Nov 14 notes

Activity 4.3.1
Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear map with standard matrix A. Sort the following items into three groups of statements: a group that means T is injective, a group that means T is surjective, and a group that means is T bijective.

Note from class: $\vec{x}$ can be thought of as an n x 1 matrix.

$T(\vec{x}) = A\vec{x}$ so can act as if I applied the transformation T to $\vec{x}$.

A. $A\vec{x} = \vec{b}$ has a solution for all $\vec{b} \in \mathbb{R}^m$
B. $A\vec{x} = \vec{b}$ has a unique solution for all $\vec{b} \in \mathbb{R}^m$
C. $A\vec{x} = \vec{b} = 0$ has a unique solution.
D. The columns of A span $\mathbb{R}^m$
E. The columns of A are linearly independent
F. The columns of A are a basis of Rm
G. Every column of RREF(A) has a pivot
H. Every row of RREF(A) has a pivot
I. m = n and RREF(A) = I

- A set of vectors are linearly independent if their RREF matrix has all pivot columns (Observation 2.5.8). RREF of the injective map's standard matrix has all pivot columns (Observation 3.4.7)
- A set of vectors spans the space if the RREF matrix has a pivot in every row (Observation 2.5.8). RREF of the surjective map's standard matrix has a pivot in each row (Observation 3.4.7)

C is injective bc you can only get the 0 vector via trivial solution
If the co-domain spans the space, we can get back any b in the domain and we have a pivot in every row and are surjective so A
If we have a unique solution then we are linearly independent and span, so B is bijective.

Bijective: F,I,B
Injective: E,G,C
Surjective: D,H,A

Activity 4.3.2

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by the standard matrix
$A = \begin{bmatrix} 2 & -1 & 0 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{bmatrix}$ Write an augmented matrix representing the system of equa-

tions given by $T(\vec{x}) = \vec{0}$, that is $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, then solve to find the kernel of T.

$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right)$ would be

$\begin{bmatrix} 2x & -1y & 0z \\ 2x & 1y & 4z \\ 1x & 1y & 3z \end{bmatrix}$

Is there a non-trivial way to get the zero vector?

$\begin{bmatrix} 2 & -1 & 0 & 0 \\ 2 & 1 & 4 & 0 \\ 1 & 1 & 3 & 0 \end{bmatrix}$ RREF:

```
Matrix(QQ,[
[2,-1,0,0],
[2,1,4,0],
[1,1,3,0]
]).rref()
[1 0 1 0]
[0 1 2 0]
[0 0 0 0]
```

Separate out the x, y, z

$x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, z = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

means
1x + 0y + 1z = 0, or x = -z
0x + 1y + 2z = 0, or y = -2z

$ket(T) = span \left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} \right\}$

Definition 4.4.3, Says that if there' sa unique soution to the eqn ax=b, then its bijection
so there's only 1 x that works set the inverse of t to be the exact x A inverse is that inverse matrix of A, so A is invertible

Activity 4.3.4

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by the standard matrix $A = \begin{bmatrix} 2 & -1 & 6 \\ 2 & 1 & 3 \\ 1 & 1 & 4 \end{bmatrix}$ Write an augmented matrix representing the system of equations given by $T(\vec{x}) = \vec{e}_1$, that is $A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, then solve $T(\vec{x}) = \vec{e}_1$ to find $T^{-1}(\vec{e}_1)$

Create an augmented matrix while solving for the standard basis vectors, then go from there

$$\begin{bmatrix} 2 & -1 & 6 & 1 \\ 2 & 1 & 3 & 0 \\ 1 & 1 & 4 & 0 \end{bmatrix}$$

$$T(\vec{e_1}) = T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) =$$

```
Matrix(QQ,[
[2,-1,6,1],
[2,1,3,0],
[1,1,4,0]
]).rref()
[1 0 1 0]
[0 1 2 0]
[0 0 0 0]
```

$$\begin{bmatrix} 1 \\ -5 \\ 1 \end{bmatrix} \begin{bmatrix} -2 \\ 14 \\ -3 \end{bmatrix} \begin{bmatrix} 3 \\ 18 \\ 4 \end{bmatrix}$$

observation 4.3.5 - row reduce entire matrix at once

so 1, -5, 1 is is something

activity 4.3.6 - set up the rref and the matrix on the right is the a inverse

activity 4.3.7 - is this invertible? give your reasons

must be square and must row-reduce to I.

we must cover the whole co-domain and the element in the domain and co-domain must have a 1-1 unique counterpart

A transformation must be bijective for its matrix to be invertible.

The RREF of the matrix must have all pivot rows and columns, thus also be a basis.

To find the inverse, set up an augmented matrix where you set the matrix equal to the standard basis vectors and solve to get the solution.

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The solution column is the inverse of the standard basis vector you have just computed the solutions for.