Oct 31 notes

Activity 3.3.4
Let T:$\mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation given by the standard matrix.
$T = \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 3x + 4y - z \\ x + 2y + z \end{bmatrix}$
a. set the three equations to 0
$3x + 4y - z = 0$
$x + 2y + z = 0$

Computing the RREF

```
(Matrix(QQ,[
[3,4,-1,0],
[1,2,1,0]
])).rref()
[ 1  0 -3  0]
[ 0  1  2  0]
```

This means:
1x = 3z
y = -1z
z = z

Activity 3.3.5
Let T:$\mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation given by the standard matrix.
$T = \left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} 2x + 4y + 2z - 4w \\ -2x - 4y + z + w \\ 3x + 6y - z - 4w \end{bmatrix}$ Again, compute RREF

```
(Matrix(QQ,[
[2,4,2,-4],
[-2,-4,1,1],
[3,6,-1,-4]
])).rref()
[ 1  0 -3  0]
[ 0  1  2  0]
```

-2,1,0,0 1 vector in our basis, kernel is 1 dimensional

Activity 3.3.6 Which of these subspaces of R3 describes the set of all vectors that are the result of using t to transform R2 vectors?
Our answer is B bc we couldn't get anything in the z row because we can't get

anything other than 0 in z row as a linear combination of T(xy) since it's always 0

Let T:V→W be a linear transformation, the image of R is an important subspace of W defined by

Activity 3.3.8
B is kernel, our answer is C bc we have x and y (not just a,a).
Activity 3.3.9
(Matrix(QQ,[[3,4,7,1], [-1,1,0,2], [2,1,3,-1]])).rref() [1 0 1 -1] [0 1 1 1] [0 0 0 0]
A - not linearly independent Im(t) is image of (T). bc the standard matrix is the identity matrix, you can get any linear combination in R4

Any vector can be rewritten as a linear combination

The image of T is the span of the columns of A. Remove the vectors creating non-pivot columns in RREF A to get a basis for the image.
we dont know if its a basis yet, but we do know it spans

bc they are linear combinations of vectors of T it is linearly dependent.

Observation 3.3.10 explains this a little more
Fact 3.3.11 the kernel is the solution set for the homogeneous system given the augmented matrix, use the coefficients of its free variables to get a basis for the kernel.

THe image T is the span of the columns of A, remove the vectors creating non-pivots to get a basis for the image.

3.3.13 pivot columns tells us about the image, non-pivot columns is the basis of kernel