Solution to Cambridge Checkpoint Mathematics Midterm 1: Stage 9

Solutions

1. $\sqrt{5} \times \sqrt{5} = 5$, which is an integer.

2. (a) $x = 4.5 \times 10^5 = 450000$
   (b) The three numbers in order of size, smallest first are: $y < x < z$

3. (a) $3^3 \times 3^4 = 3^{3+4} = 3^7$
   (b) $9^3 \div 9^5 = 9^{3-5} = 9^{-2}$

4. (a) $2a + b = 2(1) + (-1) = 2 - 1 = 1$
   (b) $3x^2 + 2y^3 = 3(-2)^2 + 2(2)^3 = 3 \times 4 + 2 \times 8 = 12 + 16 = 28$
   (c) Rearranging the given equation gives $g = \frac{20}{10-h} = \frac{20}{10-2} = -4$

5. (a) The perimeter of the rectangle is $2(x+2x+2) = 2(3x+2) = 6x+4$
   (b) The area of the rectangle is $x \times (2x + 2) = 2x^2 + 2x$

6. The simplified expression is $10x^5$

7. $0.85 \div 10^{-1} = 8.5$

8. $0.045 \div 10^{-2} = 4.5$

9. (a) $6 \times 0.3 = 1.8$
   (b) $15 \times 0.02 = 0.3$
   (c) $0.64 \div 0.08 = 8$
   (d) $1.6 \div -0.04 = -40$

10. (a) For the percentage problem, the increase by 20% can be represented by multiplying

$$150\,(100\% + 20\%) = 150\left(\frac{100}{100} + \frac{20}{100}\right) = 150\left(\frac{120}{100}\right) =$$

$$150\left(\frac{6}{5}\right) = 30 \times 6 = 180$$

The subsequent increase by 10% is represented by multiplying the intermediate result:

$$180\,(100\% + 10\%) = 180\left(\frac{100}{100} + \frac{10}{100}\right) = 180\left(\frac{110}{100}\right) =$$

$$180\left(\frac{11}{10}\right) = 18 \times 11 = \boxed{198}$$

$\boxed{OR}$

$$150 \times \frac{6}{5} \times \frac{11}{10} = 198$$

(b) The decrease by 90% can be represented by multiplying the original value by

$$50\,(100\% - 90\%) = 50\,(10\%) = 50\left(\frac{10}{100}\right) = 50\left(\frac{1}{10}\right) = \frac{50}{10} = 5$$

The subsequent decrease by 80% is represented by multiplying the intermediate result by

$$5\,(100\% - 80\%) = 5\,(20\%) = 5\left(\frac{20}{100}\right) = 5\left(\frac{1}{5}\right) = \frac{5}{5} = \boxed{1}$$

$$50 \times \frac{1}{10} \times \frac{1}{5} = 1$$

11. (a) $5x + 10 = -5$
    (b) $5x + 10 - 10 = -5 - 10$ (subtracting 10 from both sides)
    (c) $5x = -15$ (simplifying)
    (d) $\frac{5x}{5} = \frac{-15}{5}$ (dividing both sides by 5)
    (e) $x = -3$ (simplifying)

    (a) $2x + 6 = 48 - 40x$
    (b) $2x + 40x + 6 = 48 - 40x + 40x$ (adding 40x to both sides)
    (c) $42x + 6 = 48$ (combining like terms)
    (d) $42x + 6 - 6 = 48 - 6$ (subtracting 6 from both sides)
    (e) $42x = 42$ (simplifying)
    (f) $\frac{42x}{42} = \frac{42}{42}$ (dividing both sides by 42)
    (g) $x = 1$ (simplifying)

12. Bonus:

    (a) The greatest value of the sum of the digits of a three-digit number is obtained when the number is 999. The sum of the digits of 999 is $9 + 9 + 9 = 27$. The greatest value of the sum of the digits of this number is then $2 + 7 = 9$. Therefore, the greatest value of the sum of the digits of the number made from the sum of the digits of a three-digit number is 9.

    (b) Let's denote the current age of the son as S and the daughter as D. We have the following system of equations:

$$
\begin{aligned}
S+2 &= 2(S-2) \\
S+2 &= 2S-4 \quad \text{(by distributing 2)} \\
-S+S+2 &= 2S-S-4 \quad \text{(by subtracting S from both sides)} \\
2 &= S-4 \quad \text{(combining like terms)} \\
2+4 &= S-4+4 \quad \text{(by adding 4 to both sides)} \\
S &= 6 \quad \text{(simplifying)} \\
D+3 &= 3(D-3) \\
D+3 &= 3D-9 \quad \text{(by distributing 3)} \\
-D+D+3 &= 3D-D-9 \quad \text{(by subtracting D from both sides)} \\
3 &= 2D-9 \quad \text{(combining like terms)} \\
3+9 &= 2D-9+9 \quad \text{(by adding 9 to both sides)} \\
12 &= 2D \quad \text{(simplifying)} \\
12/2 &= 2D/2 \quad \text{(dividing by 2 from both sides)} \\
D &= 6 \quad \text{(simplifying)}
\end{aligned}
$$

So, the solutions are $S = 6$ and $D = 6$.

The solution of this system gives $S = 6$ and $D = 6$. Hence, the son and daughter are the same age.