RAP survey - September 19 2022

Recall from Definition 1.2.6 in `https://teambasedinquirylearning.github.io/linear-algebra/2022/LE2.html` that for a matrix to be in reduced row echelon form it should follow these rules:

1. The leading term (first non-zero) must be a 1. This is called a **pivot**.
2. Each pivot should be to the right of every higher pivot – so they should be oriented such that they generally start in the upper left section of the matrix and end in the lower right.
3. All terms in the same column of a pivot should be zeroes.
4. All rows of zeroes should be at the bottom of the matrix.

Recall from Definition 1.2.3 in `https://teambasedinquirylearning.github.io/linear-algebra/2022/LE2.html` that the following row operations are OK and will produce equivalent matrices:

1. Swapping two rows
2. Multiplying a row by a non-zero constant
3. Adding a constant multiple of one row to another row

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \\ -3 & -6 & -4 \\ 2 & 4 & 0 \\ 2 & 4 & 1 \end{bmatrix}$$

My general strategy is to:
1. Try to zero out the rows I can zero out.
2. Try to get at least 1 row to be two 0s and one 1.
3. Use the row of 0s and 1s to adjust other rows as needed.

Step 1. Change row 2 (R2 + -1*R4). Subtract row 4 from row 2. This gets us a little closer to a [0,0,1] in row 2. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 5 \\ -3 & -6 & -4 \\ 2 & 4 & 0 \\ 2 & 4 & 1 \end{bmatrix}$$

Step 2. Change row 5 (R5 + -2*R1). Subtract 2 x row 1 from row 5. Again, this is getting us closer to [0,0,1] in row 5. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 5 \\ -3 & -6 & -4 \\ 2 & 4 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

Step 3. Change row 4. (R4 + -2*R1) Subtract 2 x row 1 from row 4 . Again, this is getting us closer to [0,0,1] in row 4. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 5 \\ -3 & -6 & -4 \\ 0 & 0 & -4 \\ 0 & 0 & -3 \end{bmatrix}$$

Step 4. Change row 5 (R5 + -1*R4). Subtract row 4 from row 5. Great, now row 5 is [0,0,1]. This makes it easier to add multiples of row 5 to other rows to change only the number in column 3. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 5 \\ -3 & -6 & -4 \\ 0 & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 5. Change row 2 (R2 + -5*R5). Add -5 * row 5 to row 2 , zeroing it out. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ -3 & -6 & -4 \\ 0 & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 6. Change row 4 (R4 + 4*R5). Add 4 * row 5 to row 4, zeroing it out. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ -3 & -6 & -4 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 7. Change row 3 (R3 + 3*R1). Add 3 * row 1 to row 3. This gets us closer to zeroing out row 3. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Step 8. Change row 3 (R3 + -2*R5). ** Add -2 * row 5 to row 3, zeroing it out . This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 9. Change rows 2 and 5, swapping them (R2 <-> R5). Swap row 2 and 5 so that all the zero rows are to the bottom and the pivots are moving generally down and to the right. This is permissible because of 1.2.3 rule 1.

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$