

The runtimes of each iteration ran about as predicted. When using randomized quicksort, the act of deciding the pivot definitely added to the running times of the program. This can be seen by comparing $T(N)'''$ or $T(N)'$ to $T(N)$, where each of these runs theoretically has the same runtime $O(n \log n)$, however $T(N)$ sets the pivot to the last element whereas $T(N)'''$ and $T(N)'$ use random quicksort. Random quicksort requires a calculation for each time it is called, thus the run time is longer. However, those runtimes are still significantly smaller than $T(N)''$ for when N is greater than 1000. I chose .15 for the $c1$ constant and .1 for the $c2$ constant, to make the graph somewhat readable. All the runtimes were within the bounds of $O(n^2)$ and $O(n)$, with all but $T(N)''$ running at about $O(n \log n)$, as expected. $T(N)''$ ran at worst case and definitely pushed towards $O(n^2)$ run time.

The chart below was imported from Excel, all the run times are in microseconds. I could not get more precise measurements than microseconds so many of the $N = 10$ run I believe are rounded up, but that doesn't affect the data too much. Average run is the blue column.

	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	AVERAGE RUN
$T(N)$ 10	1	2	1	1	1	1.2
$T(N)$ 100	15	17	15	15	17	15.8
$T(N)$ 1000	231	202	223	225	233	222.8
$T(N)$ 10000	2955	3066	3201	3230	3125	3115.4
$T(N)'$ 10	9	9	9	11	11	9.8
$T(N)'$ 100	105	117	105	132	104	112.6
$T(N)'$ 1000	1247	1160	1295	1178	1190	1214
$T(N)'$ 10000	13191	12565	12620	12812	12682	12774
$T(N)''$ 10	1	2	1	1	1	1.2
$T(N)''$ 100	103	103	106	105	112	105.8
$T(N)''$ 1000	10632	10374	10213	10739	10684	10528.4
$T(N)''$ 10000	1047337	1036540	1039302	1044948	1027761	1039177.6
$T(N)'''$ 10	13	13	14	13	16	13.8
$T(N)'''$ 100	85	105	123	115	111	107.8
$T(N)'''$ 1000	967	1201	990	941	1226	1065
$T(N)'''$ 10000	8994	9057	9892	10681	12830	10290.8

n	$c1(N^2)$	$c1$	n	$c2(N)$	$c2$
10	15	0.15	10	1	0.1
100	1500	0.15	100	10	0.1
1000	150000	0.15	1000	100	0.1
10000	1500000	0.015	10000	1000	0.1

The graph below was created by putting data into Excel and graphing it. The run time is in microseconds along the y-axis and the number of elements is on the x-axis. $O(n^2)$ is the top bounding line for the graph, while $O(n)$ graphs the lower bound.

	N = 10	N = 100	N = 1000	N = 10000
$T(N)$	1.2	15.8	222.8	3115.4
$T(N)'$	9.8	112.6	1214	12774
$T(N)''$	1.2	105.8	10528.4	1039177.6
$T(N)'''$	13.8	107.8	1065	10290.8

RUN TIMES

run time in microseconds

- $O(n^2)$
- $O(n)$
- $T(n)$
- $T(n)'$
- $T(N)''$
- $T(n)'''$

	10	100	1000	10000
$O(n^2)$	15	1500	150000	1500000
$O(n)$	1	10	100	1000
$T(n)$	1.2	15.8	222.8	3115.4
$T(n)'$	9.8	112.6	1214	12774
$T(N)''$	1.2	105.8	10528.4	1039177.6
$T(n)'''$	13.8	107.8	1065	10290.8