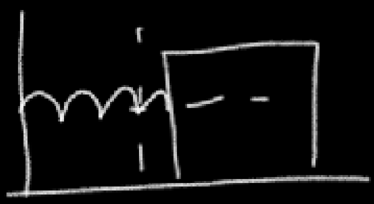
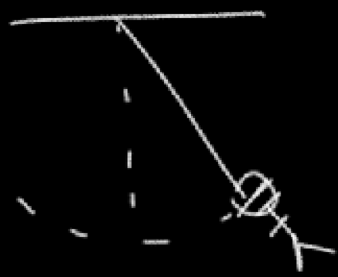
Harmonic oscillator for Dummies





traps ≠ gay

steel
fuel
can't
melt
jet
$\boxed{B}$eams

$$\ddot{x}(t) + w^2 x(t) = 0$$

$$\ddot{x} = \frac{d^2x}{dt^2}$$

anzats

$$x(t) = A e^{\lambda t} + B e^{-\lambda t}$$

$$\dot{x}(t) = A\lambda e^{\lambda t} - \lambda B e^{-\lambda t}$$

$$\ddot{x}(t) = A\lambda^2 e^{\lambda t} + B\lambda^2 e^{-\lambda t}$$

$$A\lambda^2 e^{\lambda t} + B\lambda^2 e^{-\lambda t} + Aw^2 e^{\lambda t} + Bw^2 e^{-\lambda t} = 0$$

$$A e^{\lambda t}(\lambda^2 + w^2) + B e^{-\lambda t}(\lambda^2 + w^2) = 0$$

??? $$\underbrace{(\lambda^2 + w^2)}\underbrace{(A e^{\lambda t} + B e^{-\lambda t})}_{x(t)} = 0$$

$$\lambda^2 + w^2 = 0$$

$$\lambda = \pm\sqrt{-1}\,w \quad \text{imaginary...}$$

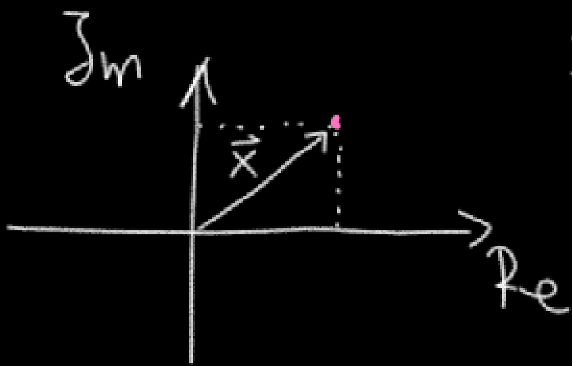
$\lambda = \pm\sqrt{-1}\,\omega$ imaginary unit

$\lambda = i\omega \qquad i^2 = -1$

$$x(t) = Ae^{i\omega t} + Be^{-i\omega t} \in \mathbb{C}\,?$$

Is the position imaginary?

It sure is for my scientific career



$x = \mathrm{Re}(x) + \mathrm{Im}(x)\cdot i$

what are you 5?

$x = r(\cos\varphi \pm i\sin\varphi)$

$x = re^{\pm i\varphi}$ Euler formula <inside>

$$\cos\varphi = \frac{1}{2}\left(e^{i\varphi} + e^{-i\varphi}\right)$$

$$\sin\varphi = \frac{1}{2i}\left(e^{i\varphi} - e^{-i\varphi}\right)$$

Real solutions

↓

fixed A,B relation

$\underline{B = A}\ \left(= \frac{1}{2}\tilde{A}\right)$ great convention, buddy

$$x(t) = A\left(e^{i\omega t} + e^{-i\omega t}\right)$$
$$= \tilde{A}\cos(\omega t)$$

$$B = -A \left(= -\frac{1}{2i}\tilde{B}\right)$$

$$x(t) = A\left(e^{i\omega t} - e^{-i\omega t}\right)$$
$$= \tilde{B}\sin(\omega t)$$

Combining the real solutions:

$$x(t) = \tilde{A}\cos(\omega t) + \tilde{B}\sin(\omega t)$$

- you could also use it as an asantz at the beginning
- $\tilde{A}, \tilde{B}$ are fixed with initial conidions on position and velocity
- Usually the harmonic motion concerns one of the sine, cosine functions