the signal and noise spectra *beforehand*, so that the
[...] can be determined. This nonlinear technique
[...]ept that the Wiener filter's frequency response is
[...]gment, based on the spectrum *of that segment*. In
[...]s frequency response changes from segment-to-
[...] by the characteristics of the signal itself.

[...]s in implementing this (and other) nonlinear
[...]verlap-add method for filtering long signals is not
[...]cy response changes, the time domain waveform of
[...]ger align with the neighboring segments. This can
[...]mbering that audio information is encoded in
[...]change over time, and not in the shape of the time
[...]cal approach is to divide the original time domain
[...]egments. After processing, a smooth window is
[...]ver-lapping segments before they are recombined.
[...] transition of the frequency spectrum from one

[...]chnique is called **homomorphic signal processing**.
[...] *the same structure*. Addition is not the only way
[...]nce can be combined with a signal of interest;
[...]olution are also common means of mixing signals
[...]combined in a nonlinear way (i.e., anything other
[...]ot be separated by linear filtering. Homomorphic
[...]eparate signals combined in a nonlinear way by
[...]me linear. That is, the problem is converted to the
[...]r system.

[...]n audio signal transmitted via an AM radio wave.
[...]ms change, the received amplitude of the signal
[...]resulting in the loudness of the received audio
[...]er time. This can be modeled as the audio signal,
[...]ng *multiplied* by a slowly varying signal, $g[\ ]$, that
[...]g gain. This problem is usually handled in an
[...]n *automatic gain control* (AGC), but it can also be
[...]DSP.

[...]he input signal, $a[\ ]\times g[\ ]$, is passed through the
[...]n the identity, $\log(x+y) = \log x + \log y$, this results
[...]combined by addition, i.e., $\log a[\ ] + \log g[\ ]$. In
[...]m is the homomorphic transform that turns the
[...]ltiplication into the linear problem of *addition*.

[...]re separated by a conventional linear filter, that is,
[...]sed, while others are rejected. For the AGC, the
[...]composed of very low frequencies, far below the
[...]nd of the voice signal. The logarithm of these

$a[\ ]\times g[\ ] \rightarrow$ Logarithm $\rightarrow$ Linear Filter $\rightarrow$ Anti-Logarithm $\rightarrow a[\ ]$

$\log a[\ ] + \log g[\ ]$ $\log a[\ ]$

FIGURE 22-11
Homomorphic separation of multiplied signals. Taking the logarithm of the input signal transforms components that are *multiplied* into components that are *added*. These components can then be separated by linear filtering, and the effect of the logarithm undone.

In effect, $\log a[\ ] + \log g[\ ]$ is converted into $\log a[\ ]$. In the last step, the logarithm is undone by using the exponential function (the anti-logarithm, or e^x), producing the desired output signal, $a[\ ]$.

Figure 22-12 shows a homomorphic system for separating signals that have been *convolved*. An application where this has proven useful is in removing echoes from audio signals. That is, the audio signal is convolved with an impulse response consisting of a delta function plus a shifted and scaled delta function. The homomorphic transform for convolution is composed of two stages, the *Fourier transform*, changing the convolution into a multiplication, followed by the *logarithm*, turning the multiplication into an addition. As before, the signals are then separated by linear filtering, and the homomorphic transform undone.

An interesting twist in Fig. 22-12 is that the linear filtering is dealing with frequency domain signals in the same way that time domain signals are usually processed. In other words, the time and frequency domains have been swapped from their normal use. For example, if FFT convolution were used to carry out the linear filtering stage, the "spectra" being multiplied would be in the *time domain*. This role reversal has given birth to a strange jargon. For instance, *cepstrum* (a rearrangement of *spectrum*) is the Fourier transform of the logarithm of the Fourier transform. Likewise, there are *long-pass* and *short-pass* filters, rather than low-pass and high-pass filters. Some authors even use *Quefrency Alanysis* and *liftering*.
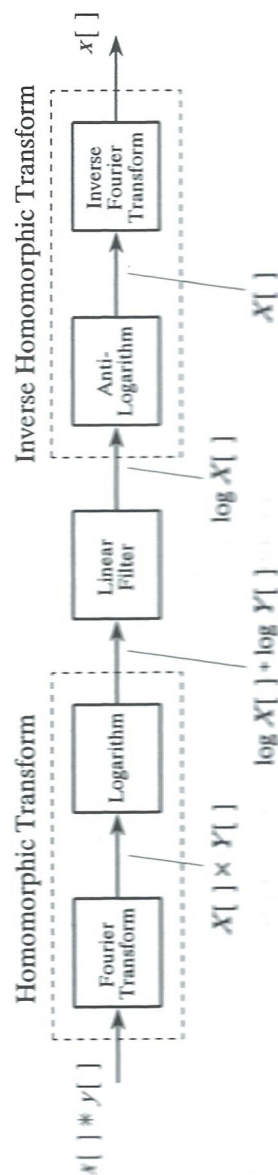
Homomorphic Transform   Inverse Homomorphic Transform

$x[\ ]*y[\ ] \rightarrow$ Fourier Transform $\rightarrow$ Logarithm $\rightarrow$ Linear Filter $\rightarrow$ Anti-Logarithm $\rightarrow$ Inverse Fourier Transform $\rightarrow x[\ ]$

$X[\ ]\times Y[\ ]$ $\log X[\ ] + \log Y[\ ]$ $\log X[\ ]$ $X[\ ]$

FIGURE 22-12

Alan V. Oppenheim *and* Ronald W. Schafer

From Frequency to Quefrency: A History of the Cepstrum

In their classic paper entitled *The Quefrency Alanysis of Time Series for Echos: Cepstrum, Pseudo-Auto-covariance, Cross-Cepstrum, and Saphe Cracking* [1], Bogert, Healy and Tukey were perhaps in a very playful frame of mind when they coined the term *cepstrum*, along with a complete glossary that included, in addition to those in the title of their paper, terms such as *rahmonics* and *liftering*. The motivation for this strange new terminology, where familiar words were paraphrased by interchanging consonants, was succinctly stated as follows: "In general, we find ourselves operating on the frequency side in ways customary on the time side and vice versa" [1].

To suggest what prompted the invention of the term cepstrum, note that a signal with a simple echo can be represented as

$$x(t) = s(t) + \alpha s(t - \tau). \quad (1)$$

The Fourier spectral density (spectrum) of such a signal is given by

$$|X(f)|^2 = |S(f)|^2[1 + \alpha^2 + 2\alpha\cos(2\pi f \tau)]. \quad (2)$$

Thus, we see from (2) that the spectral density of a signal with an echo has the form of an envelope (the spectrum of the original signal) that modulates a periodic function of frequency (the spectrum contribution of the echo). By taking the logarithm of the spectrum, this product is converted to the sum of two components; specifically

$$
\begin{aligned}
C(f) &= \log|X(f)|^2 \\
&= \log|S(f)|^2 + \log[1 + \alpha^2 \\
&\quad + 2\,\alpha\cos(2\pi f \tau)]. \quad (3)
\end{aligned}
$$

Thus, $C(f)$ viewed as a waveform has an additive periodic component whose "fundamental frequency" is the echo delay τ. In conventional analysis of time waveforms, such periodic components show up as lines or sharp peaks in the corresponding Fourier spectrum. Therefore, the "spectrum" of the log spectrum would likewise show a peak when the original time waveform contained an echo. This new "spectral" representation domain was not the frequency domain, nor was it really the time domain. So, looking to forestall confusion while emphasizing connections to familiar concepts, Bogert et al. chose to refer to it as the *quefrency domain*, and they termed the spectrum of the log of the spectrum of a time waveform the *cepstrum*. While most of the terms in the glossary at the end of the original paper have faded into the background, the term *cepstrum* has survived and become part of the digital signal processing lexicon.

In the early 1960s, totally unrelated to, and independent of, the work by Bogert et al., Al Oppenheim was pursuing his doctoral research on a class of nonlinear signal processing techniques inspired by the concept of homomorphic (i.e., linear in a generalized sense) mappings between algebraic groups and vector spaces. His dissertation, "Superposition in a Class of Nonlinear Systems" [2] completed at MIT in May, 1964, developed a theory for nonlinear signal processing referred to as homomorphic systems. The use of such systems for signal processing was termed homomorphic filtering.

The essential idea of homomorphic system theory was that many signal processing operations satisfy the same algebraic postulates as addition. Therefore, homomorphic mappings between signal spaces in which these other operations play the role of signal (vector) addition are, in essence, linear mappings in a generalized sense. This suggested a new approach to a variety of problems in separating signals that had been nonadditively combined, such as through convolution or multiplication. Various potential applications of homomorphic signal separation were actively considered, primarily for deconvolution and demultiplication.

The first step in the development of homomorphic filtering for deconvolution was to work out both the basic theory and ways to implement homomorphic systems for convolution. It was shown in Al Oppenheim's Ph.D. dissertation that all homomorphic systems have a canonic representation consisting of a cascade of three systems. The first system is an invertible nonlinear

Theirs is a historical collaboration. It started in 1965, it continues to-date, and it is planned to continue for at least another 40 years. From this collaboration resulted the coauthored books *Digital Signal Processing* (1975), *Discrete-Time Signal Processing* (1989, 1999), *Computer-Based Exercises for Signal Processing Using MATLAB 5* (1998), and numerous research articles, all of which testify to the creative enthusiasm and challenging work of their authors. Their numerous research and teaching awards, including the IEEE Education Medal received by each clearly acknowledge their great love and talent for both teaching and research. The quiz is easy, Dear Reader, and you may have guessed the answer right away. Our guests today, in a double feature of the "DSP History" column, are Dr. Al Oppenheim and Dr. Ron Schafer.

Al Oppenheim was born on 11 November 1937, in New York City. He obtained his S.B. (1961), S.M. (1961), and Sc.D. (1964) degrees from the Massachusetts Institute of Technology (MIT), Cambridge. He has been with the Department of Electrical Engineering and Computer Science at MIT for his entire academic career, with a two-year leave of absence at MIT Lincoln Laboratory (1967–1969) and two years part time as an associate division head at MIT Lincoln Laboratory (1978–1980). Dr. Oppenheim's research focuses on algorithms for signal processing motivated by applications in speech processing, image processing, acoustics, communications, and more recently drifting toward biology. In addition to the books mentioned earlier, he also coauthored: *Signals and Systems* (1982, 1997), authored two widely used video lecture series, and edited and coedited other books. Dr. Oppenheim is the recipient of numerous awards for excellence, including the IEEE Third Millenium Medal (2000), the IEEE Centennial Medal (1984), the IEEE Education Medal (1988), and election to the National Academy of Engineering (1987). In collaborators, he appreciates "creative, out-of-the-box thinking, willingness to assume that any idea is a potentially good idea, ability to listen and be open-minded, but also to challenge in a friendly and constructive way." His hobbies include a variety of sports pursued intensely. Over the years these have included skiing, squash, tennis, offshore sailing and racing, windsurfing, biking, and flying. His current nonprofessional passions are magic (illusions), windsurfing, and flying (he received his private pilot's license in 2003). He is a playful soul, even more so on St. Patrick's day, when (in true Irish spirit), his friends and colleagues call him "O'Ppenheim."

Ron Schafer was born on 17 February 1938 in Tecumseh, Nebraska. He obtained the B.S.E.E. (1961) and M.S.E.E. (1962) degrees from the University of Nebraska, Lincoln, and the Ph.D. degree (1968) from MIT. After a career-defining six years with Bell Telephone Laboratories (1968–1974), he joined the Georgia Institute of Technology (Georgia Tech) in 1974 as John and Marilu McCarty Professor. In 2004, he retired from Georgia Tech as Professor Emeritus and took a position as distinguished technologist at Hewlett-Packard Laboratories in Palo Alto, California. Ron's work has focused on speech processing, image processing, and applications of DSP to medicine and biology. In addition to the books mentioned earlier, he also coauthored *Digital Processing of Speech Signals* (1978), *DSP First: A Multimedia Approach* (1998), and *Signal Processing First* (2003). He is the recipient of several awards for excellence, including the IEEE Emanuel R. Piore Award (1980), the IEEE Education Medal (1992), the IEEE Signal Processing Society Education Award (2000), and election to the National Academy of Engineering (1992). In addition to his almost four-decade collaboration with Al, he has had long, fruitful, and greatly valued collaborations over many years with Larry Rabiner, Tom Barnwell, Russ Mersereau, and Jim McClellan. Ron appreciates in collaborators their ability to push and motivate him and their tolerance of his tendency to procrastinate. To his own children and a multitude of students, he has often quoted an instruction from the MIT catalog of 1963: "A student is expected to study at least three hours outside of class for every hour in the classroom." To his chagrin, this suggestion was not generally met with an enthusiastic response. Dr. Schafer's hobbies include photography, fly fishing, reading, walking, and volunteering.

We invite you to follow Dr. Al Oppenheim and Dr. Ron Schafer in their pursuit of reconstructing the captivating history of the cepstrum. Spell-checkers are not recommended...

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characteristic operator (system) that maps a nonadditive combination operation such as convolution into ordinary addition. The second system is a linear system obeying additive superposition, and the third system is the inverse of the first nonlinear system. Thus, for signals combined by convolution, a homomorphic deconvolution system maps convolution into addition, then addition into addition, and finally addition into convolution. For convolution (denoted $*$), the characteristic operator $D_*[\ ]$ has the property that $D_*[x_1 * x_2] = \hat{x}_1 + \hat{x}_2$, where $D_*[x_1] = \hat{x}_1$ and $D_*[x_2] = \hat{x}_2$. One intuitively appealing approach to the implementation of the nonlinear mapping $D_*[\ ]$ is through the complex-valued logarithm of the complex-valued Fourier transform. That is, if two signals are convolved, their

Fourier transforms are multiplied, and a suitably defined complex logarithm will produce the sum of two log Fourier transforms. The inverse Fourier transform of a sum is the sum of the individual inverse transforms, so the cascade of — Fourier transform → complex logarithm → inverse Fourier transform — maps convolution into a sum of corresponding signals. Interest in this implementation fortunately coincided nicely with the publication of the 1965 paper by Cooley and Tukey [3] on efficient implementation of the Fourier transform on digital computers, i.e., the fast Fourier transform (FFT). The FFT provided an efficient means of implementing the Fourier transform computations needed to implement the nonlinear mapping $D_*[\ ]$.

It was a fortuitous discussion in 1965 between Jim Flanagan of Bell Telephone Laboratories and Al Oppenheim that connected the work going on at MIT to the development of the cepstrum at Bell Laboratories. After hearing about homomorphic deconvolution from Oppenheim, Flanagan noted that the characteristic system for homomorphic convolution was reminiscent of the spectrum of the log of the spectrum (i.e., the cepstrum) as proposed by Bogert et al. Furthermore, he suggested looking at work by Michael Noll [4], [5] in the *Journal of Acoustical Society of America*. Noll credits Manfred Schroeder (who was aware of the work of Bogert et. al.) with suggesting to him that it might be interesting to apply cepstrum analysis on a short-time basis to speech signals. In the *Journal of Acoustical Society of America* papers, Noll applied the cepstrum as a basis for pitch detection. The problem of pitch detection is very similar to detecting echo times in the sense that the basic speech model consists of representing speech as the convolution of the

vocal tract impulse response with the quasi-periodic train of glottal pulses. The basic idea of cepstral pitch detection is illustrated in Figures 1 and 2.

The success of cepstral pitch detection suggested to us the strong potential of applying homomorphic deconvolution to deconvolve speech, i.e., to separate the vocal tract impulse response, the glottal pulse shape, and the periodic excitation. Bogert et al. had introduced the concept of *liftering*, i.e., linear filtering of the log spectrum, as a way of emphasizing the periodic component of the log spectrum so as to enhance the detectability of echos [1]. The concept of homomorphic filtering clearly suggested how liftering could be used to actually separate the various convolutional components, such as separating the vocal tract filter response from the periodic excitation spectrum. By applying a low-pass lifter to the cepstrum in Figure 2 to extract the low quefrency components below the first rahmonic peak, the slowly varying curve (in red, upper graph) results. The low quefrency components thus correspond to the resonance structure of the vocal tract and high frequency falloff of the speech spectrum due to the glottal pulse.

This example shows that the general concept of homomorphic deconvolution is fundamentally different from echo or pitch detection in a number of ways. One, of course, is that the ultimate objective in homomorphic deconvolution is signal separation and recovery rather than detection; i.e., the nonlinear mapping is followed by linear filtering and subsequently by the inverse of the nonlinear mapping. While the spectral density as used by Bogert et al. loses phase information, the objective of homomorphic deconvolution requires that phase information be retained or reconstructed. Consequently, in homomorphic deconvolution the complex-valued Fourier transform and complex logarithm must in general be used. To retain both the relationship to the work of Bogert at al. and the distinction, the term *complex cepstrum* was eventually applied to the nonlinear mapping in homomorphic deconvolution, and the term power cepstrum or real cepstrum used for the cepstrum as originally defined by Bogert et al.

Shortly after Al Oppenheim joined the MIT faculty and became aware of the work in [1] and [4], he and Ron Schafer were introduced to each other by Tom Stockham, a
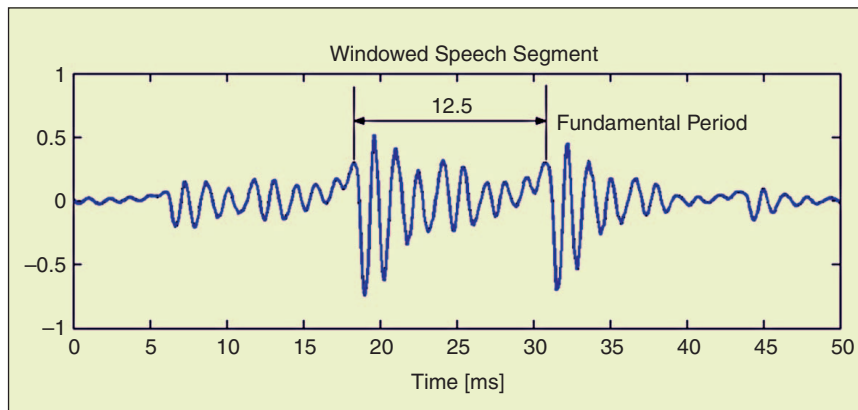
▲ 1. A segment of voiced speech with a Hamming window during a voiced (vowel-like) time interval of 50 ms. Short-time analysis of speech involves the analysis of a succession of such segments taken sequentially in time. At this particular time in a vowel sound, the pitch period is approximately 12.5 ms.
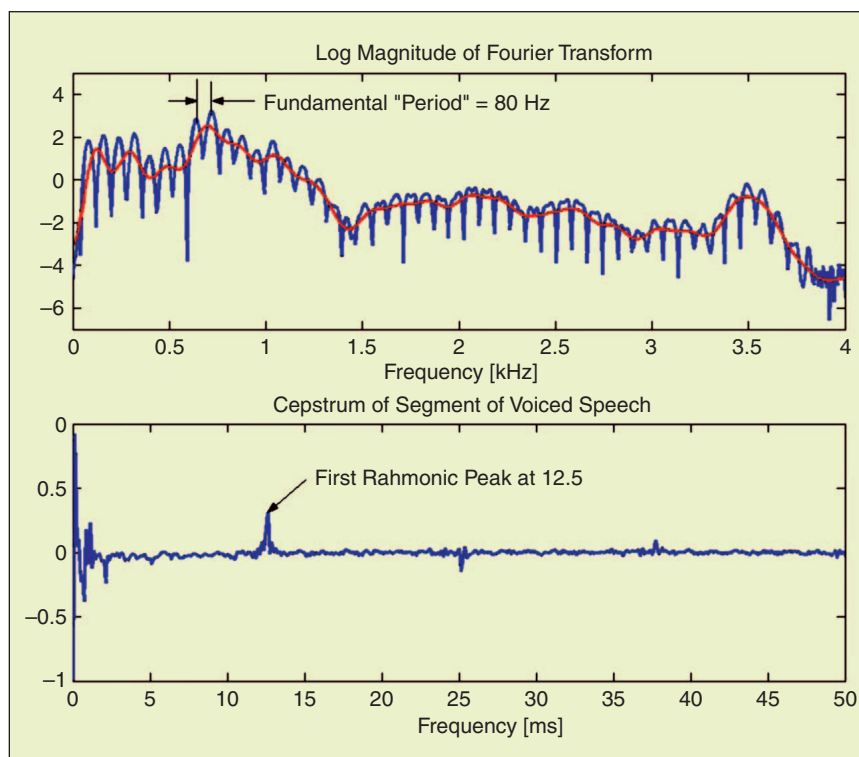
▲ *2. Log magnitude (in blue, upper graph), lowpass liftered (linearly filtered) log magnitude (in red, upper graph), and cepstrum (in blue, bottom graph) of the segment of voiced speech illustrated in Figure 1. The rapidly varying curve in the upper graph is the log magnitude of the discrete-time Fourier transform of the segment of speech in Figure 1. The lower plot is the cepstrum, i.e., the inverse discrete Fourier transform of the log magnitude in the upper plot. Note the peaks at "rahmonics" of 1/80 = 12.5 ms, the fundamental quefrency of the quasi-periodic ripples in the upper graph. As can be seen by comparing the speech segment in Figure 1 to this cepstrum, this fundamental quefrency is also the period (pitch period) of the time waveform. The search for such peaks in the cepstrum was the basis for Noll's pitch detection algorithm.*

faculty member at MIT and a member of Oppenheim's dissertation committee. At that time, Ron Schafer was a graduate student in search of an interesting doctoral dissertation topic. Thus began a close and wonderfully productive collaboration between a young professor and an enthusiastic graduate student, exploring and developing the theoretical concepts, techniques, and applications of the complex cepstrum. The resulting doctoral dissertation "Echo Removal by Discrete Generalized Linear Filtering" [6] by Ron Schafer at MIT in 1968, focused on the issues of the dis-crete-time formulation of the complex cepstrum, phase computation, recursion relations, and applications to echo removal from speech. In our development of the complex cepstrum, a variety of alternate implementations of the complex cepstrum and relationships between the power cepstrum and complex cepstrum were developed, both in general and for minimum-phase, maximum-phase and all-pass sequences. The work also led to the interpretation in the cepstral domain of the Hilbert transform relationship between Fourier transform magnitude and phase for minimum phase signals.

The early work on the application of homomorphic filtering to deconvolution and demultiplication and consequently on the extension of the power cepstrum to the complex cepstrum, was described in the *Proceedings of the IEEE* [7] and also developed and described more extensively in Chapter 10 of our first jointly authored textbook [8] published in 1975. As an interesting side note, throughout the various stages of proofreading of this book, we constantly had to maintain vigilance to be certain that this "strange" term *cepstrum* wasn't inadvertently "corrected" to what seemed to be more appropriate. (Our text editing program also continues to complain as we write this article). We breathed a sigh of relief when the last page proofs were returned to the publisher. When the first printing of the book appeared, it was clear that a particularly diligent proofreader at the publisher had caught the "error" at the last instant and *cepstrum* had been reversed to *spectrum* throughout. This unusual term has caused linguistic difficulties in other ways. Since it was meant as a paraphrase of the term spectrum, its natural pronunciation would be *kepstrum*. On the other hand, based on its spelling the natural pronunciation would be *sepstrum*, and there have been lively debates on which should be used. The adopted pronunciation among those aware of the origin is *kepstrum*, and in fact some authors have chosen to spell it as *kepstrum* to make the pronunciation clear.

Applications of the cepstrum, complex cepstrum, and homomorphic deconvolution have been explored in a variety of areas including audio processing, speech processing, geophysics, radar, medical imaging, and others. Early in the development of the concept of homomorphic filtering, Tom Stockham had shown particular

interest in exploring potential applications to both demultiplication and deconvolution. The work on homomorphic demultiplication led to novel methods for dynamic range compression, as disclosed in a patent by Stockham and Oppenheim [9], and signal enhancement for both audio and images, the latter of which was discussed by Stockham in [10]. An early and very novel application of the cepstrum to deconvolution was the work by Stockham et al. [11] directed at blind deconvolution such as the restoration of old phonograph recordings. They focused in particular on restoration of recordings by Enrico Caruso. The objective in the restoration of these old recordings was to compensate for the undesireable frequency response of the "morning glory" recording horn used in that era. Since the recording horn was retuned daily and since its frequency response was typically subject to temperature and humidity variations, the compensation could not simply be based on modern day measurements of an archived recording horn, i.e., the spectral characteristics would have to be estimated from the recording itself. Stockham's essential idea in the Caruso restoration was to use log spectral averaging (essentially cepstral averaging) and subtraction of the log spectral average of a modern recording of the same arias. The result was then used as an estimate of the log spectrum of the recording horn which was to be compensated for. This idea of log spectrum, or equivalently, cepstrum averaging has been useful in many other applications for estimating and compensating for a convolutional distortion.

Also in the context of audio processing, speech processing developed as a major application area for the cepstrum, both in the form of the power cepstrum and the complex cepstrum. For example, the

cepstral pitch detector utilizing the power cepstrum became widely used. Homomorphic deconvolution was also successfully applied by us and others in the late 1960s to separate the glottal excitation and the vocal tract impulse response for speech modeling in general and for speech compression in particular.

The cepstrum also plays a significant role in many speech recognition systems. Specifically, the cepstral coefficients have been found empirically to be a more robust, reliable feature set for speech recognition and speaker identification than linear predictive coding (LPC) coefficients or other equivalent parameter sets. Cepstral parameter extraction in speech recognizers is, in some instances, based on converting LPC parameters to cepstral coefficients by utilizing the recursion relationship [7], [8] that we developed in our early research to obtain the complex cepstrum of minimum-phase signals without the need for explicit computation of the Fourier transform or phase unwrapping. An alternative use of the cepstrum in speech recognition is the mel-frequency cepstrum [12]. The mel-frequency cepstrum is based on calculating the cepstrum from the logarithm of the spectrum obtained from a filter bank with center frequencies and bandwidths determined by a constant mel-frequency interval. Cepstral averaging and time-differencing of cepstra are now standard methods of removing the effects of linear filtering prior to recognition.

While speech processing has been one of the more successful applications of cepstral analysis and homomorphic deconvolution, applications in other areas have been explored with considerable success. As mentioned previously, detection of echoes in seismic signals was the motivating application in the original work of Bogert et. al. Following the

publication of [7], there was considerable interest in exploring homomorphic deconvolution in the context of seismology and exploration geophysics. For example, the complex cepstrum was used in the early 1970s in determining seismic wavelets and in deconvolving seismic traces [13], [14].

There are many other innovations and applications that can be traced to the cepstrum and its subsequent development, but space does not permit an exhaustive discussion or bibliography. This is not surprising in light of the ubiquity of convolution as a model for what goes on in our physical world. It is almost certain that we have not seen the end of new ideas and new applications of cepstrum analysis and homomorphic filtering. For example, our original theoretical development showed that the complex cepstrum of a signal with a rational z-transform could be obtained in terms of the roots of the numerator and denominator [8]. This avoids the issue of phase computation completely. Steiglitz and Dickinson demonstrated this in [15] for polynomial z-transforms of only moderate length. However, a recent article [16] by Sitton, Burrus, Fox, and Treitel in *IEEE Signal Processing Magazine* has shown that polynomials of order up to 1 million can be accurately rooted using methods based on the FFT. At the time the complex cepstrum was introduced and developed, factoring such large polynomials was not even considered. With such powerful new tools it may be time to take another look at the complex cepstrum in the context of new applications.

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dsp history *continued from page 99*

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