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Recall from **Definition 1.2.6** in `https://teambasedinquirylearning.github.io/linear-algebra/2022/LE2.html` that for a matrix to be in **reduced row echelon form** it should follow these rules:

**1.** The leading term (first non-zero) must be a 1. This is called a **pivot.**
**2.** Each **pivot should be to the right of every higher pivot** – so they should be oriented such that they generally start in the upper left section of the matrix and end in the lower right.
**3.** All terms in the same **column** of a pivot should be **zeroes**.
**4.** All **rows of zeroes** should be at the **bottom** of the matrix.

Recall from **Definition 1.2.3** in `https://teambasedinquirylearning.github.io/linear-algebra/2022/LE2.html` that the following row operations are OK and will produce equivalent matrices:

**1.** Swapping two rows
**2.** Multiplying a row by a non-zero constant
**3.** Adding a constant multiple of one row to another row

**Starting Matrix:**

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \\ -3 & -6 & -4 \\ 2 & 4 & 0 \\ 2 & 4 & 1 \end{bmatrix}$$

**My general strategy is to:**
**1.** Try to zero out the rows I can zero out.
**2.** Try to get at least 1 row to be two 0s and one 1.
**3.** Use the row of 0s and 1s to adjust other rows as needed.

**Step 1. Change row 2 (R2 + -1R4).**
Subtract row 4 from row 2. This gets us a little closer to a [0,0,1] in row 2. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 5 \\ -3 & -6 & -4 \\ 2 & 4 & 0 \\ 2 & 4 & 1 \end{bmatrix}$$

**Step 2. Change row 5 (R5 + -2R1).** Subtract 2 x row 1 from row 5. Again, this is getting us closer to [0,0,1] in row 5. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 5 \\ -3 & -6 & -4 \\ 2 & 4 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

**Step 3. Change row 4. (R4 + -2R1).** Subtract 2 x row 1 from row 4 . Again, this is getting us closer to [0,0,1] in row 4. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 5 \\ -3 & -6 & -4 \\ 0 & 0 & -4 \\ 0 & 0 & -3 \end{bmatrix}$$

**Step 4. Change row 5 (R5 + -1R4).** Subtract row 4 from row 5. Great, now row 5 is [0,0,1]. This makes it easier to add multiples of row 5 to other rows to change only the number in column 3. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 5 \\ -3 & -6 & -4 \\ 0 & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

**Step 5. Change row 2 (R2 + -5R5).** Add -5 * row 5 to row 2 , zeroing it out. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ -3 & -6 & -4 \\ 0 & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

**Step 6. Change row 4 (R4 + 4R5).** Add 4 * row 5 to row 4, zeroing it out. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ -3 & -6 & -4 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Step 7. Change row 3 (R3 + 3R1).** Add 3 * row 1 to row 3. This gets us closer to zeroing out row 3. This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Step 8. Change row 3 (R3 + -2R5).** Add -2 * row 5 to row 3, zeroing it out . This is permissible because of 1.2.3 rule 3.

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Step 9. Change rows 2 and 5, swapping them (R2 <-> R5).** Swap row 2 and 5 so that all the zero rows are to the bottom and the pivots are moving generally down and to the right. This is permissible because of 1.2.3 rule 1.

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$