Cambridge IGCSE Mathematics: Midterm 1 Solutions
Year 10

J. Dittenber, M.S.

This examination has 20 questions, for a total of 61 points and 10 bonus points.

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You must show all of your work for credit

Full Name in English: ____________________

Nickname: ____________________

You must show all of your work for credit

1. (18 points) Fill in the table. If the result is irrational, put an I in the cell. The first row has been done for you.

Solution For decimals, if you do not have a calculator, turn them into fractions

$$0.04 = \frac{4}{100}$$

$$\left(\frac{4}{100}\right)^2 = \left(\frac{4}{10^2}\right)^2 = \frac{16}{10^4} = 16 \times 10^{-4} = 0.0016$$

$$0.04 = \frac{4}{100} = \sqrt{\frac{4}{100}} = \frac{\sqrt{4}}{\sqrt{100}} = \frac{2}{10} = 2 \times 10^{-1} = 0.2$$

n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$	$\frac{a}{b}$
0.2	0.04	I	0.008	I	$\frac{2}{10}$
64	4096	8	262144	4	$\frac{64}{1}$
0.04	0.0016	0.2	0.000064	I	$\frac{4}{100}$
7	49	I	343	I	$\frac{7}{1}$

2. (15 points) Fill in the table by rounding the numbers to the degree of accuracy mentioned.

n	Hundreds	Tens	Ones	Tenths	Hundredths
2751.367	2800	2750	2751	2751.4	2751.37
213.00523	200	210	213	213.0	213.01
48756.123	48800	48760	48756	48756.1	48756.12

3. (9 points) Fill in the table by rounding the numbers to the number of significant figures mentioned.

n	1 s.f.	2 s.f.	3 s.f.
2751.367	3000	2800	2750
213.00523	200	210	213
48756.123	50000	49000	48800

4. (1 point) Write the information on the following line as an inequality. *Note: there is a hole at -1 and a dot at 5.
   Solution: $-1 < x \leq 5$

5. (1 point) Evaluate

$$5 + 1 \times (-10)^2 - 5$$

$$5 + 1 \times 100 - 5$$
$$= 5 + 100 - 5$$

   Solution: 100

6. (1 point) Show all of your working to work out $\frac{1}{4}$ of 80 Solution: 20

7. (1 point) Write the following as percentages. Show all of your working.

   a. $\dfrac{2}{5}$
      Solution: $\frac{2}{5} \times \frac{20}{20} = \frac{40}{100} = 0.40 = 40\%$

   b. $\dfrac{3}{20}$
      Solution: $\frac{3}{20} \times \frac{5}{5} = \frac{15}{100} = 0.15 = 15\%$

   c. $\dfrac{6}{50}$
      Solution: $\frac{6}{50} \times \frac{2}{2} = \frac{12}{100} = 0.12 = 12\%$

8. (2 points) Evaluate each of the following:

   a. $2\dfrac{3}{4} - \dfrac{1}{3}$

      Solution: Convert the mixed number to an improper fraction: $2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$.
      Convert fractions to have a common denominator by finding the LCM. The LCM(3,4) is 12: $\frac{11}{4} \times \frac{3}{3} = \frac{33}{12}$ and $\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$.
      Subtract the fractions: $\frac{33}{12} - \frac{4}{12} = \frac{29}{12}$.
      Convert back to mixed number form: $\frac{29}{12} = 2\frac{5}{12}$.
      Therefore, $2\frac{3}{4} - \frac{1}{3} = 2\frac{5}{12}$.

   b. $3\dfrac{1}{2} \times \dfrac{5}{8}$

      Solution: Convert the mixed number to an improper fraction: $3\frac{1}{2} = \frac{6}{2} + \frac{1}{2} = \frac{7}{2}$.
      Multiply the fractions: $\frac{7}{2} \times \frac{5}{8} = \frac{35}{16}$.
      Convert back to mixed number form: $\frac{35}{16} = 2\frac{3}{16}$.
      Therefore, $3\frac{1}{2} \times \frac{5}{8} = 2\frac{3}{16}$.

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9. (2 points) A ruler 60 cm long is broken into three parts in the ratio 8:9:13. How long are the three parts?

   Solution: The total ratio is $8 + 9 + 13 = 30$. The length of the parts are: $60 \times \frac{8}{30} = 16$ cm, $60 \times \frac{9}{30} = 18$ cm, $60 \times \frac{13}{30} = 26$ cm

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10. (1 point) Simplify the following using indices

$$\frac{2^3 \times 8^2}{4^{-2}}$$

Rewrite in as same base.

$$\frac{2^3 \times (2^3)^2}{(2^2)^{-2}}$$

$$\frac{2^3 \times 2^6}{2^{-4}} = 2^3 \times 2^6 \times 2^4 = 2^{3+6+4}$$

Solution: 2^{13}

11. (1 point) Circle the correct answer: What is 23458000 in standard form?

    A. 23.458×10^7

    B. 2.3458×10^{-7}

    C. 2.3458×10^7

    D. 2.3458×10^8

12. (1 point) A person deposited \$5,000 in the bank. The constant, annual rate of interest was 2%. How many years had to pass to earn the simple interest gained of \$500. Show all of your working. Note: The simple interest formula is

$$I = \frac{Prt}{100}$$

Solution: Using the formula $I = \frac{Prt}{100}$, we get $500 = \frac{5000 \times 2 \times t}{100}$ which simplifies to $t = 5$ years

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13. (1 point) A journey to school takes a girl 55 minutes. What time does she leave home at 0838? Solution: She arrives at $09:33$.

$$\begin{array}{r}
\text{Departure time} : 08:38 \\
\text{Travel duration} : 00:55 \\
\hline
\text{Raw arrival time} : 08:38 + 00:55 = 08:93
\end{array}$$

$$\text{Extra hour} : \frac{93}{60} = 1 \text{ hour, remainder 33 minutes}$$

$$\text{Converted arrival time} : 08:93 = 09:33$$

14. (1 point) Factorise fully:

Working:

Factor out the common factor of $4x$:

$$4x^3 + 20x = 4x(x^2 + 5)$$

Final Answer:

$4x(x^2 + 5)$

15. (1 point) Expand and simplify where possible

    Working:

    a) Using distributive property, we have:

    $(x+2)(x+1) = x^2 + x + 2x + 2 = x^2 + 3x + 2$

    Final Answer for a):

    $x^2 + 3x + 2$

    Working:

    b) Using distributive property, we have:

    $(2x+3)(x-1) = 2x^2 - 2x + 3x - 3 = 2x^2 + x - 3$

    Final Answer for b):

    $2x^2 + x - 3$

16. (2 points) Factorise fully using any method:

    Working:

    a) This is a perfect square trinomial, so it factorizes to the square of a binomial:

    $x^2 + 4x + 4 = x^2 + 2x + 2x + 4 = (x^2 + 2x) + (2x + 4) = x(x+2) + 2(+x+2) = (x+2)^2$

    Final Answer for a):

    $(x+2)^2$

    Working:

    b) By finding two numbers that multiply to $2 \times 6 = 12$ and add to 7, which are 3 and 4, the trinomial factorizes to:

    $2x^2 + 7x + 6 = 2x^2 + 3x + 4x + 6 = (2x^2 + 3x) + (+4x + 6) = x(2x+3) + 2(2x+3) = (2x+3)(x+2)$

    Final Answer for b):

    $(2x+3)(x+2)$

17. (1 point) Solving using any method:

$$\begin{aligned}
2x^2 + 2x - 3 &= 0 \\
2x^2 + 2x &= 3 \\
2(x^2 + \frac{1}{2}x) &= 3 \\
(x^2 + \frac{1}{2}x) &= \frac{3}{2} \quad \text{(Dividing entire equation by 2)} \\
x^2 + x + \frac{1}{4} &= \frac{3}{2} + \frac{1}{4} \quad \text{(Adding } (\frac{1}{2})^2 \text{ to both sides)} \\
(x + \frac{3}{2})^2 &= \frac{7}{4} \quad \text{(Simplifying)} \\
x + \frac{1}{2} &= \pm\sqrt{\frac{7}{4}} \quad \text{(Taking square root of both sides)} \\
x &= -\frac{1}{2} \pm \frac{\sqrt{7}}{2} \quad \text{(Isolating } x\text{)}
\end{aligned}$$

18. (1 point) Solve using the quadratic formula, which is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

. . . .

$$2x^2 - 7 = 3x$$

Working:

Rearranging the equation to the standard form $ax^2 + bx + c = 0$, we have:

$2x^2 - 3x - 7 = 0$

Applying the quadratic formula where $a = 2$, $b = -3$, $c = -7$, we get:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 * 2 * (-7)}}{2 * 2} = \frac{3 \pm \sqrt{65}}{4}$$

Final Answer:

$x = \frac{3+\sqrt{65}}{4}, \frac{3-\sqrt{65}}{4}$

Bonus

19. (5 points) The sum of three numbers is 96. The first number is 6 times the third number, and the third number is 40 less than the second number. What is the difference between the first and second numbers?

    Solution Working:

    Let x be the third number. It follows that the first number is $6x$, and the second number is $x + 40$.

    We have
    $$6x + (x + 40) + x = 8x + 40 = 96,$$
    from which $x = 7$.

    Therefore, the first number is 42, and the second number is 47. Their absolute value of the difference is $|42 - 47| = \boxed{5}$.

    Final Answer:

    5

20. (5 points) How many ways are there to split the integers 1 through 14 into 7 pairs such that in each pair, the greater number is at least 2 times the lesser number?

    Solution

    Clearly, the integers from 8 through 14 must be in different pairs, and 7 must pair with 14.

    Note that 6 can pair with either 12 or 13. From here, we consider casework:

    If 6 pairs with 12, then 5 can pair with one of $10, 11, 13$. After that, each of $1, 2, 3, 4$ does not have any restrictions. This case produces $3 \cdot 4! = 72$ ways. If 6 pairs with 13, then 5 can pair with one of $10, 11, 12$. After that, each of $1, 2, 3, 4$ does not have any restrictions. This case produces $3 \cdot 4! = 72$ ways. Together, the answer is $72 + 72 = \boxed{144}$.