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A New Detection Algorithm for OFDM System without Cyclic Prefix

Chunguang Wang, Zheng Zhou

College of Telecom Engineering, Beijing University of Posts and Telecommunications
Beijing China

Abstract-**In OFDM system, cyclic prefix (CP) is necessary to combat inter-symbol interference (ISI) and inter-carrier interference (ICI). Employing CP simplifies the channel equalization, but results in a considerable bandwidth loss. In this paper, a multi-carrier detection algorithm is proposed for OFDM system without CP, which implements the successive detection by decision-feedback equalizer (DFE) based on minimum mean-square error (MMSE) criterion. Simulation results show that the proposed scheme has higher bandwidth efficiency and better performance in bit error rate (BER).**

Keywords-**OFDM, decision feedback equalizer, successive detection**

I. INTRODUCTION

OFDM is an attractive technology for high-speed data transmission. In OFDM system, the entire bandwidth is partitioned into a large number of parallel and independent subchannels, which increase the symbol duration and reduces ISI. Therefore, OFDM is an effective technique for combating multipath fading [1].

Cyclic prefix (CP) is added in front of each symbol. If the CP length is longer than channel impulse response, the ISI and ICI will be eliminated. Although employing CP greatly simplifies receiver signal process, it also results in considerable loss in the efficiency of bandwidth utilization.

There have been several studies to increase the efficiency of bandwidth usage for the conventional OFDM system. The first is to increase the number of sub-carriers so that the proportion of the CP can be reduced. This method narrows the bandwidth of each sub-carrier and makes OFDM systems more sensitive to the frequency offset. Another way is to utilize time domain equalizer (TEQ), which shortens the effective channel to be less than the CP length [2]. But it still needs CP, and it isn't approximate for the wireless channel varying fast. Residual ISI cancellation (RISIC) method [3], which uses tail cancellation and cyclic restoration, is effective in combating ISI. This method doesn't outperform the traditional OFDM systems with enough CP.

In this paper, we propose a multi-carrier detection algorithm for OFDM system without CP. The proposed algorithm can efficiently eliminate ISI and ICI and the bandwidth efficiency is greatly increased. It first removes ISI from the previous symbol and then detects the signals of subcarriers successively, which is analogous to the successive interference cancellation (SIC) for multiuser detection in DS-CDMA [9]. The already available decisions are used as feedback to eliminate their contribution to the ICI. Different from the successive detection proposed in [6], which detects the signals by iteration, our algorithm implements the successive detection by DFE. The feedback matrix can be obtained by Cholesky decomposition of a particular matrix.

The rest of this paper is organized as follows: Section 2 gives the system model. Section 3 gives the analysis of ISI and ICI caused by the absence of CP. In Section 4, the successive detection algorithm based on decision feedback for OFDM system without CP is proposed. Simulation results and the performance analysis are given in Section 5 and the conclusion is drawn in Section 6.

II. SYSTEM MODEL

In the OFDM transmitter, the data sequence is partitioned into a number of parallel streams. Inverse Fast Fourier Transform (IFFT) is used as an efficient means of implementing the multicarrier modulation [1]:

$$x_{k,n} = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} s_{k,i} e^{j2\pi i n/N} \quad n = 0,1,\cdots,N-1 \tag{1}$$

where $x_{k,n}$ stands for the nth sample in the kth OFDM symbol, $s_{k,i}$ is the transmitted complex data on the ith subcarrier during kth symbol period, N is the size of DFT.

Let $\mathbf{x}(k)$ denote the kth OFDM symbol $\mathbf{x}(k) = [x_{k,0}, x_{k,1} \cdots x_{k,N-1}]^T$, and $\mathbf{s}(k)$ denote $\mathbf{s}(k) = [s_{k,0}, s_{k,1} \cdots s_{k,N-1}]^T$. After the IFFT, the last m samples of each block are added to the start of the block as cyclic prefix (CP).

The OFDM symbols are transmitted through the multipath fading channel corrupted by noise sequence $\mathbf{v}(k)$, which is additive white Gaussian noise (AWGN) with the variance σ^2. And we assume that the noise is independent from the source sequence $\mathbf{s}(k)$. The channel impulse response is denoted as $\mathbf{h} = [h_0, h_1, \cdots, h_L]^T$, L is the channel order.

At the receiver, CP is removed from the received signal and the result is passed through a serial-to-parallel converter to obtain: $\mathbf{r}(k) = [r_{k,0}, \mathrm{r}_{k,1} \cdots \mathrm{r}_{k,N-1}]^T$. DFT of $\mathbf{r}(k)$ is computed and the result $\mathbf{u}(k)$ is represented by: $\mathbf{u}(k) = [u_{k,0}, \mathrm{u}_{k,1} \cdots \mathrm{u}_{k,N-1}]^T$. Let Q represent the DFT matrix, $\mathbf{Q}^*$ is the IDFT matrix, where $*$ denotes complex conjugate, so:

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$$\mathbf{x}(k) = \mathbf{Q}^{*}\mathbf{s}(k) \tag{2}$$

$$\mathbf{u}(k) = \mathbf{Q}\mathbf{r}(k) \tag{3}$$

III. ANALYSIS OF ISI AND ICI

If we assume that $N >> L$, as true in practice, the ISI comes from only one previous OFDM symbol. We can infer that $\mathbf{r}(k)$ can be expressed in matrix form as follows [4]:

$$\mathbf{r}(k) = (\mathbf{H}_0 - \mathbf{H}_1)\mathbf{x}(k) + \mathbf{H}_2\mathbf{x}(k-1) + \mathbf{v}(k) \tag{4}$$

where

$$\mathbf{H}_0 = \begin{bmatrix} h_0 & 0 & \cdots & h_L & \cdots & \cdots & h_2 & h_1 \\ h_1 & h_0 & \cdots & 0 & h_L & \cdots & h_3 & h_2 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \cdots & \vdots \\ h_{L-1} & h_{L-2} & \ddots & h_0 & 0 & \ddots & & h_L \\ h_L & h_{L-1} & \ddots & h_1 & h_0 & \ddots & & 0 \\ \vdots & \ddots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & \cdots & & h_L & h_{L-1} & \cdots & h_1 & h_0 \end{bmatrix} \tag{5}$$

$$\mathbf{H}_1 = \begin{bmatrix} 0 & \cdots & h_L & h_{L-1} & \cdots & h_{m+1} & \overbrace{0 \; \cdots \; 0}^{m} \\ 0 & \cdots & 0 & h_L & \cdots & h_{m+2} & 0 \; \cdots \; 0 \\ \vdots & \cdots & \vdots & \ddots & \ddots & \ddots & \vdots \; \cdots \; \vdots \\ 0 & \cdots & 0 & 0 & \ddots & h_L & 0 \; \cdots \; 0 \\ \vdots & \cdots & \vdots & \ddots & \ddots & \ddots & \vdots \; \cdots \; \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \; \cdots \; \vdots \end{bmatrix} \tag{6}$$

$$\mathbf{H}_2 = \begin{bmatrix} 0 & \cdots & 0 & h_L & h_{L-1} & \cdots & h_{m+1} \\ 0 & \cdots & 0 & 0 & \cdots h_L & \cdots & h_m \\ \vdots & \cdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \ddots & \ddots & 0 & h_L \\ \vdots & \cdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots \; 0 \end{bmatrix} \tag{7}$$

For conventional OFDM system, the length of the cyclic prefix m is larger than the channel order L. In this case, both $\mathbf{H}_1$ and $\mathbf{H}_2$ are zero and:

$$\mathbf{u}(k) = \mathbf{Q}\mathbf{H}_0\mathbf{Q}^*\mathbf{s}(k) + \mathbf{V}(k) \tag{8}$$

where $\mathbf{V}(k)$ is the Fourier Transform of noise $\mathbf{v}(k)$.

Since $\mathbf{H}_0$ is a circulant matrix, $\mathbf{\Lambda} = \mathbf{Q}\mathbf{H}_0\mathbf{Q}^*$ is a $N \times N$ diagonal matrix, so

$$\mathbf{u}(k) = \mathbf{\Lambda}\mathbf{s}(k) + \mathbf{V}(k) \tag{9}$$

As $\mathbf{\Lambda}$ is a diagonal matrix, (9) indicates that the equalization process in OFDM system only needs one-tap frequency domain equalizer (FEQ) to accomplish the channel compensation. However, this simplicity results in considerable loss in bandwidth efficiency. The bandwidth efficiency is measured by $\frac{N}{N+m}$, for a heavily disperse channel, a large m is required to make $\mathbf{H}_1$ and $\mathbf{H}_2$ zero. The large m will severely degrade the bandwidth efficiency.

In order to increase bandwidth efficiency, we consider the OFDM system without CP. In this case, both $\mathbf{H}_1$ and $\mathbf{H}_2$ are nonzero, ICI and ISI are present in the received data. After FFT demodulation, the signal becomes:

$$\begin{aligned} \mathbf{u}(k) &= \mathbf{Q}\mathbf{H}_0\,\mathbf{x}(k) - \mathbf{Q}\mathbf{H}_1\mathbf{x}(k) + \mathbf{Q}\mathbf{H}_2\mathbf{x}(k-1) + \mathbf{V}(k) \\ &= \mathbf{\Lambda}\mathbf{s}(k) - \mathbf{Q}\mathbf{H}_1\mathbf{Q}^*\mathbf{s}(k) + \mathbf{Q}\mathbf{H}_2\mathbf{Q}^*\mathbf{s}(k-1) + \mathbf{V}(k) \end{aligned} \tag{10}$$

The second and third terms of the right hand in (10) are the unwanted ICI and ISI respectively. The last component is the effect contributed by noise.

IV. MMSE-DF DETECTION ALGORITHM

From the above analysis, for OFDM system without CP, both ISI and ICI are present. In this section, we propose a multicarrier detection algorithm to eliminate the effect of ISI and ICI.

Equation (4) is written as:

$$\begin{aligned} \mathbf{r}(k) &= (\mathbf{H}_0 - \mathbf{H}_1)\mathbf{x}(k) + \mathbf{H}_2\mathbf{x}(k-1) + \mathbf{v}(k) \\ &= (\mathbf{H}_0 - \mathbf{H}_1)\mathbf{Q}^*\mathbf{s}(k) + \mathbf{H}_2\mathbf{Q}^*\mathbf{s}(k-1) + \mathbf{v}(k) \\ &= \mathbf{P}_0\mathbf{s}(k) + \mathbf{P}_1\mathbf{s}(k-1) + \mathbf{v}(k) \end{aligned} \tag{11}$$

where

$$\mathbf{P}_0 = (\mathbf{H}_0 - \mathbf{H}_1)\mathbf{Q}^* \tag{12}$$

$$\mathbf{P}_1 = \mathbf{H}_2\mathbf{Q}^* \tag{13}$$

Based on expression of (11), we can first remove ISI from the previous symbol and then compensate for the distortion of the current symbol caused by ICI. We assume that the decision of previous symbol $\hat{\mathbf{s}}(k-1)$ is correct, so subtracting the feedback part from the received symbol results the ISI-free signal:

$$\mathbf{y}(k) = \mathbf{r}(k) - \mathbf{P}_1\hat{\mathbf{s}}(k-1) = \mathbf{P}_0\mathbf{s}(k) + \mathbf{v}(k) \tag{14}$$

From (14), ISI from the previous symbol is eliminated from $\mathbf{r}(k)$, but $\mathbf{y}(k)$ still has the ICI and noise component.

In [7], Y. Sun proposed the linear MMSE equalization method to reduce ICI. The equalizer matrix is designed as:

$$\mathbf{G} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \lambda\mathbf{I})^{-1} \quad (15)$$

where $\mathbf{H}$ is channel matrix, $\lambda = \frac{1}{SNR}$, $\mathbf{I}$ is $N \times N$ identity matrix. This method detects all the subcarrier's data simultaneously, but it can only make a trade-off between ICI and noise.

In the following section, we propose the nonlinear multi-carrier detection (MCD) algorithm based on decision feedback, which is analogous to the successive interference cancellation (SIC) for multiuser detection in DS-CDMA [9]. Instead of detecting all the data simultaneously, the proposed method detects the data one-by-one, the already available decision is used as feedback to eliminate their contribution to the ICI.

The detection algorithm has a decision feedback structure. We represent the feedforward filter by $\mathbf{W}$ and represent the feedback filter by $\mathbf{B}$. The DFE structure is depicted in Fig. 1.

We assume that the successive detection order is from N to 1. For the OFDM symbol indexed by i, the detection procedure is described as follows: the Nth subcarrier's signal is detected first, then the estimate $\hat{\mathbf{s}}(N)$ is weighted by the last column of $\mathbf{B}$ and is subtracted from the ith symbol, that means the ICI caused by Nth subcarrier to other subcarriers is removed. Then the $(N-1)$th subcarrier's signal is recovered next, and the estimate $\hat{\mathbf{s}}(N-1)$ is weighted by the second last column of $\mathbf{B}$ and is subtracted. This procedure is carried out until all the subcarrier's signals have been detected. The already available decisions are used as feedback to eliminate their contribution to the ICI.

We can infer that if the detection order is from N to 1, the feedback matrix $\mathbf{B}$ should be upper triangular.

From Fig. 1, we can get:

$$\mathbf{z}(i) = \mathbf{W}\mathbf{y}(i) = \mathbf{W}\mathbf{P}_0\mathbf{s}(i) + \mathbf{W}\mathbf{v}(i) \quad (16)$$

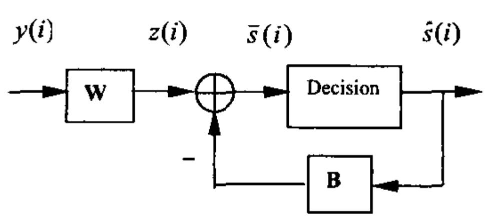

Fig. 1. The DFE structure

$$\bar{\mathbf{s}}(i) = \mathbf{z}(i) - \mathbf{B}\hat{\mathbf{s}}(i) \quad (17)$$

$$\hat{\mathbf{s}}(i) = Q(\bar{\mathbf{s}}(i)) \quad (18)$$

where $Q(\bullet)$ is the decision device.

Under the assumption that the previous decision is correct, we obtain:

$$\mathbf{e}(i) = \hat{\mathbf{s}}(i) - \mathbf{s}(i) = \mathbf{W}\mathbf{y}(i) - (\mathbf{B} + \mathbf{I}_{N\times N})\mathbf{s}(i) \quad (19)$$

Our object is to find the feedforward filter matrix $\mathbf{W}$ and the feedback matrix $\mathbf{B}$ based on the MMSE criterion, that is, select $\mathbf{W}$ and $\mathbf{B}$ to minimize the mean square error $E\left\{|\mathbf{e}(i)|^2\right\}$[5].

First, we assume that $\mathbf{B}$ is fixed and we obtain the matrix $\mathbf{W}$ which minimizes $E\left\{|\mathbf{e}(i)|^2\right\}$. Using the orthogonality principle, $\mathbf{e}(i)$ should be orthogonal to $\mathbf{y}(i)$ [8], which yields:

$$\begin{aligned} &E\left\{\mathbf{e}(i)\mathbf{y}^H(i)\right\} = \mathbf{0}_{N\times N} \\ &\Rightarrow \mathbf{W}E\left\{\mathbf{y}(i)\mathbf{y}^H(i)\right\} = (\mathbf{B} + \mathbf{I}_{N\times N})E\left\{\mathbf{s}(i)\mathbf{y}^H(i)\right\} \end{aligned} \quad (20)$$

According to (14) and the assumption that the additive noise is independent of the transmitted data, we obtain:

$$\begin{aligned} \mathbf{R}_{sy} &= E\left\{\mathbf{s}(i)\mathbf{y}^H(i)\right\} = E\left\{\mathbf{s}(i)[\mathbf{P}_0\mathbf{s}(i) + \mathbf{v}(i)]^H\right\} \\ &= \mathbf{R}_{ss}\mathbf{P}_0^H = \mathbf{R}_{ys}^H \end{aligned} \quad (21)$$

$$\mathbf{R}_{yy} = E\left\{\mathbf{y}(i)\mathbf{y}^H(i)\right\} = \mathbf{P}_0\mathbf{R}_{ss}\mathbf{P}_0^H + \mathbf{R}_{vv} \quad (22)$$

From (20), (21) and (22) we can infer that:

$$\begin{aligned} \mathbf{W} &= (\mathbf{B} + \mathbf{I}_{N\times N})\mathbf{R}_{sy}\mathbf{R}_{yy}^{-1} \\ &= (\mathbf{B} + \mathbf{I}_{N\times N})\mathbf{R}_{SS}\mathbf{P}_0^H(\mathbf{P}_0\mathbf{R}_{ss}\mathbf{P}_0^H + \mathbf{R}_{vv})^{-1} \end{aligned} \quad (23)$$

$\mathbf{e}(i)$ can be calculated using (19) and (23)

$$\mathbf{e}(i) = (\mathbf{B} + \mathbf{I}_{N\times N})\varphi(i) \quad (24)$$

where

$$\varphi(i) = \mathbf{R}_{sy}\mathbf{R}_{yy}^{-1}\mathbf{y}(i) - \mathbf{s}(i) \quad (25)$$

The autocorrelation of $\varphi(i)$ is:

$$\begin{aligned} \mathbf{R}_{\varphi\varphi} &= E\left\{[\mathbf{R}_{sy}\mathbf{R}_{yy}^{-1}\mathbf{y}(i) - \mathbf{s}(i)][\mathbf{R}_{sy}\mathbf{R}_{yy}^{-1}\mathbf{y}(i) - \mathbf{s}(i)]^H\right\} \\ &= \mathbf{R}_{ss} - \mathbf{R}_{sy}\mathbf{R}_{yy}^{-1}\mathbf{R}_{ys} \\ &= \mathbf{R}_{ss} - \mathbf{R}_{ss}\mathbf{P}_0^H(\mathbf{P}_0\mathbf{R}_{ss}\mathbf{P}_0^H + \mathbf{R}_{vv})^{-1}\mathbf{P}_0\mathbf{R}_{ss} \end{aligned} \quad (26)$$

Thorough the matrix inversion lemma [8]:

$$(\mathbf{A} - \mathbf{C}\mathbf{B}^{-1}\mathbf{D})^{-1} = \mathbf{A}^{-1} + \mathbf{A}^{-1}\mathbf{C}(\mathbf{B} - \mathbf{D}\mathbf{A}^{-1}\mathbf{C})^{-1}\mathbf{D}\mathbf{A}^{-1} \quad (27)$$

we obtain:

$$\mathbf{R}_{\varphi\varphi} = (\mathbf{R}_{ss}^{-1} + \mathbf{P}_0{}^H \mathbf{R}_{vv}^{-1} \mathbf{P}_0)^{-1} \quad (28)$$

Thus the covariance of $\mathbf{e}(i)$ is given by:

$$\mathbf{R}_{ee} = E\{\mathbf{e}(i)\mathbf{e}^H(i)\} = (\mathbf{B} + \mathbf{I}_{N\times N})\mathbf{R}_{\varphi\varphi}(\mathbf{B} + \mathbf{I}_{N\times N})^H$$
$$= (\mathbf{B} + \mathbf{I}_{N\times N})(\mathbf{R}_{ss}^{-1} + \mathbf{P}_0{}^H \mathbf{R}_{vv}^{-1}\mathbf{P}_0)^{-1}(\mathbf{B} + \mathbf{I}_{N\times N})^H \quad (29)$$

As $E\{|\mathbf{e}(i)|^2\} = tr\{\mathbf{R}_{ee}\}$, the minimization of MSE is equal to minimizing $tr\{\mathbf{R}_{ee}\}$ under the constraint that $\mathbf{B} + \mathbf{I}_{N\times N}$ is upper triangular with unit diagonal. Consider the Cholesky factorization of $\mathbf{R}_{\varphi\varphi}$:

$$\mathbf{R}_{ss}^{-1} + \mathbf{P}_0{}^H R_{vv}^{-1} \mathbf{P}_0 = \mathbf{U}^H \mathbf{D}\mathbf{U} \quad (30)$$

where $\mathbf{U}$ is a upper triangular with unit diagonal.

By setting

$$\mathbf{B} = \mathbf{U} - \mathbf{I}_{N\times N} \quad (31)$$

we get the feedback matrix $\mathbf{B}$. Substitute $\mathbf{B}$ into (23), we can get the feedforward filter $\mathbf{W}$.

V. SIMULATION RESULTS

In this section, we give the simulation of the proposed method and compare its performance with the conventional OFDM with sufficient CP and linear MMSE method.

The simulation parameters are as following: the modulation scheme used for this simulation is 16-QAM. 64 sub-carriers are used for each OFDM symbol, among which there are 52 data sub-carriers. Total 20 MHz bandwidth is used to operate in 5GHz band, thus the sub-carrier frequency spacing is 0.315MHz and the IFFT/FFT period is 3.2 μs.

For OFDM system with CP, the CP length is set to be 16, which is longer than the channel length. The efficiency of bandwidth usage is 75%. The OFDM system without CP, which applies the proposed algorithm, has a bandwidth efficiency of 100%.

The simulation is done over SUI-1 (Stanford University Interim) channel model. This channel model consists of 3 taps, each having independent Rayleigh-fading amplitude distribution. The root mean square (RMS) delay spread is $\tau_{RMS} = 0.111$.

Fig. 2 gives the bit error rate (BER) comparison between conventional OFDM system, OFDM with linear MMSE and OFDM system based on the proposed successive algorithm. It shows that the OFDM system based on proposed method can achieve lower BER than the other two schemes. The reason that the proposed method can obtain lower BER is: in each step of the successive detection, one subcarrier's data is subtracted from the received signal after its detection. This enables to

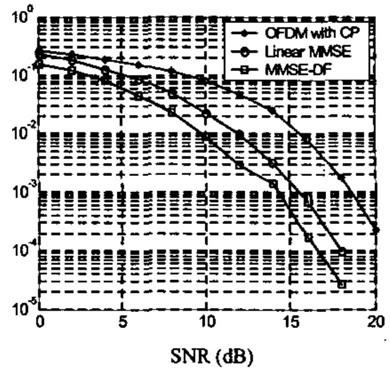

Fig. 2. BER performance

reduce the interference and noise influence for the other subcarriers and therefore increase the reliability of the decision.

VI. CONCLUSION

In this paper, we propose a successive detection algorithm for OFDM system without CP. In this algorithm, the signals are detected successively and the previous decision is used as feedback to eliminate its contribution to ISI and ICI. This scheme can remove the need of cyclic prefix, so leads to high spectral efficiency. Simulation results show that with the proposed method, an OFDM system without CP actually outperforms a conventional OFDM system with CP and OFDM system with linear MMSE equalizer not only in bandwidth efficiency, but also in bit error rate (BER).

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