proj2_c

October 5, 2016

```
In [1]: %pylab inline
```

```
Populating the interactive namespace from numpy and matplotlib
```

```
In [2]: import matplotlib.pyplot as plt
```

1 Application

In the third and final portion of project 2 I applied what I learned in part 2 of newtons method to the nonlinear equation

$$e \sin(w) - w = t$$

where $e = \sqrt{1 - \frac{b^2}{a^2}}$, t is time and w is the angle around the orbit. To use Newton's root finding method I simplified the equation to

$$e \sin(w) - w - t = 0$$

and e becomes $\sqrt{1 - \frac{1.25^2}{2.0^2}}$. To solve the function for $f(w)$ some additional functionality must be added to the Kepler class equation. Time must be changed for each separate running of newtons method without time being passed as a parameter. I solved this issue with an array of the values and a counter inside the Kepler equation class. After each successful completion of newtons method. I incremented the time counter by one and found the next time to be used. To start the newtons method solve I used $w = 0$ for the first iteration and used the resultant w for the subsequent newton's method calls. Once w was found from the equation

$$e \sin(w) - w - t_i = 0$$

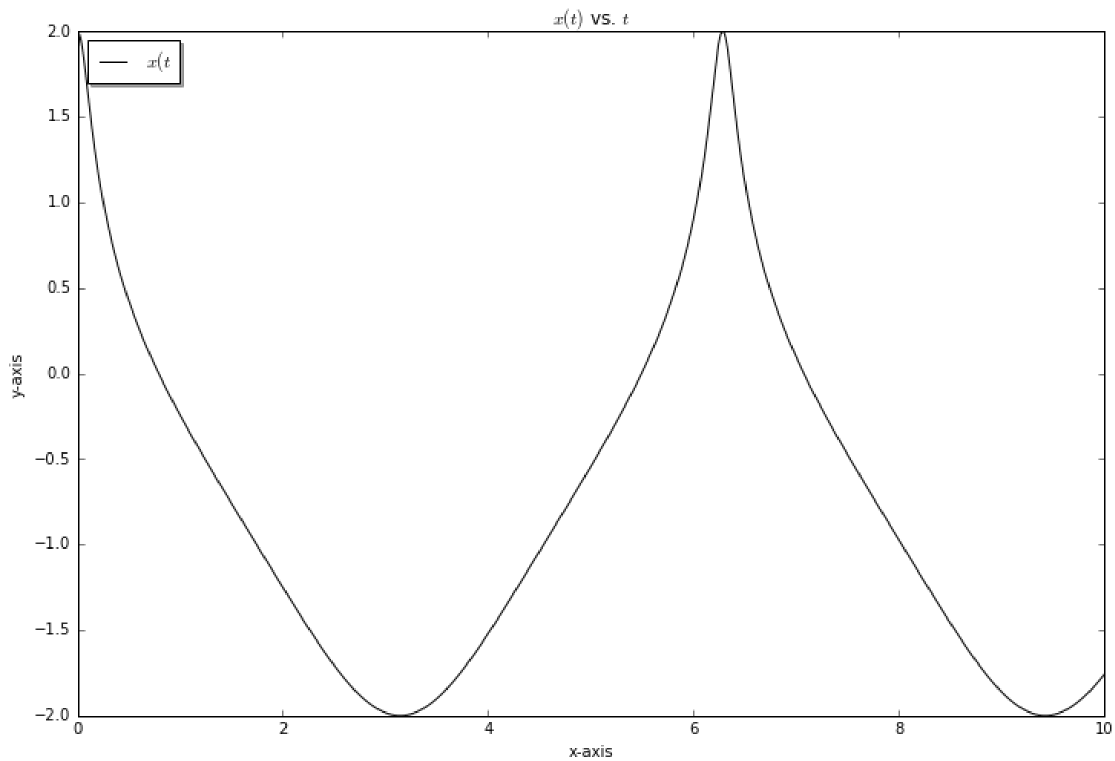
I was able to find the radial position of the object with the function

$$r(w) = \frac{ab}{\sqrt{(b\cos(w))^2 + (a\sin(w))^2}}$$

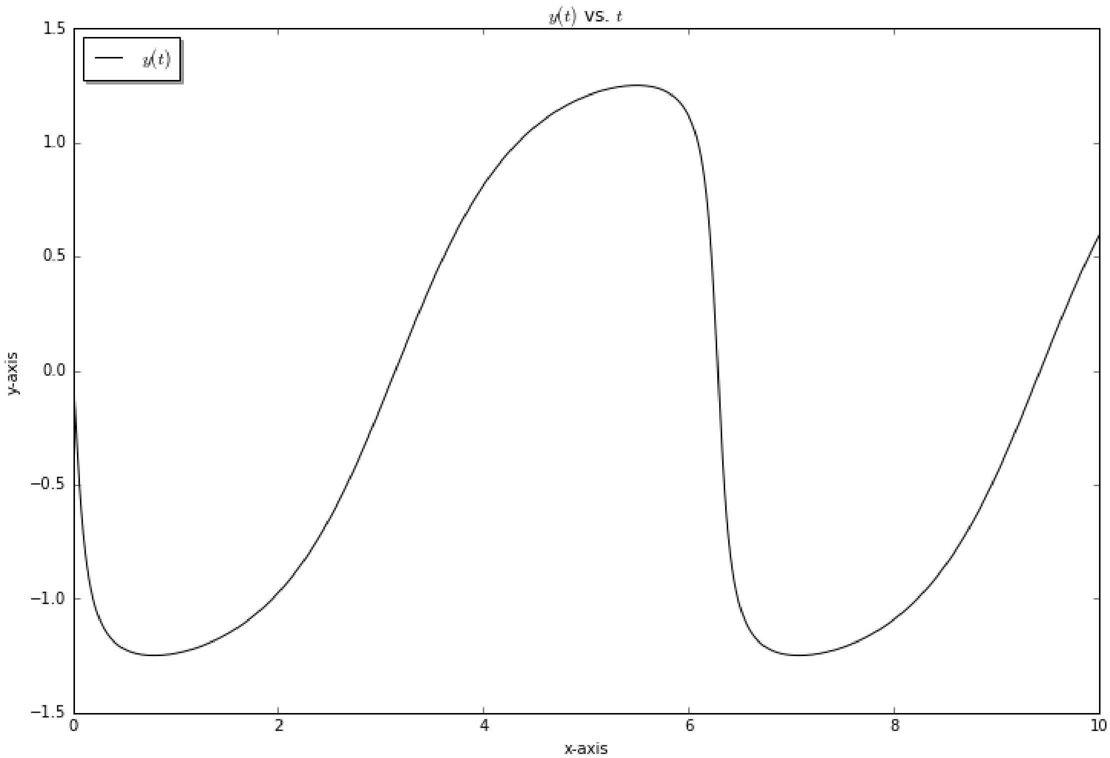
Finally I was able to find the Cartesian coordinates of the object at time t with the functions $x(t) = r\cos(w)$ and $y(t) = r\sin(2)$. Finally I loaded the t value and the (x,y) coordinate into respective files named **t.txt**, **x.txt**, **y.txt**.

```
In [3]: t = open("t.txt").read();
        t = t.split("\n");
        t.remove("");

        x = open("x.txt").read();
        x = x.split("\n");
        x.remove("");

        y = open("y.txt").read();
        y = y.split("\n");
        y.remove("");
```

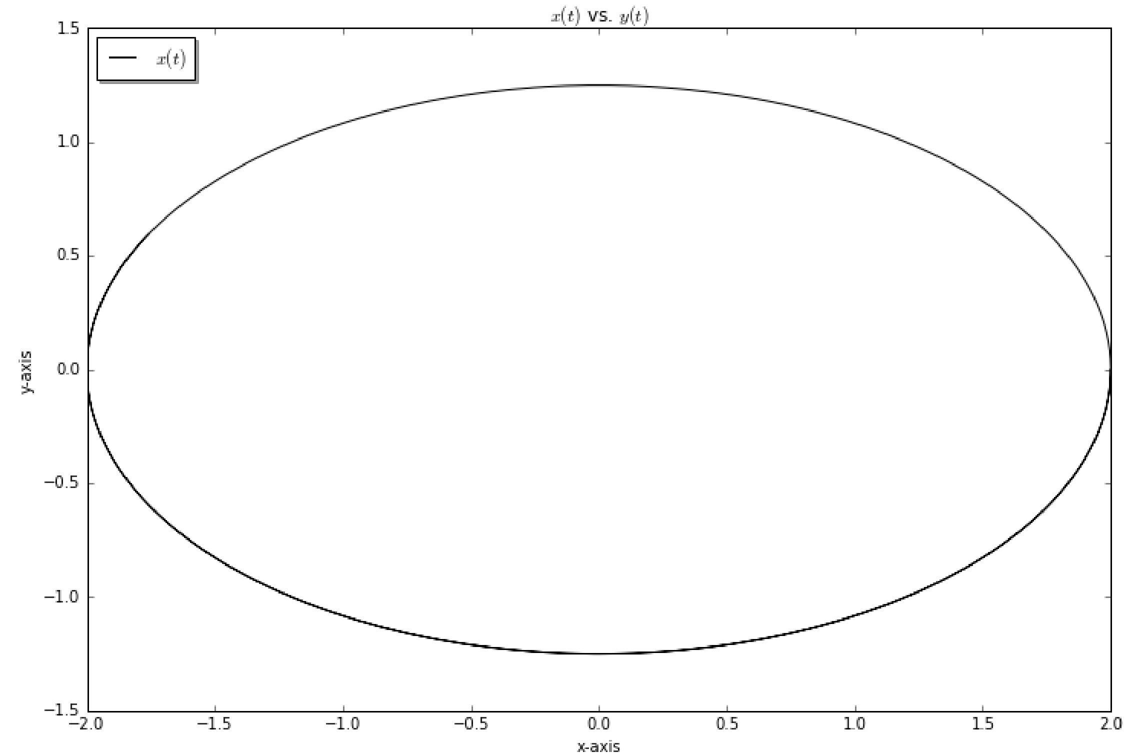
```
In [7]: fig = plt.figure(figsize=(12,8))
        ax = fig.add_subplot(111)

        ax.plot(t,x,"k-", label="$x(t$")
        ax.set_title("$x(t)$ vs. $t$")
        ax.set_xlabel("x-axis")
        ax.set_ylabel("y-axis")

        legendPlot1 = ax.legend(loc="upper left", shadow=True)

        plt.show()
```



In [8]:
```
fig = plt.figure(figsize=(12,8))
ax = fig.add_subplot(111)

ax.plot(t,y,"k-", label="$y(t)$")
ax.set_title("$y(t)$ vs. $t$")
ax.set_xlabel("x-axis")
ax.set_ylabel("y-axis")

legendPlot1 = ax.legend(loc="upper left", shadow=True)

plt.show()
```



In [9]:
```
fig = plt.figure(figsize=(12,8))
ax = fig.add_subplot(111)

ax.plot(x,y,"k-", label="$x(t)$")
ax.set_title("$x(t)$ vs. $y(t)$")
ax.set_xlabel("x-axis")
ax.set_ylabel("y-axis")

legendPlot1 = ax.legend(loc="upper left", shadow=True)

plt.show()
```



When our Cartesian points are graphed in relation to each other (not in relation to time) the orbit of our object becomes visible. This really shows the power of computational science because thousands of equations were solved to generate the coordinates for this graph and without optimizations and newton's quadratic convergence the solution would be much more difficult to compute.

```
In [10]: %cat kepler.cpp

/* Project - Project 2_Part c
 * Prof - Dr Xu
 * Name - Jake Rowland
 * Date - 10/6/16
 * Purpuse - Use Newton's method of root finding for orbital path prediction
*/

#include <cmath>
#include <iostream>
#include <fstream>

#include "fcn.hpp"
#include "newton.cpp"

//Create the function f(x) = e * sin(w) - w - t
class Kepler: public Fcn
```

```
{
private:
        //define variables needed for each iteration of f(x)
        double t[10001];
        double e;
        int run;

        //Function to find all values of t
        void defT()
        {
                std::ofstream tOut("t.txt", std::ios::out);
                for(int i = 0; i < 10001; i++)
                {
                        t[i] = (double)i / 1000;
                        tOut << t[i] << "\n";
                }
        }

public:
        //Constructor to create constant e
        Kepler(double a, double b)
        {
                e = sqrt(1 - (pow(b,2)/pow(a,2)));
                run = 0;
                defT();
        }

        //Operation that calculates the fuction f(x)
        double operator()(double w)
        {
                return e * sin(w) - w - t[run];
        }

        //Increment run for next run
        void runIncr()
        {
                run = run + 1;
        }
};

//Derivative of f(x)
class KeplerDerv: public Fcn
{
private:
        //Definition of e for f'(x)
        double e;

public:
```

```
        //Constructor which defines e
        KeplerDerv(double a, double b)
        {
                e = sqrt(1 - (pow(b,2)/pow(a,2)));
        }

        //Operation that calculates f'(x)
        double operator()(double w)
        {
                return e * cos(w) - 1;
        }
};

int main()
{
        //Define a and b
        double a = 2.0;
        double b = 1.25;

        //Define the two values
        Kepler f(a,b);
        KeplerDerv fd(a, b);

        //Define tolerance
        double tol = pow(10,-5);

        //Define x and y array
        double x[10001];
        double y[10001];

        //Define first guess
        double w = 0.0;

        //Define .txt writers
        std::ofstream xOut("x.txt", std::ios::out);
        std::ofstream yOut("y.txt", std::ios::out);

        //For all spots in x[] and y[]
        for(int i = 0; i < 10001; i++)
        {
                //Define w' as root of newton(w)
                double w = Newton::newton(f, fd, w, 6, tol, false);

                //Calculate r with w
                double r = (a*b) / (sqrt( pow((b * cos(w)),2) + pow((a * sin(w)),2) ));

                //Calc (x,y) cordinates with r and w
                x[i] = r * cos(w);
```

```
            y[i] = r * sin(w);

            //Write to file
            xOut << x[i] << "\n";
            yOut << y[i] << "\n";

            //Increment t in function f
            f.runIncr();
        }
}
```