Control Systems EE2S21 – Lecture 9

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DCSC / DIAM

Today:

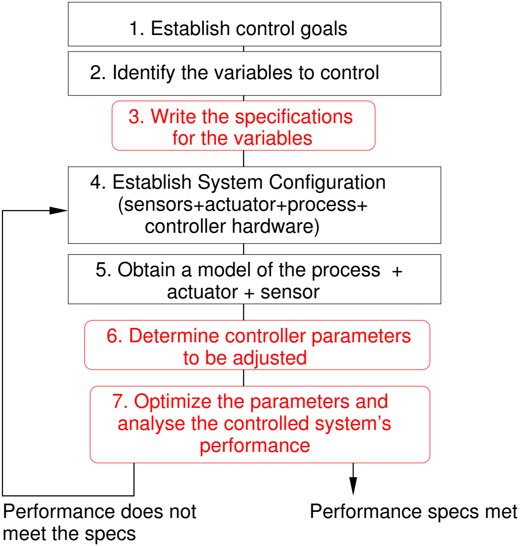
Control design cycle

What can control do?

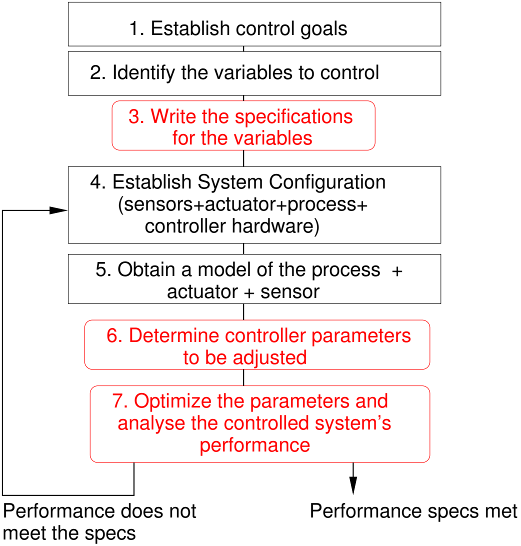
What are we going to do?

Study material: Chapter 4 (FCDS) — Lecture slides

The Control Design Cycle

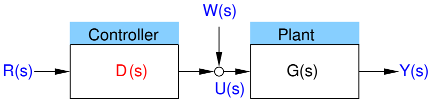


The Control Design Cycle

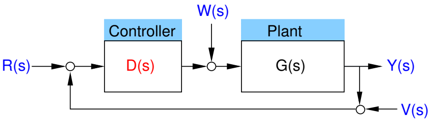


Summary: Given a model of the system to be controlled (process, sensors, actuators) and design goals, find a controller or determine that none exists.

Open-loop



Closed-loop



with

- R – the reference input
- W – the disturbance input
- U – the (controlled) plant input
- Y – the (controlled) plant output
- V – the measurement error

Contents of this lecture

What can (feedback) control achieve?

1. Tracking a reference signal
2. Disturbance rejection
3. Insensitivity for system (parameter) changes (model uncertainty)
4. Stability of the controlled system

Case Study 1: DC motor

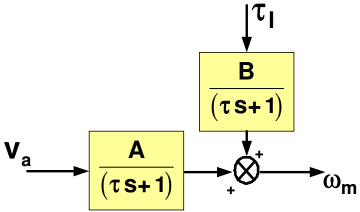
If the inductance is very small:

$$J_m \ddot{\theta}_m + b\dot{\theta}_m = K_t i + \tau_\ell$$
$$K_t \dot{\theta}_m + Ri = v_a$$

with motor voltage v_a, motor speed $\omega_m = \dot{\theta}_m$, and ext. torque τ_ℓ

Through Laplace transform and substitution:

$$\Omega_m(s) = \frac{A}{\tau s + 1} V_a(s) + \frac{B}{\tau s + 1} T_\ell(s)$$

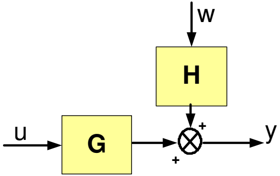


τ is the time constant

Open-loop versus closed-loop (feedback) control

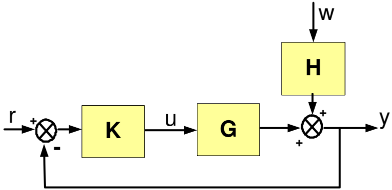
Open-loop

$$y = Gu + Hw \quad (u = Kr)$$



Closed-loop

$$y = \frac{KG}{1+KG}r + \frac{H}{1+KG}w$$



For r a step-input and w zero for DC motor:

$$Y(s) = \frac{KA}{\tau s + 1} \cdot \frac{1}{s}$$

$$Y(s) = \frac{\frac{KA}{1+KA}}{\frac{\tau}{1+KA}s + 1} \cdot \frac{1}{s} := \frac{T_{\text{cl}}}{S_{\text{cl}}\tau s + 1} \cdot \frac{1}{s}$$

1. Tracking a reference input (a step)

Open-loop: $Y(s) = \frac{KA}{\tau s+1} \cdot \frac{1}{s} + \frac{B}{\tau s+1} W(s)$

Steady-state value of $y(t)$ (for $w(t) = 0$):

$$y_{ss} = \lim_{t \to \infty} y(t) \overset{\text{FVT}}{=} \lim_{s \to 0} s \frac{KA}{\tau s + 1} \cdot \frac{1}{s} = KA$$

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If $K = \frac{1}{A}$, then $y_{ss} = 1$

$\boxed{\text{Property of open-loop control: System inversion required}}$

1. Tracking a reference input (a step)

Closed-loop: $Y(s) = \frac{T_{cl}}{S_{cl}\tau s + 1} \cdot \frac{1}{s} + \frac{S_{cl}B}{S_{cl}\tau s + 1} W(s)$

Steady-state value of $y(t)$ (for $w(t) = 0$):

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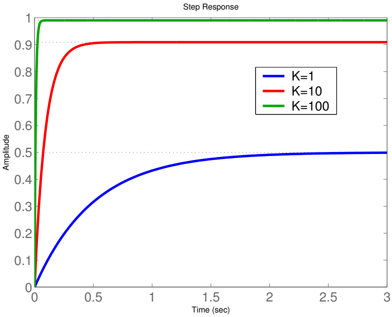
If $|KA| \gg 1$

$$y_{ss} \approx 1$$

Property of closed-loop control: Feedback changes dynamics (often improves it at the cost of lower stability)



2. Disturbance rejection

Goal: $y_{ss} = r_{ss}$

Open-loop: Choose $u_{ss} = \frac{1}{A} r_{ss} \implies y_{ss} = r_{ss} + \underbrace{B w_{ss}}_{\text{error}}$

Closed-loop:

$$y_{ss} = T_{cl} r_{ss} + B S_{cl} w_{ss}$$

For $|KA| >> 1, \rightarrow T_{cl} \approx 1$ and $S_{cl} \approx 0 \rightarrow y_{ss} = r_{ss}$

$\boxed{\text{Feedback reduces the effect of disturbances on the output}}$

3. Effect of system changes: Open-loop

Consider the steady-state open-loop transfer:

$$y_{ss} = Au_{ss} + Bw_{ss}$$

Let system gain change as $A \to A + \delta A$ with input selected by open-loop rule: $u_{ss} = \frac{1}{A} r_{ss}$, then controlled steady-state value of output reads:

$$y_{ss} = \left(1 + \frac{\delta A}{A}\right) r_{ss} + Bw_{ss}$$

Even without noise: 10% error in A leads to 10% error in output signal

3. Effect of system changes: Closed-loop

$$y_{ss} = \underbrace{\frac{KA}{1+KA}}_{T_{cl}} r_{ss} + \frac{B}{1+KA} w_{ss}$$

For $A \to A + \delta A$, we seek to determine the change in the closed-loop transfer T_{cl}:

$$T_{cl} + \delta T_{cl} = \frac{K(A+\delta A)}{1+K(A+\delta A)}$$

$$\delta T_{cl} = \frac{dT_{cl}}{dA}\delta A \Rightarrow \frac{\delta T_{cl}}{T_{cl}} = \underbrace{\frac{1}{1+KA}}_{\text{sensitivity function}} \frac{\delta A}{A}$$

For $|KA| \gg 1$ effect of gain changes on T_{cl} diminishes.

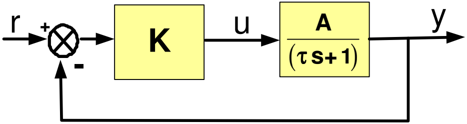
> Feedback reduces the effect of gain changes on the output with a factor $1/(1+AK)$

4. Speed of response / tracking to a reference input

Open-loop: For $Y(s) = \frac{KA}{\tau s+1} R(s) \overset{R(s)=\frac{1}{s}}{\Longrightarrow} y(t) = KA(1 - e^{-\frac{1}{\tau} t})$.

Response determined by time constant τ or pole $s = -1/\tau$.

Closed-loop: Transfer $r \to y$



$$T_{\text{cl}}(s) = \frac{K \frac{A}{\tau s+1}}{1 + K \frac{A}{\tau s+1}} = \frac{KA/(1+KA)}{\frac{\tau}{1+KA} s + 1}$$

Time constant of closed loop: $\frac{\tau}{1+AK} \ll \tau$

Intermediate conclusion: Static case

Feedback can:

1. reduce the sensitivity of the system (plant + controller) to parameter variations.
2. reduce the effect of disturbance on the controlled output.
3. improve the reference tracking (increased speed of response + reduced steady state error)

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1. reduce the sensitivity of the system (plant + controller) to parameter variations.
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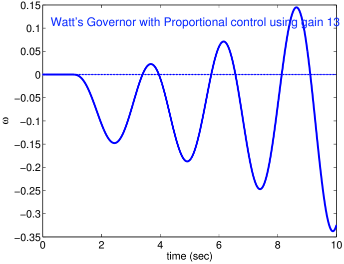
3. improve the reference tracking (increased speed of response + reduced steady state error)

BUT be aware of the loss of stability



Case Study 2: The Spirit Mars Rover

Spirit

- 1.6m, 174 kg, 44m/day
- Landed on 9 p.m. PST (3.1.2004)

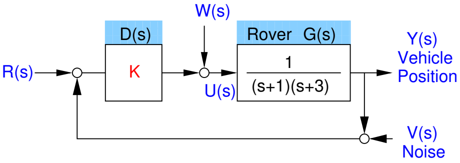
Sojourner

- 65cm, 10 kg, 10m/12 weeks

Goal: To illustrate an *advanced* control design problem formulation

Key Players in Desirable Control Specification

Spirit Mars Rover feedback scheme



$$Y(s) = \frac{D(s)G(s)}{1 + D(s)G(s)} R(s) + \frac{G(s)}{1 + D(s)G(s)} W(s) - \frac{D(s)G(s)}{1 + D(s)G(s)} V(s)$$

$$U(s) = \frac{D(s)}{1 + D(s)G(s)} R(s) + \frac{1}{1 + D(s)G(s)} W(s) - \frac{D(s)}{1 + D(s)G(s)} V(s)$$

Nomenclature:

Sensitivity function $S(s) = \frac{1}{1+D(s)G(s)} \left(= \frac{s^2+4s+3}{s^2+4s+3+K} \right)$, and

Complementary Sensitivity function $T(s) = \frac{D(s)G(s)}{1+D(s)G(s)} \left(= \frac{K}{s^2+4s+3+K} \right)$

Intuitive Controller Design Specifications

Consider the output equation of the "standard" closed loop:

$$Y(s) = \underbrace{\frac{D(s)G(s)}{1 + D(s)G(s)}}_{T(s)} R(s) + G(s)\underbrace{\frac{1}{1 + D(s)G(s)}}_{S(s)} W(s) - T(s)V(s)$$

Then the requirements of 'good' output tracking are:

Intuitive Controller Design Specifications

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Then the requirements of 'good' output tracking are:

1. 'Good' reference tracking: $y(t) \approx r(t)$ — requires $T(s) \approx 1$
2. 'Good' disturbance rejection: $y(t) \approx 0$ for $w(t) \neq 0$ — requires $S(s) \approx 0$
3. 'Insensitivity' to noise $y(t) \approx 0$ for $v(t) \neq 0$ — requires $T(s) \approx 0$

Conflicting requirements: 1(2) and 3 are conflicting!

Resolving the conflict

The resolution of the conflict in the controller specification requirements is possible when the different requirements only need to hold in different frequency bands.

To study the frequency dependency, we look at the tracking of a reference input first.

Tracking a reference input

Consider the output equation of the "standard" closed loop:

$$Y(s) = \underbrace{\frac{D(s)G(s)}{1 + D(s)G(s)}}_{T(s)} R(s) + G(s) \underbrace{\frac{1}{1 + D(s)G(s)}}_{S(s)} W(s) - T(s)V(s)$$

Then $E(s) = R(s) - Y(s)$ satisfies

$$\boxed{E(s) = S(s)R(s) - G(s)S(s)W(s) + T(s)V(s)}$$

When $r(t)$ is a sinusoid of frequency ω_0 $\left[\frac{\text{rad}}{\text{sec}}\right]$ (and amplitude 1) and $w(t) \equiv v(t) \equiv 0$, the maximum amplitude error is

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$$|E(j\omega_0)| = |S(j\omega_0)|$$

Thus, to reduce the error to 1% of the input, we must make $|S(j\omega)| \leq 0.01$ for $\omega = \omega_0$!

Good Tracking & Noise + Disturbance rejection

$$\boxed{E(s) = S(s)R(s) - G(s)S(s)W(s) + T(s)V(s)}$$

1. Tracking sinusoids in a limited frequency band: This frequency band of interest can be specified via the weighting function W_{Track}

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   The tracking goal is $\boxed{|W_{\text{Track}}(j\omega)S(j\omega)| \ll 1 \quad \forall\omega}$

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2. Noise reduction: The noise generally is dominant in a high frequency band. This band can be specified via a weighting function W_{Noise} and the requirement reads
   $\boxed{|W_{\text{Noise}}(j\omega)T(j\omega)| \ll 1 \quad \forall\omega}$

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3. Disturbance reduction: $\boxed{|W_{\text{Dist}}(j\omega)GS(j\omega)| \ll 1 \quad \forall\omega}$

Summary Advanced Control Design Procedure

Let the ∞-norm of a complex vector function $P(j\omega)$ be defined as

$$\|P(j\omega)\|_\infty = \max_\omega \max_i |P_i(j\omega)|$$

then the controller design problem can be formulated as the following multi-criteria optimization problem:

$$\min_{D(s)} \left\| \begin{bmatrix} W_{\text{Track}}(j\omega)S(j\omega) \\ W_{\text{Dist}}(j\omega)G(j\omega)S(j\omega) \\ W_{\text{Noise}}(j\omega)T(j\omega) \end{bmatrix} \right\|_\infty$$

Towards a more classical, alternative control design approach?

Why?

Towards a more classical, alternative control design approach?

Why? Because the solution to the multi-criteria design problem is far from trivial:

1. Even if one has a numerical black-box that solves the problem, it would deliver little insight in the choices for making the trade-offs (in the weighting functions) and the achievable limits of performance dictated by the properties of the system!
2. Building insight/intuition is very difficult because of the non-linear dependency of the cost function on the controller parameters.
3. Its solution is extremely hard (NP-hard) when one is interested in simple controllers with PID structure (still used in 90% of the cases),

$$D(s) = k_\mathrm{p} + \frac{k_\mathrm{i}}{s} + k_\mathrm{d} s$$

So let us start with simple things first!

The classical control design approach

Why an attractive starter:

1. Centered on geometric pictures (root locus, Bode plots, $\cdots$)
2. Simple mathematics (complex numbers, elementary function theory)
3. Focused on 1st-order and 2nd-order systems (with time delay) AND tracking specific inputs (such as a step)
   $\rightarrow$ covers many interesting, modern applications