Nov 9 notes

Note: no notes for Nov 7 because it was an office hours session.

Observation 4.1.1
If $T : \mathbb{R}^n \to \mathbb{R}^m$ and $S : \mathbb{R}^m \to \mathbb{R}^k$ are linear maps, then the composition map S $\circ$ T is a linear map from $\mathbb{R}^n \to \mathbb{R}^k$

The outputs/inputs have to match in order to make a composition map. Activty 4.1.2 will work because we have an R2 intermediate.

Activity 4.1.2
$T : \mathbb{R}^3 \to \mathbb{R}^2$ is B = $\begin{bmatrix} 2 & 1 & -3 \\ 5 & -3 & 4 \end{bmatrix}$
$S : \mathbb{R}^2 \to \mathbb{R}^4$ is A = $\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 3 & 5 \\ -1 & -2 \end{bmatrix}$
What are the domain and codomain of the composition map S $\circ$ T?

Definition 3.1.2 says given a linear transformation T : V $\to$ W , V is called the domain of T and W is called the co-domain of T.

The composition map is $\mathbb{R}^3 \to \mathbb{R}^4$, so domain is $\mathbb{R}^3$ and co-domain is $\mathbb{R}^4$

Therefore, the answer is C.

Activity 4.1.3
$T : \mathbb{R}^3 \to \mathbb{R}^2$ is B = $\begin{bmatrix} 2 & 1 & -3 \\ 5 & -3 & 4 \end{bmatrix}$
$S : \mathbb{R}^2 \to \mathbb{R}^4$ is A = $\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 3 & 5 \\ -1 & -2 \end{bmatrix}$
What size will the standard matrix of S $\circ$ T : $\mathbb{R}^3 \to \mathbb{R}^4$ be? (rows x columns)

The composition map is $\mathbb{R}^3 \to \mathbb{R}^4$, so T then S.
We work from left to right, so S $\circ$ T.
Could also just look at the size of the domains and co-domains. We are going from R3 to R4, so we go from R to L to get 4 by 3. Could also just think about 3 columns going in because there's 3 basis vectors for R3. We know we should have 4 rows bc we know at the end we'll be in R4, so each column will have 4 entries.

What does this map do to standard basis vectors?

$$S \circ T(\vec{e}_1) = S\begin{bmatrix}2\\5\end{bmatrix} = 2\begin{bmatrix}1\\0\\3\\-1\end{bmatrix} + 5\begin{bmatrix}2\\1\\5\\-2\end{bmatrix} = \begin{bmatrix}2\\0\\6\\-2\end{bmatrix} + \begin{bmatrix}10\\5\\25\\-10\end{bmatrix} = \begin{bmatrix}12\\5\\31\\-12\end{bmatrix}$$

$$S \circ T(\vec{e}_2) = S\begin{bmatrix}1\\-3\end{bmatrix} = 1\begin{bmatrix}1\\0\\3\\-1\end{bmatrix} + -3\begin{bmatrix}2\\1\\5\\-2\end{bmatrix} = \begin{bmatrix}1\\0\\3\\-1\end{bmatrix} + \begin{bmatrix}-6\\-3\\-15\\6\end{bmatrix} = \begin{bmatrix}-5\\-3\\-12\\5\end{bmatrix}$$

$$S \circ T(\vec{e}_3) = S\begin{bmatrix}-3\\4\end{bmatrix} = -3\begin{bmatrix}1\\0\\3\\-1\end{bmatrix} + 4\begin{bmatrix}2\\1\\5\\-2\end{bmatrix} = \begin{bmatrix}-3\\0\\-9\\3\end{bmatrix} + \begin{bmatrix}8\\4\\20\\-8\end{bmatrix} = \begin{bmatrix}5\\4\\11\\-5\end{bmatrix}$$

$$S \circ T(\vec{e}_4) = S\begin{bmatrix}0\\0\end{bmatrix} = 0\begin{bmatrix}1\\0\\3\\-1\end{bmatrix} + 0\begin{bmatrix}2\\1\\5\\-2\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}12\\5\\31\\-12\end{bmatrix}, \begin{bmatrix}-5\\-3\\-12\\5\end{bmatrix}, \begin{bmatrix}5\\4\\11\\-5\end{bmatrix}, \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

Definition 4.1.5

We define the product AB of a m $\times$ n matrix A and a n $\times$ k matrix B to be the m $\times$ k standard matrix of the composition map of the two corresponding linear functions.

Activity 4.1.6

$S : \mathbb{R}^3 \to \mathbb{R}^2$ is A $= \begin{bmatrix}-4 & -2 & 3\\0 & 1 & 1\end{bmatrix}$

$T : \mathbb{R}^2 \to \mathbb{R}^3$ is B $= \begin{bmatrix}2 & 3\\1 & -1\\0 & -1\end{bmatrix}$

(a) Write the dimensions (rows $\times$ columns) for A, B, AB, and BA.

A = 2 x 3, B = 3 x 2, AB = 2 x 2, BA = 3 x 3
In the case of AB, we are going B to A, which means we are going from R2 to R3, so 2 columns going in from R2 and 3 rows because we are in R3.
Then, we are going back to R2, so we are going back to 2 columns and 2 rows.
In the case of BA, we are going from A to B, which means 3 columns going in and 3 rows coming out, so 3x3.

Could also just look at the size of the domains and co-domains. We are going from R3 to R4, so we go from R to L to get 4 by 3. Could also just think about 3 columns going in because there's 3 basis vectors for R3. We know we should have 4 rows bc we know at the end we'll be in R4, so each column will have 4 entries.

(b) Find the standard matrix AB of S ∘ T.
(c) Find the standard matrix BA of T ∘ S.

$$S{\circ}T(\vec{e}_1) = S\begin{bmatrix}-4\\0\end{bmatrix} = -4\begin{bmatrix}2\\1\\0\end{bmatrix} + 0\begin{bmatrix}3\\-1\\-1\end{bmatrix} = \begin{bmatrix}2\\0\\6\\-2\end{bmatrix} + \begin{bmatrix}10\\5\\25\\-10\end{bmatrix} = \begin{bmatrix}12\\5\\31\\-12\end{bmatrix}$$

The composition map is $\mathbb{R}^3 \to \mathbb{R}^4$, so T then S.
We work from left to right, so S ∘ T.

Activity 4.1.8

Let $B = \begin{bmatrix} 3 & -4 & 0 \\ 2 & 0 & -1 \\ 0 & -3 & 3 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 7 & -1 \\ 0 & 3 & 2 \\ 1 & 1 & -1 \end{bmatrix}$.

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B = [3 -4 0 ; 2 0 -1 ; 0 -3 3]
A = [2 7 -1 ; 0 3 2 ; 1 1 -1]
B*A
6 9 -11
3 13 -1
3 -6 -9
```