Oct 24 notes

Definition 3.1.1 A linear transformation (also called a linear map) is a map between vector spaces that preserves the vector space operations. More precisely, if V and W are vector spaces, a map T:V $\rightarrow$ W is called the linear transformation if:
$T(\vec{v}+\vec{w}) = T(\vec{v}) + T(\vec{w})$ for any $\vec{v}, \vec{w} \in V$
$T(c\vec{v}) = cT(\vec{v})$ for any $c \in \mathbb{R}, \vec{v} \in V$

Definition 3.1.2
Given a linear transformation T:V $\rightarrow$ W, V is called the domain of T and W is called the co-domain.

Example 3.1.3
Let T : $\mathbb{R}^3 \rightarrow \mathbb{R}^2$

Example 3.1.4
To show it is not linear, just show that one rule breaks.

Fact 3.1.5
A map between Euclidean spaces is linear exactly when every component of the output is a linear combination of the vriables of Rn.
For example, the following map is definitely linear bc x-z and 3y are linear combinations of x, y, z
So even one entry that isn't linear will break the map

Activity 3.1.6:
P is a vector space and D is a linear map
It preserves the vector space operations
The exposition in this problem is explaining how it's closed to addition and multiplication
How do we know P is a vector space?
P is infinite dimensional, so the basis is infinite.
If we have two polynomials, we add and multiply them the usual way, so its a vector
Again, these two properties tell me that i have a linear map

what is a derivative? look up derivative power rule. if you have x to the n and take the derivative, you get x$^n - x^n - 1 wherenisthecoeffciient$

$S(f(x))$
3.1.7
T definitely not, compute T both sides and see one has an extra exponent
For S, we compute the same stuff, we haven't violated the rules but haven't proven its linear yet.
fact 3.1.8 domain v codomain w, is a linear transformation then Activity 3.1.10:

verify that these are true, and verifies additivity and multiplication
If these are true, then S is linear

Activity 3.1.11: verify that these are true, and verifies additivity and multiplication
If these are true, then S is linear

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Sample 3.1.6 practice problem

Cnsider the following maps of polynomials $S : \mathcal{P}$ and $T : \mathcal{P}$ defined by:

$S(f(x) = 3xf(x)$
$T(f(x)) = 3f'(x)f(x)$
Explain why one is a linear transformation and the other is not

To show S is a linear transformation, we must show two things:

$S(f(x) + g(x)) = S(f(x)) + s(g(z))$ – e.g., closed to addition
and
$S(cf(x)) = cS(f(x))$ – e.g., closed to scalar multiplication

Right hand side is from the definition of S
$S(f(x) + g(x)) = 3x(f(x) + g(x))$

Distribute the $3x$ on the right side of equation:
$S(f(x) + g(x)) = 3xf(x) + 3xg(x)$

but note, by definition of S (note – I am confused about this step):
$S(f(x)) = 3xf(x)$ and
$S(g(x)) = 3xg(x)$

So we have
$S(f(x) + g(x)) = S(f(x)) + S(g(x))$