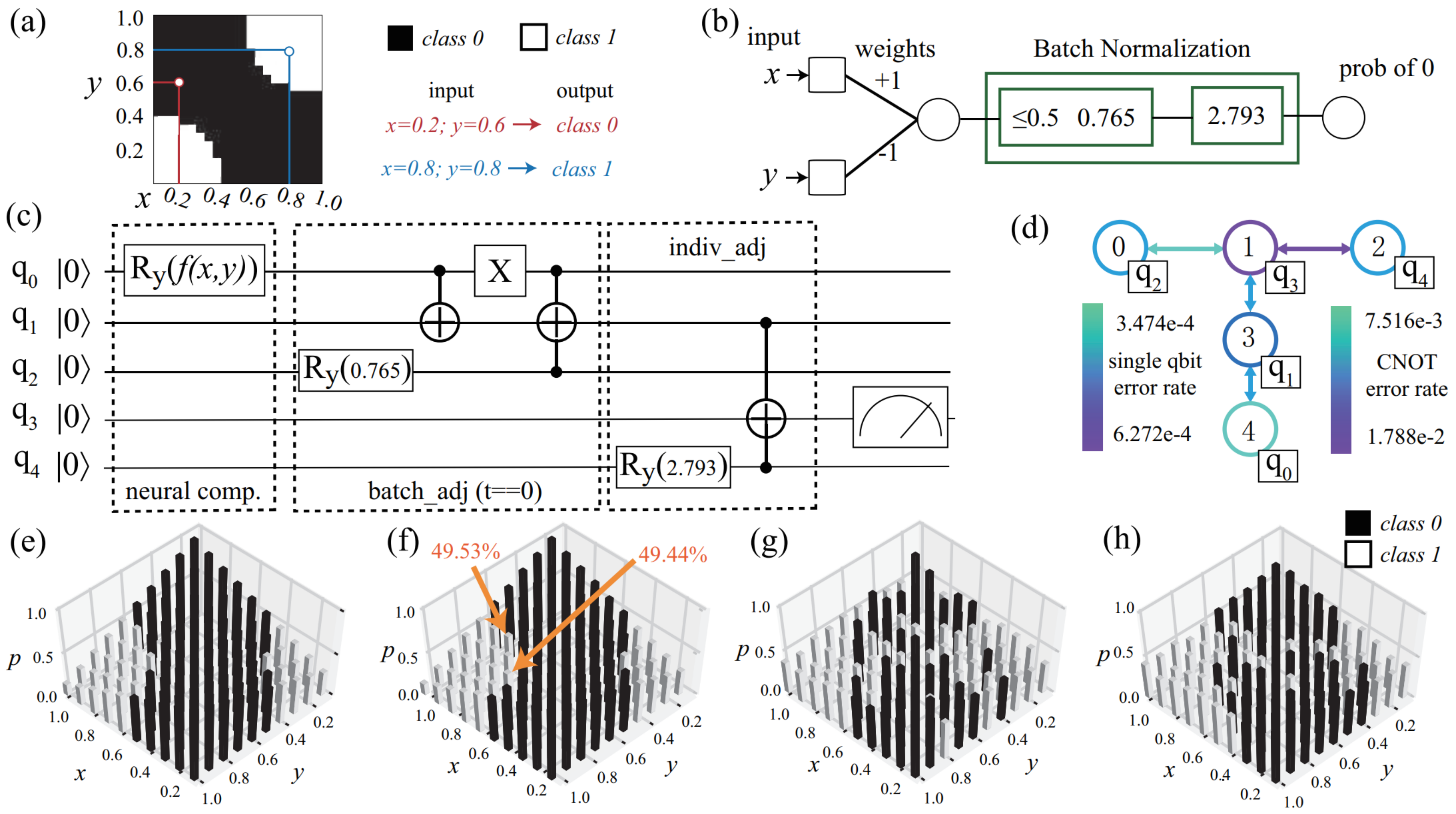
[ref of FFNN] Tacchino, F., et al., 2019. Quantum implementation of an artificial feed-forward neural network. *arXiv preprint arXiv:1912.12486*.

QF-Nets Achieve the Best Accuracy on MNIST

Dataset	w/o BN					w/ BN				
	binMLP(C)	FFNN(Q)	MLP(C)	QF-pNet	QF-hNet	binMLP(C)	FFNN(Q)	MLP(C)	QF-pNet	QF-hNet
1,5	61.47%	61.47%	69.12%	69.12%	90.33%	55.99%	55.99%	85.30%	84.56%	**96.60%**
3,6	72.76%	72.76%	94.21%	91.67%	97.21%	72.76%	72.76%	96.29%	96.39%	**97.66%**
3,8	58.27%	58.27%	82.36%	82.36%	89.77%	58.37%	58.07%	86.74%	86.90%	**87.20%**
3,9	56.71%	56.51%	68.65%	68.30%	95.49%	56.91%	56.71%	80.63%	78.65%	**95.59%**
0,3,6	46.85%	51.63%	49.90%	59.87%	89.65%	50.68%	50.68%	75.37%	78.70%	**90.40%**
1,3,6	60.04%	59.97%	53.69%	53.69%	94.68%	59.59%	59.59%	86.76%	86.50%	**92.30%**
0,3,6,9	72.68%	72.33%	84.28%	87.36%	92.85%	69.95%	68.89%	82.89%	76.78%	**93.63%**
0,1,3,6,9	50.00%	51.10%	49.00%	43.24%	87.96%	60.96%	69.46%	70.19%	71.56%	**92.62%**
0,1,2,3,4	46.96%	50.01%	49.06%	52.95%	83.95%	64.51%	69.66%	71.82%	72.99%	**90.27%**

[ref of FFNN] Tacchino, F., et al., 2019. Quantum implementation of an artificial feed-forward neural network. *arXiv preprint arXiv:1912.12486*.

On Actual IBM "ibmq_essex" Quantum Processor



Hands-On Tutorial (3)

PreP+ U_P+ U_N+ M+ PostP (MNIST)

Hands-On Tutorial (4)

PreP + U_P+ Optimized U_N+ M+ PostP (MNIST)

Hands-On Tutorial (5)

Comparison

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