Systeem- en Regeltechniek
EE2S21

Bond Graphs + Causality

Lecture 4

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DIAM / DCSC

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Energy and Power

Question posted before was if we can approach the different engineering domains in a similar way? $\Rightarrow$ Answer is yes!

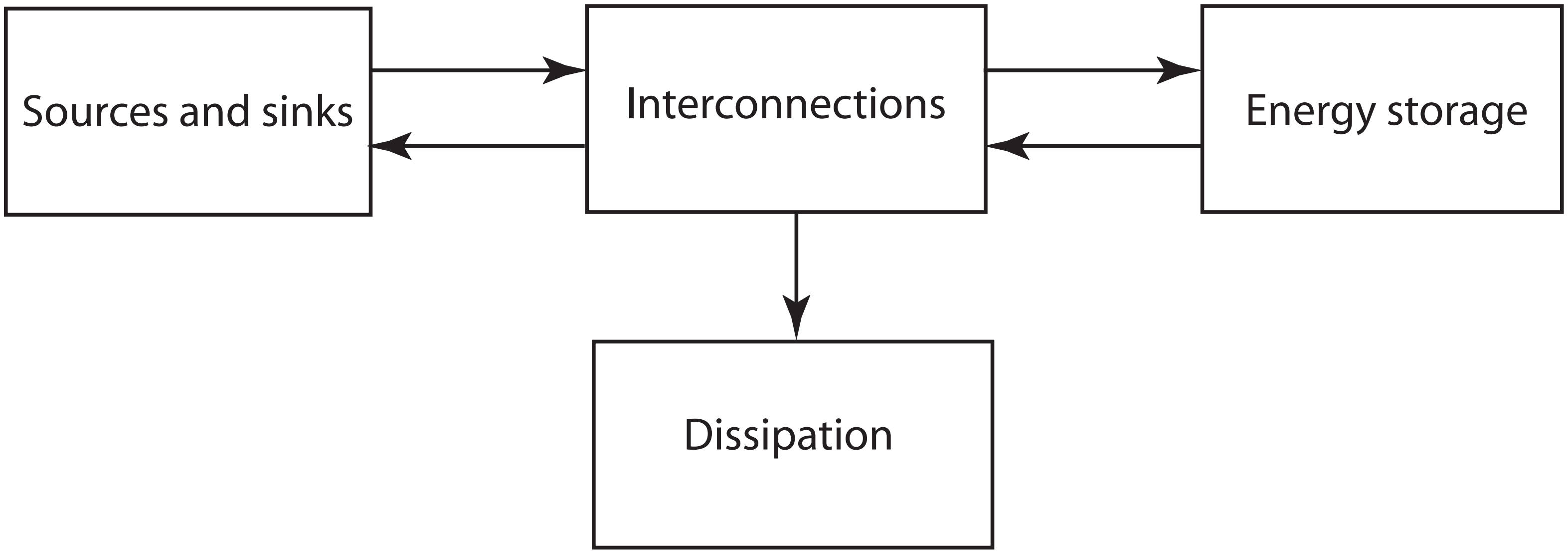
- The different domains can be treated similarly, except for the thermo-dynamical domain that is only partly similar.
- Main properties in common:

  energy supply, energy storage and energy dissipation.

- Law of energy conservation with help of power flow $P(t)$ and stored energy $E(t)$, where

$$P(t) = \frac{dE(t)}{dt} = \text{effort} \times \text{flow}.$$

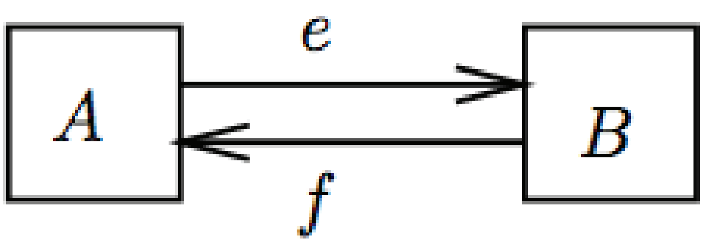
Systems as Energy Processors

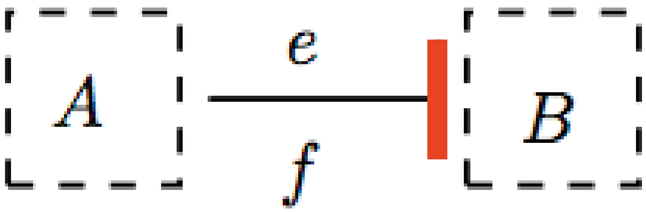


- Energy supply: Se and Sf elements
- Energy storage: I and C elements
- Energy dissipation: R elements
- Interconnections: 1-junctions, 0-junctions, TF and GY elements.

Causality Assignment

- Causality: cause and effect
- A way to determine if dynamics can be written in state-space form or represented as a block diagram
- Bond graph notation: causal strokes





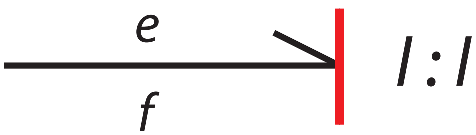
- Causal stroke where effort is the input!
- Sources:

Causality of Linear I-Elements

- Integrating causality:

$$f(t) = \frac{1}{I}\int_0^t e(s)ds + f(0),$$

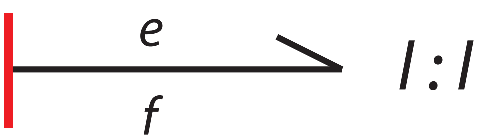
where e is input and f is output, i.e.,



- Differentiating causality:

$$e(t) = I\frac{df(t)}{dt},$$

where f is input and e is output, i.e.,



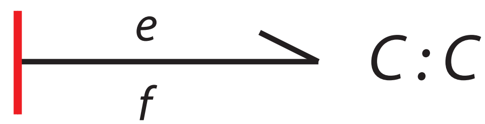
- Integrating causality is preferred! (see reader)

Causality of Linear C-Elements

- Integrating causality:

$$e(t) = \frac{1}{C}\int_0^t f(s)ds + e(0),$$

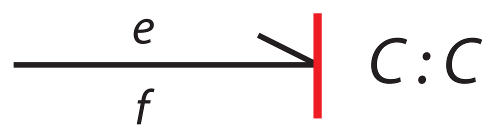
where f is input and e is output, i.e.,



- Differentiating causality:

$$f(t) = C\frac{de(t)}{dt},$$

where e is input and f is output, i.e.,



- Integrating causality is preferred! (see reader)

Causality of Linear R-Elements

R-elements possess a static relationship between e and f.

- If f is input, then e is output, i.e.,

$$e(t) = Rf(t),$$

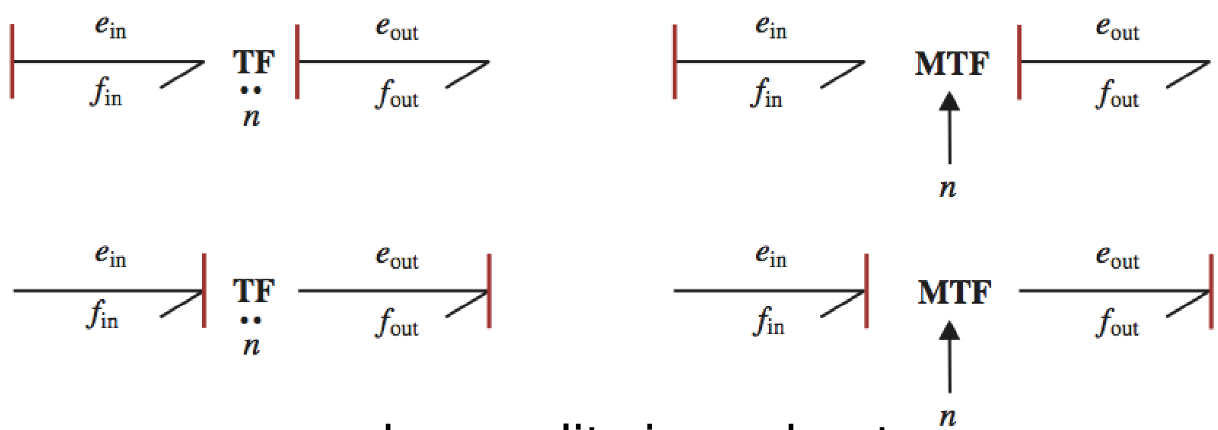
- and when e is input, then f is output, i.e.,

$$f(t) = \frac{1}{R}e(t),$$

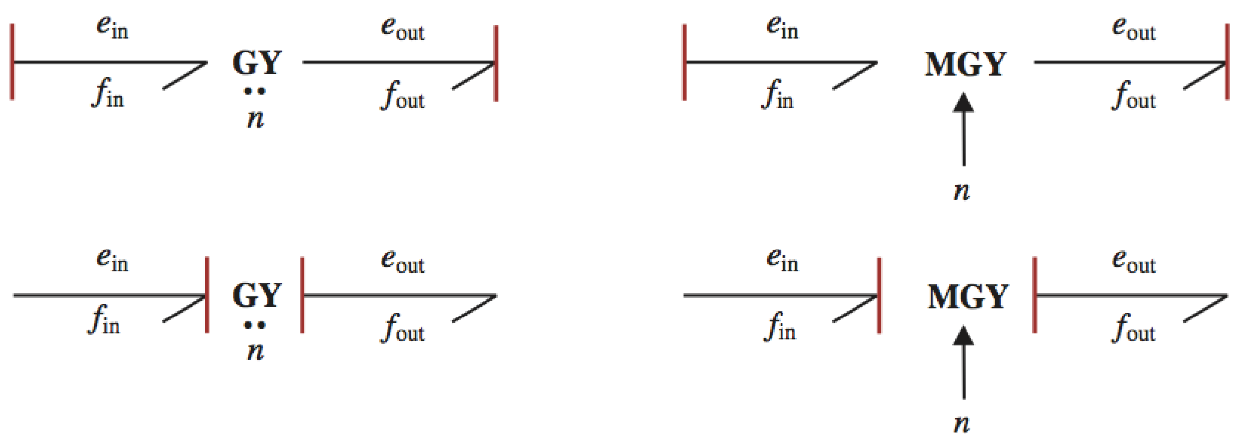
- This means we have two possible choices

Causality of TF- and GY-Elements

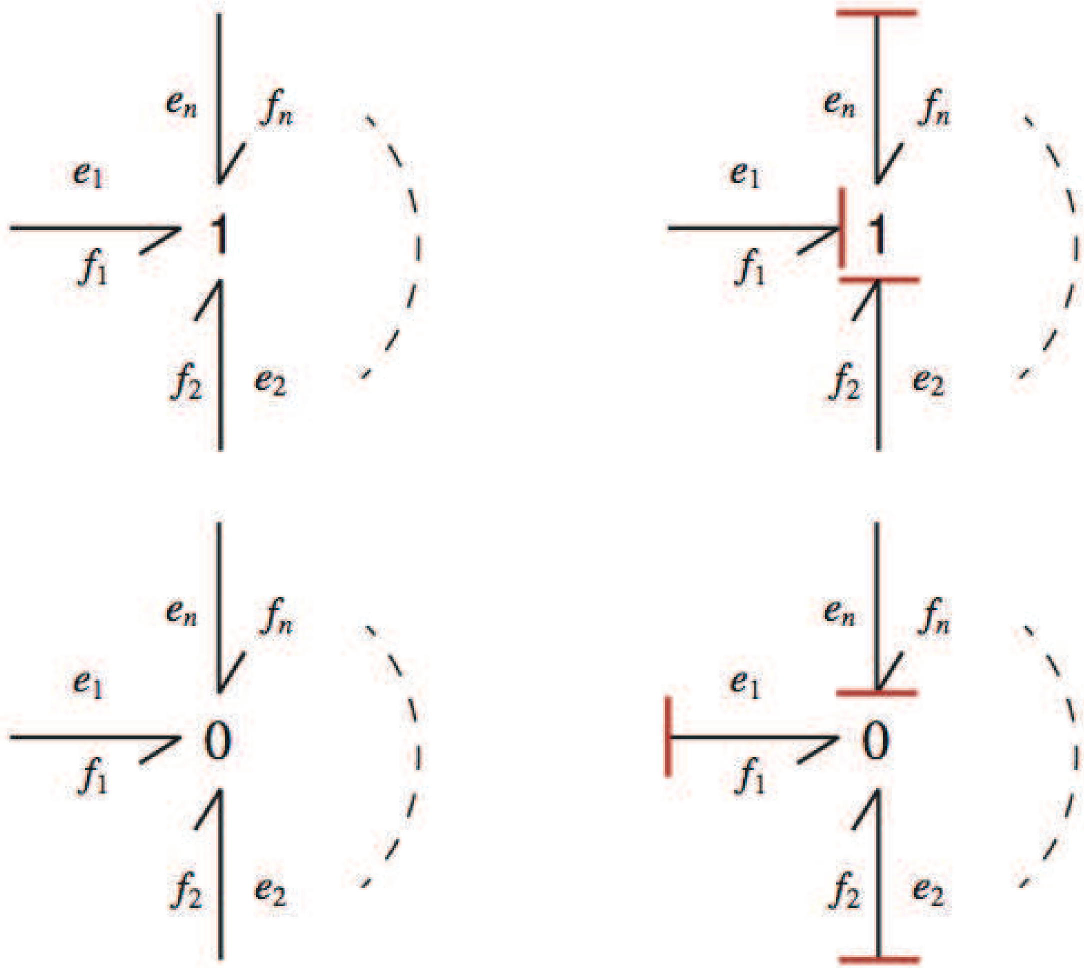
- Transformers: same causality in and out:



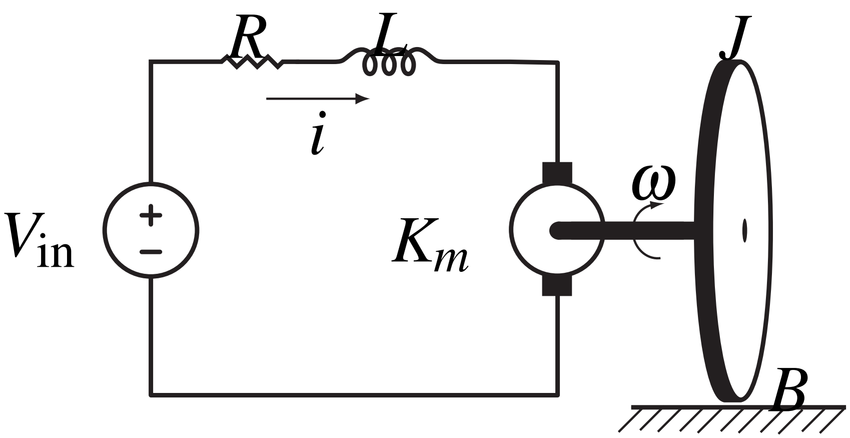
- Transformers: reversed causality in and out:



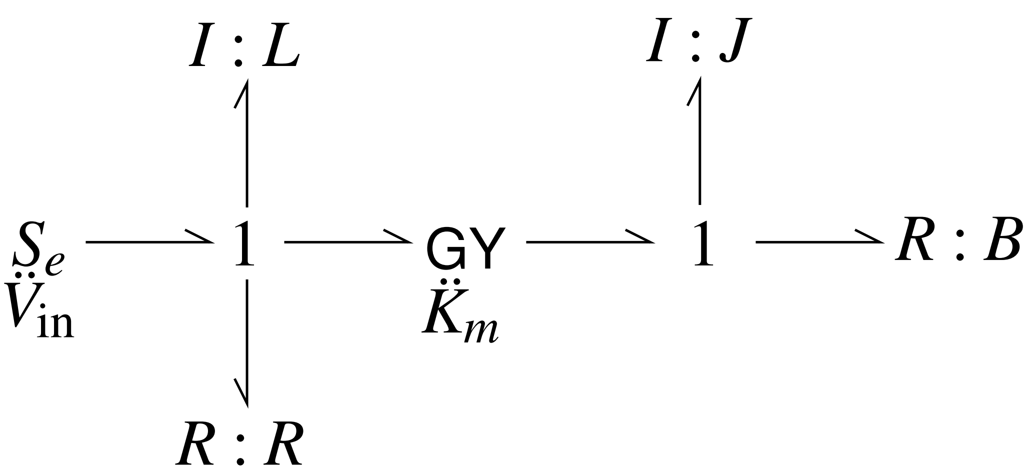
Causality of Junctions



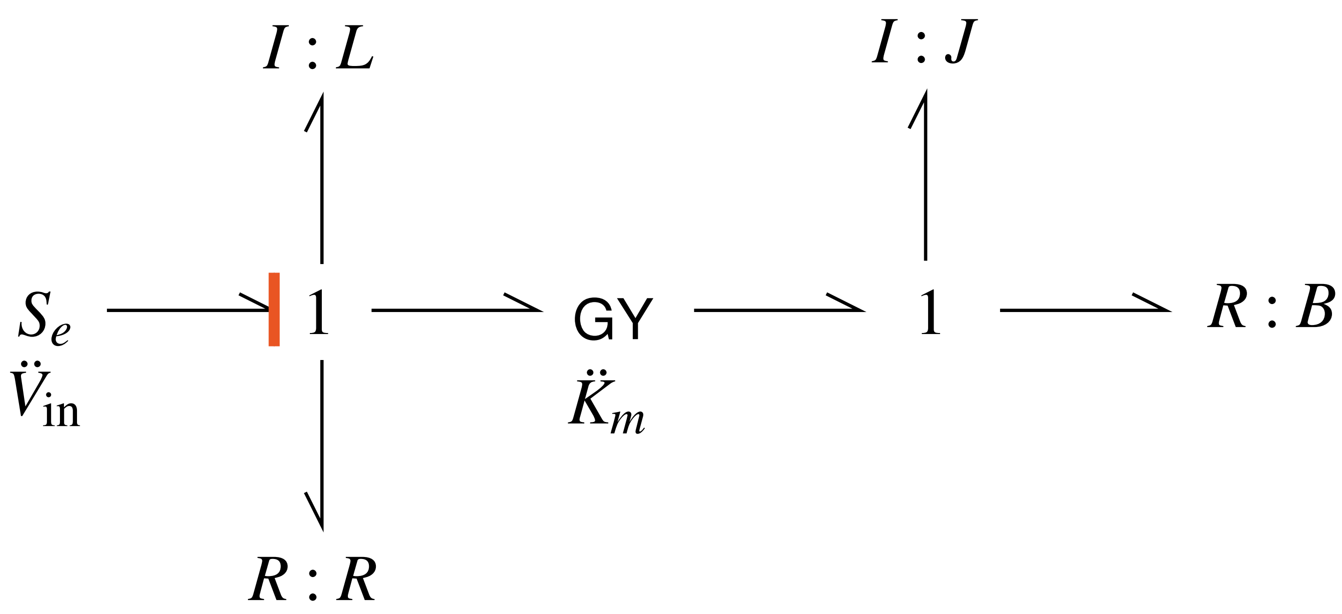
Example: DC motor



- Two states corresponding to storage elements L and J
  $\Rightarrow$ order 2
- Two dissipative elements
- One gyrator
- One source



Example: DC motor



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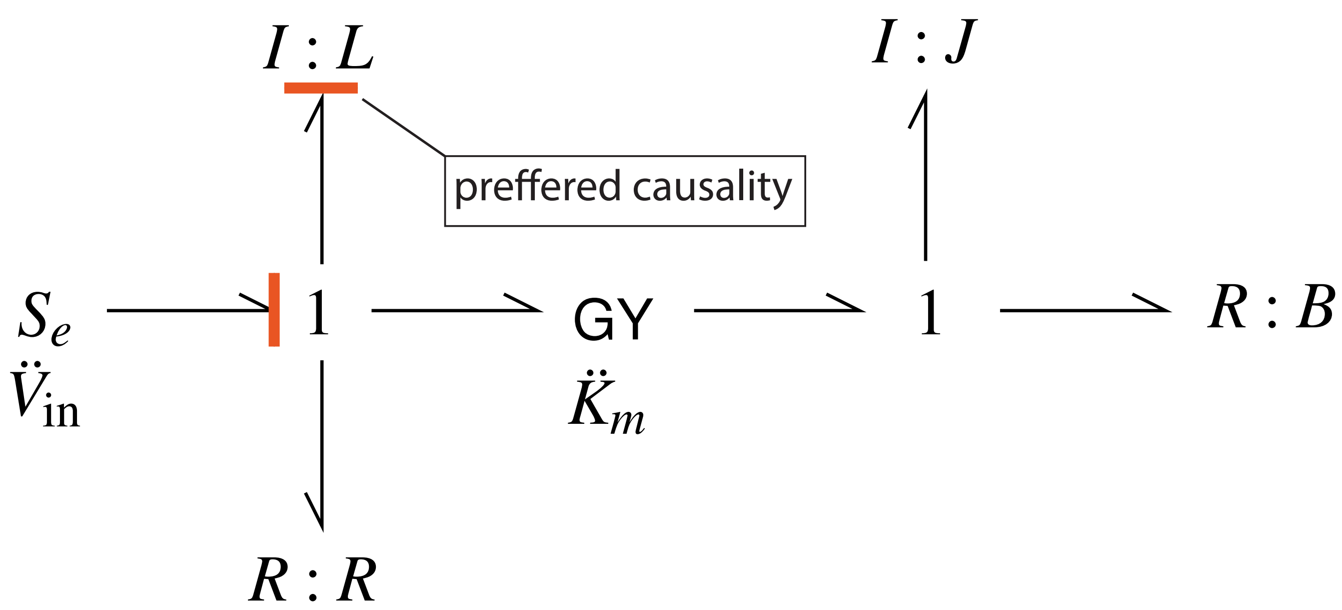
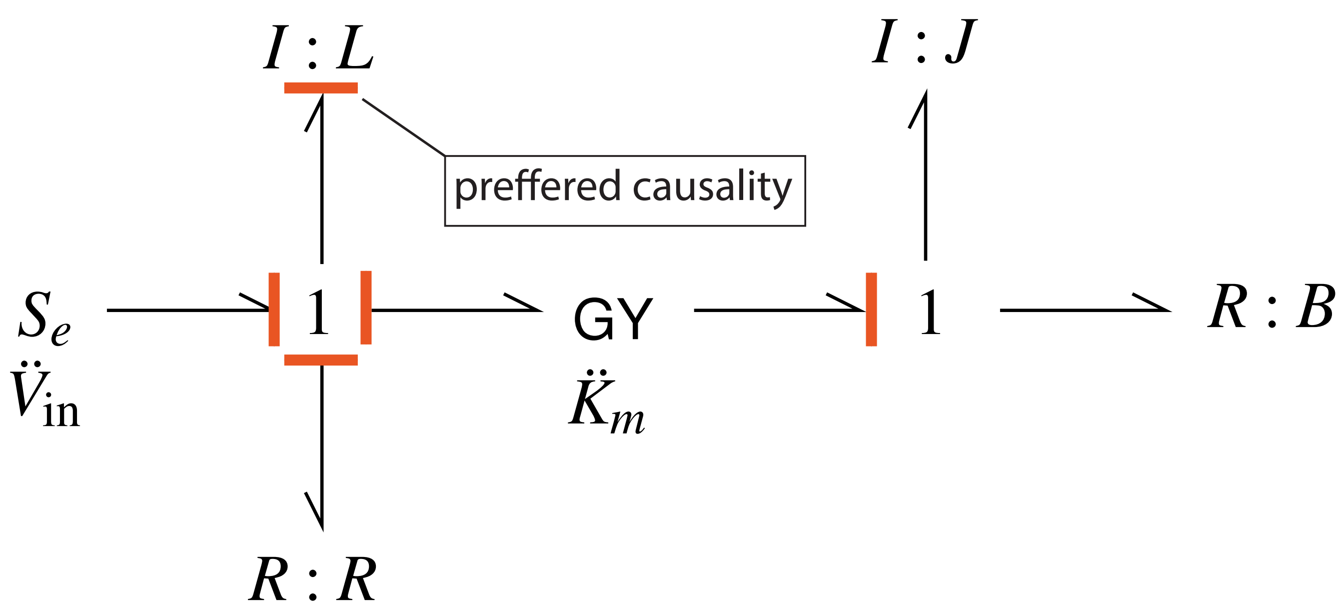
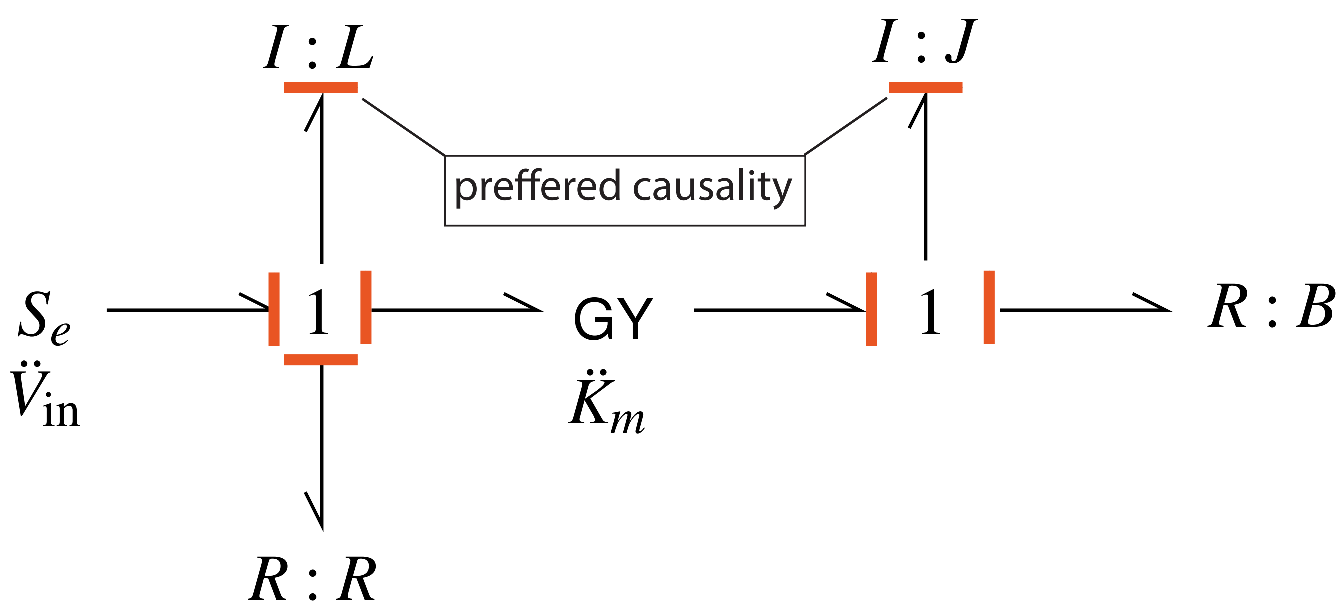
Example: DC motor



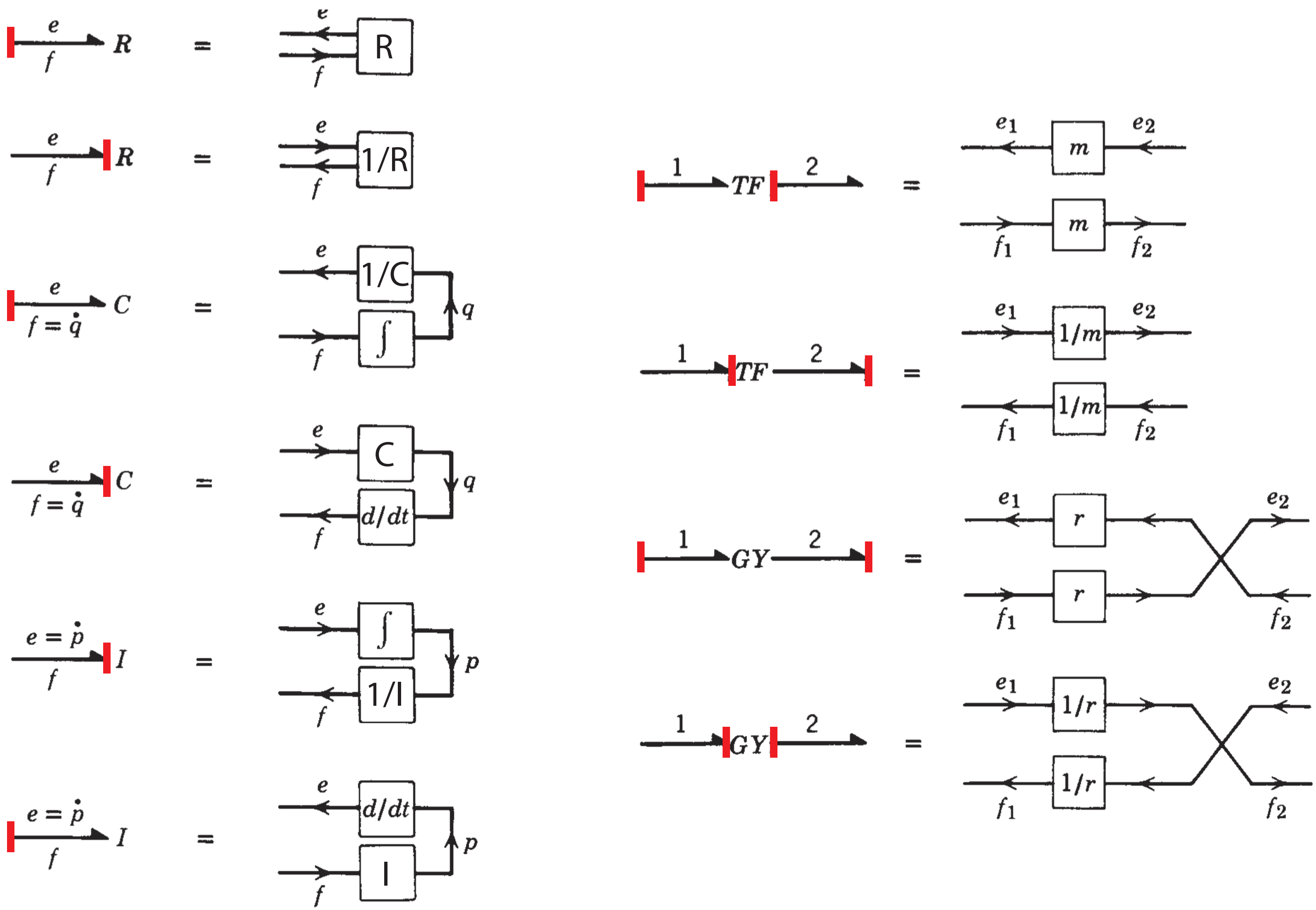
Example: DC motor



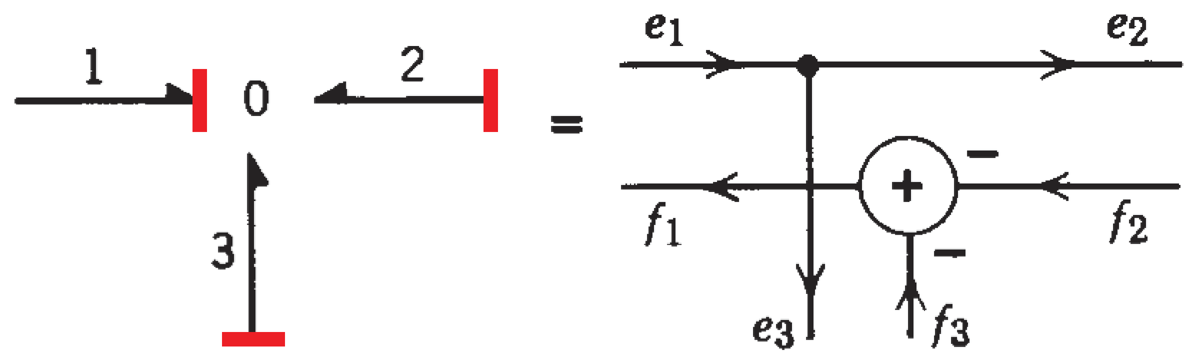
Example: DC motor

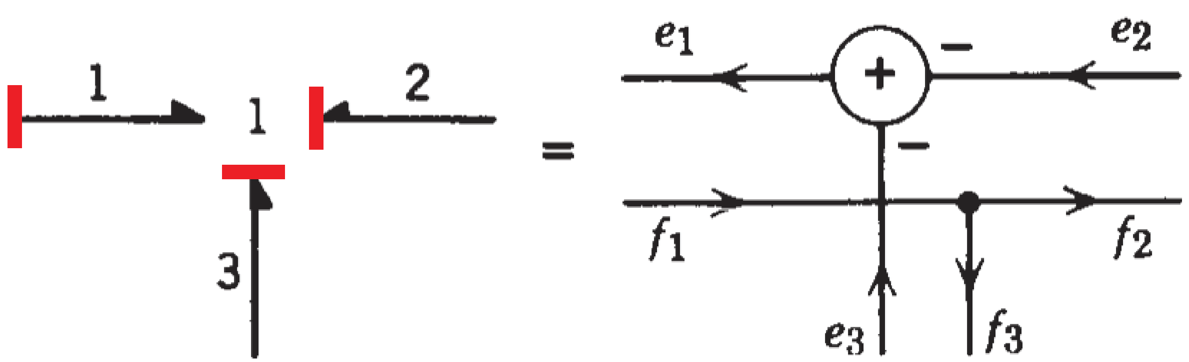


From Bonds to Blocks...

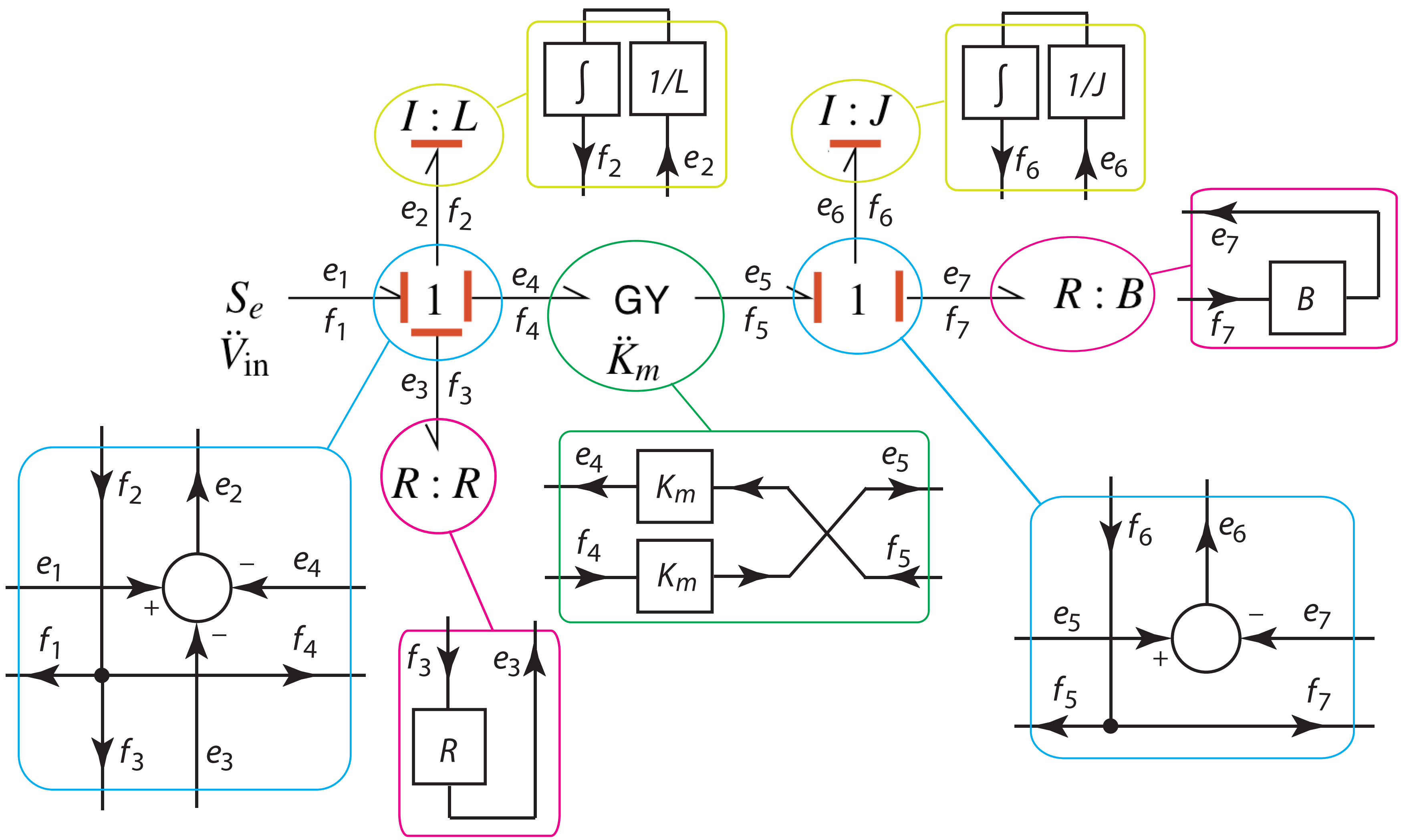


From Bonds to Blocks...

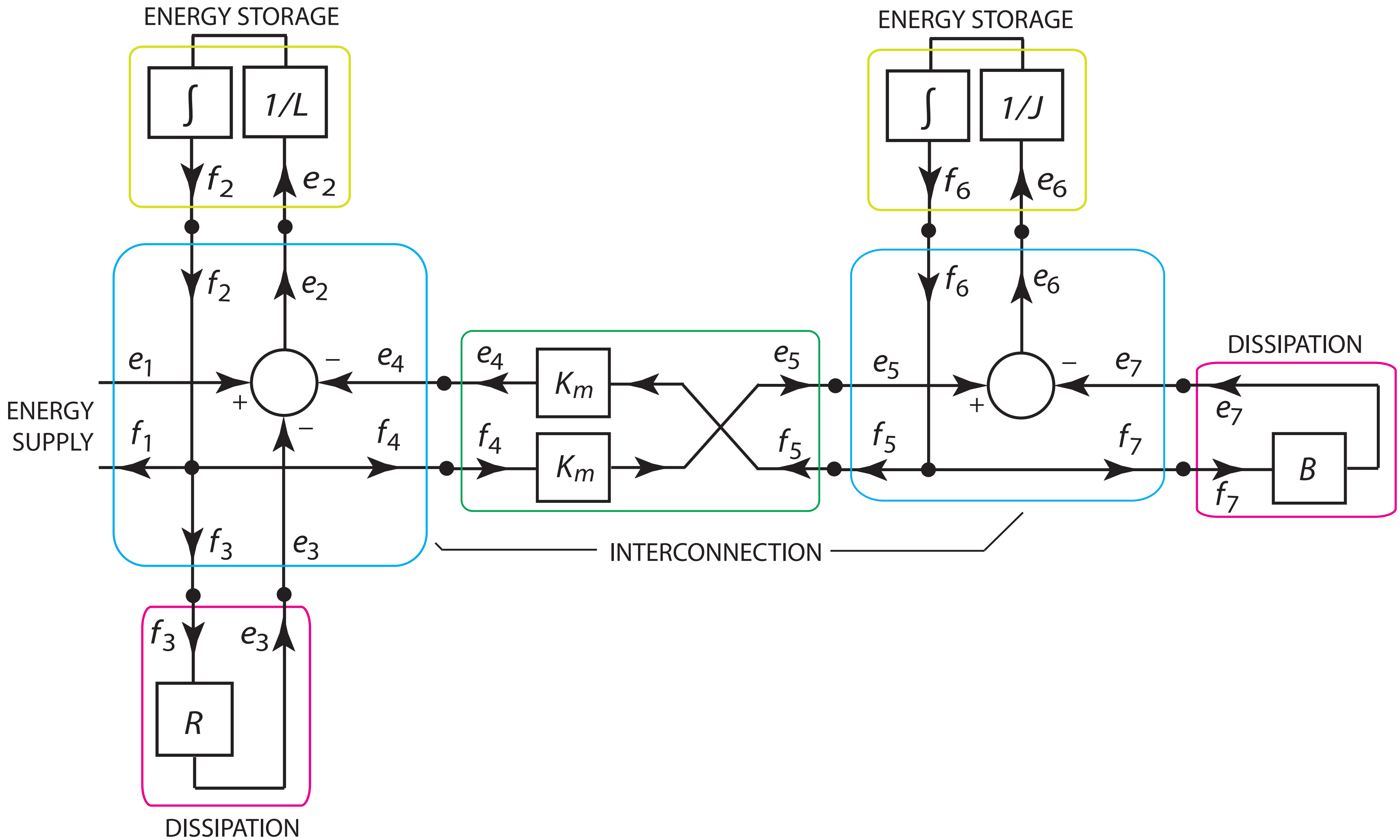




From Bonds to Blocks: DC Motor Example



From Bonds to Blocks: DC Motor Example



Causality Assignment Procedure

1a. Chose a fixed causality of a source element, assign its causality, and propagate this assignment through the graph using the causal constraints. Go on until all sources have their causalities assigned.

1b. Chose a not yet causal port with fixed causality (non-invertable equations), assign its causality, and propagate this assignment through the graph using the causal constraints. Go on until all ports with fixed causality have their causalities assigned.

2. Chose a not yet causal port with preferred causality (I- and C-elements), assign its causality, and propagate this assignment through the graph using the causal constraints. Go on until all ports with preferred causality have their causalities assigned.

Causality Assignment Procedure

3. Chose a not yet causal port with indifferent causality, assign its causality, and propagate this assignment through the graph using the causal constraints. Go on until all ports with indifferent causality have their causalities assigned.

Often, the bond graph is completely causal after step 2. Each storage element represents a state variable, and the set of equations is an explicit set of ODE's.

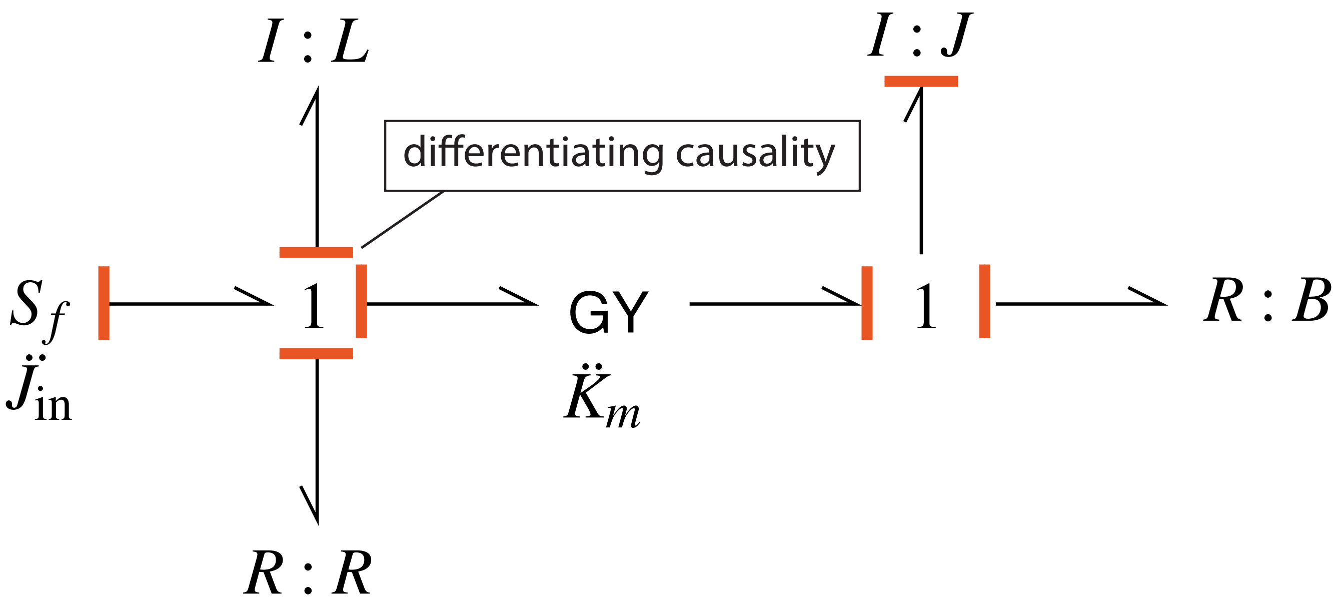
Model Insight Via Causal Analysis

If this is *not* the case, then the moment where a conflict occurs gives insight in the correctness and completeness of the model.

- Bond graph is completely causal after step 1 $\Rightarrow$ the model does not have any dynamics.
- Arises a causal conflict at step 1a or 1b, then the problem is ill-posed. For example, two effort sources connected to one 0-junction. Both sources 'want' to determine the one effort variable.
- When a conflict arises at step 2, a storage element receives a non-preferred causality.
  1. This storage element does not represent a state variable.
  2. Its initial value cannot be chosen freely.
  3. Such element is often called a dependent storage element.

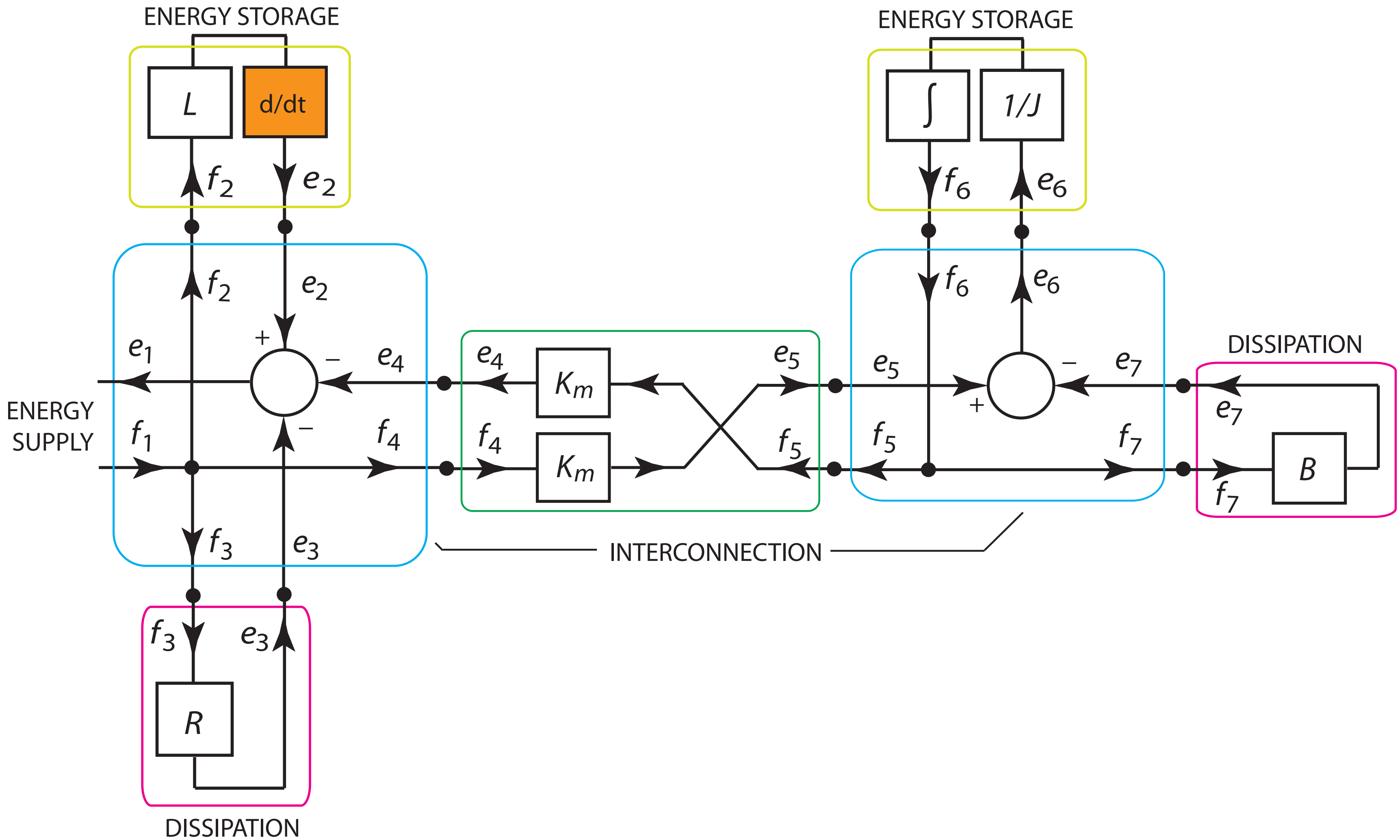
Causality Conflict

Suppose that the DC motor is driven by a current source:



We now run into trouble...

Causality Conflict



Model Insight via Causal Analysis (Cont'd)

- This indicates that a storage element was not taken into account during modelling, which should be there from physical systems viewpoint.
- When step 3 is necessary, a so-called algebraic loop is present in the graph.
  - Such loop causes the resulting set of DE's to be implicit.
  - Often this is an indication that a storage element was not modelled, which should be there from a physical systems viewpoint.
- To fix the causality conflict in the DC motor example, a C-element should be added in parallel to the current source.

Order of State Equations vs. System Order

Causal analysis also provides information on the **order of the set of equations**.

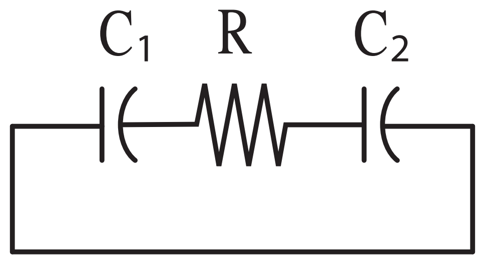
- The number of independent initial conditions equals the number of storage elements with integral causality.
- This number is called the **order of the system**.
  - For the DC motor with voltage source: system order is 2
  - For the DC motor with current source: system order is 1
- Order of the set of equations $\leq$ order of the system.
- Storage elements can depend on each other.
- Each has their own initial value, but together represent one state variable. (Their input signals are equal, or have a factor in between.)

Order of State Equations vs. System Order (cont'd)

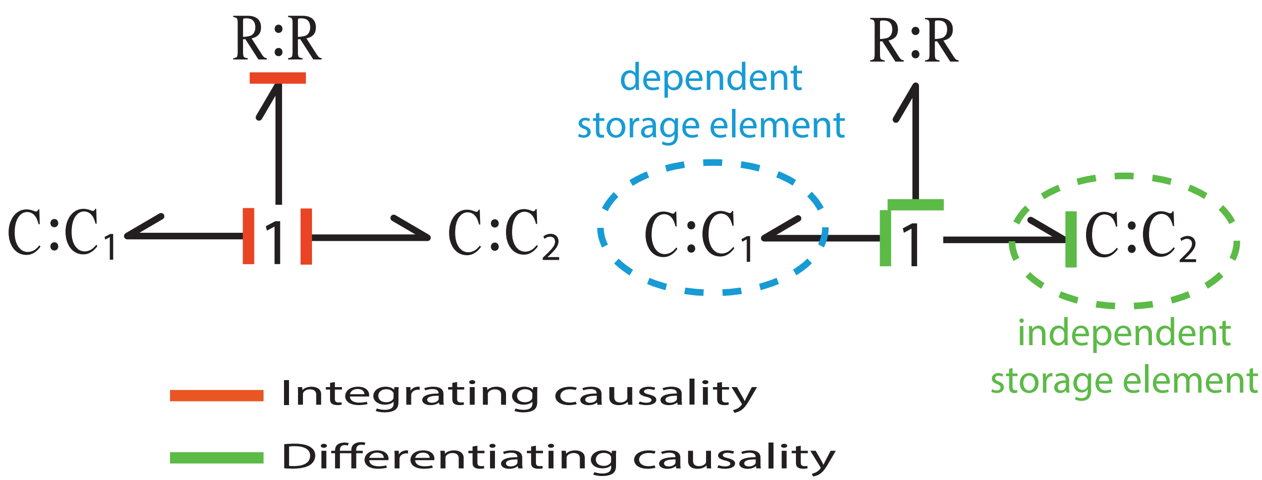
To check whether this kind of dependent storage elements show up:

- Perform causal analysis again, but now prefer differential causality.
- Those storage elements that get both differential and integral causality at each preference are the real storage elements and contribute to the state of the system.
- The order of the set of state equations is the number of storage elements that get in both cases their preferred causality.
- The storage elements that get in both cases not their preferred causality, are the dependent storage elements.
- Those storage elements that get only integral causality case using preferred causality are called semi-dependent storage elements. (This indicates that a storage element was not taken into account during modelling.)

Example 1



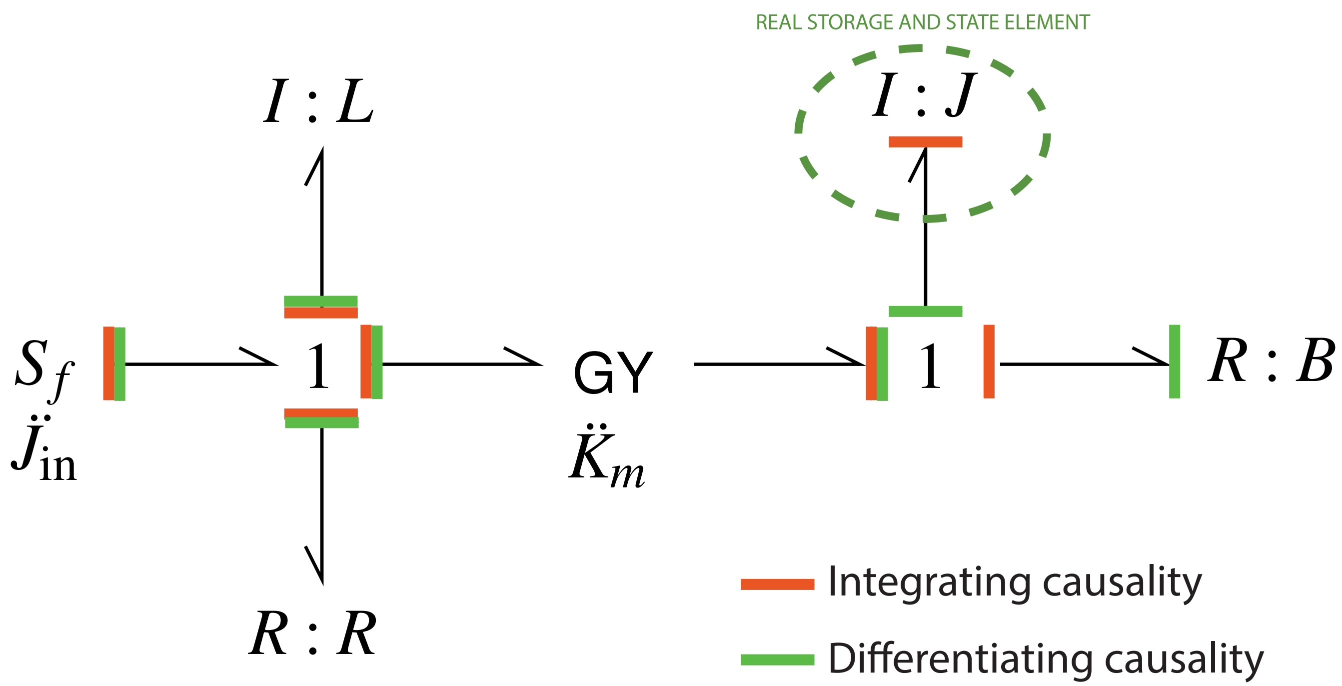
C1
R
C2



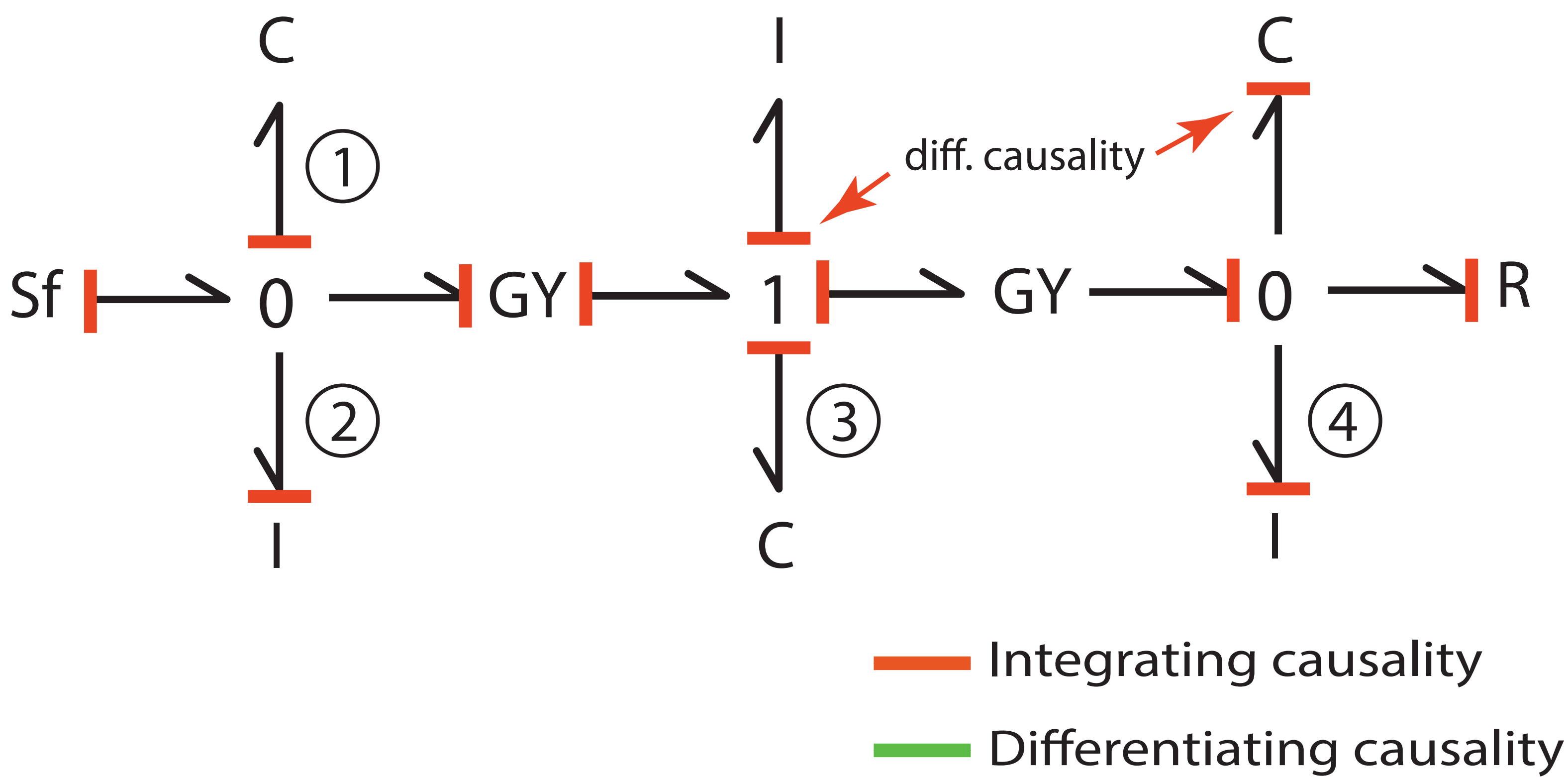
R:R
C:C1
1
C:C2
dependent storage element
R:R
C:C1
1
C:C2
independent storage element
Integrating causality
Differentiating causality

Example 2

DC motor driven by a current source:



Example 3



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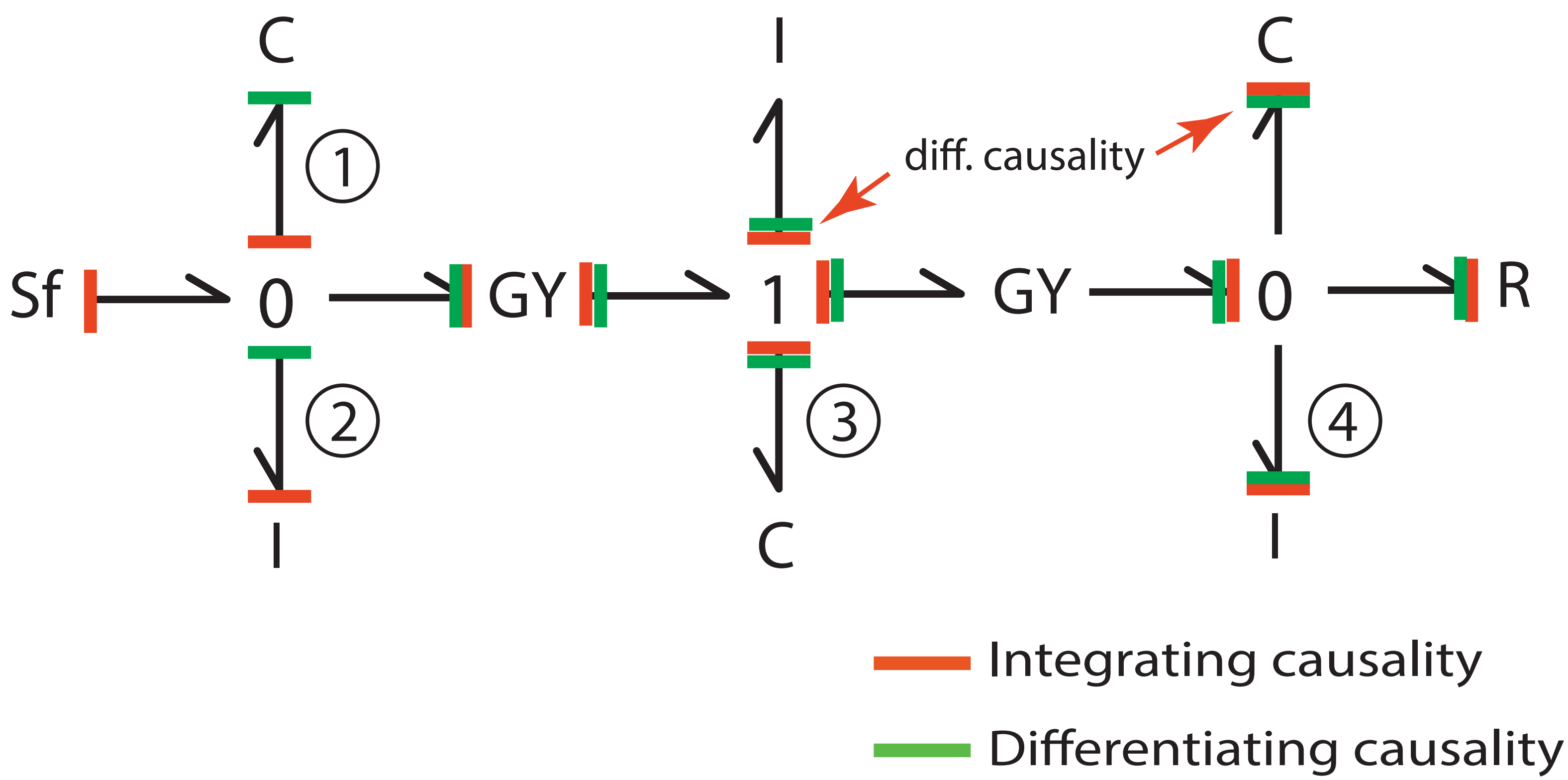
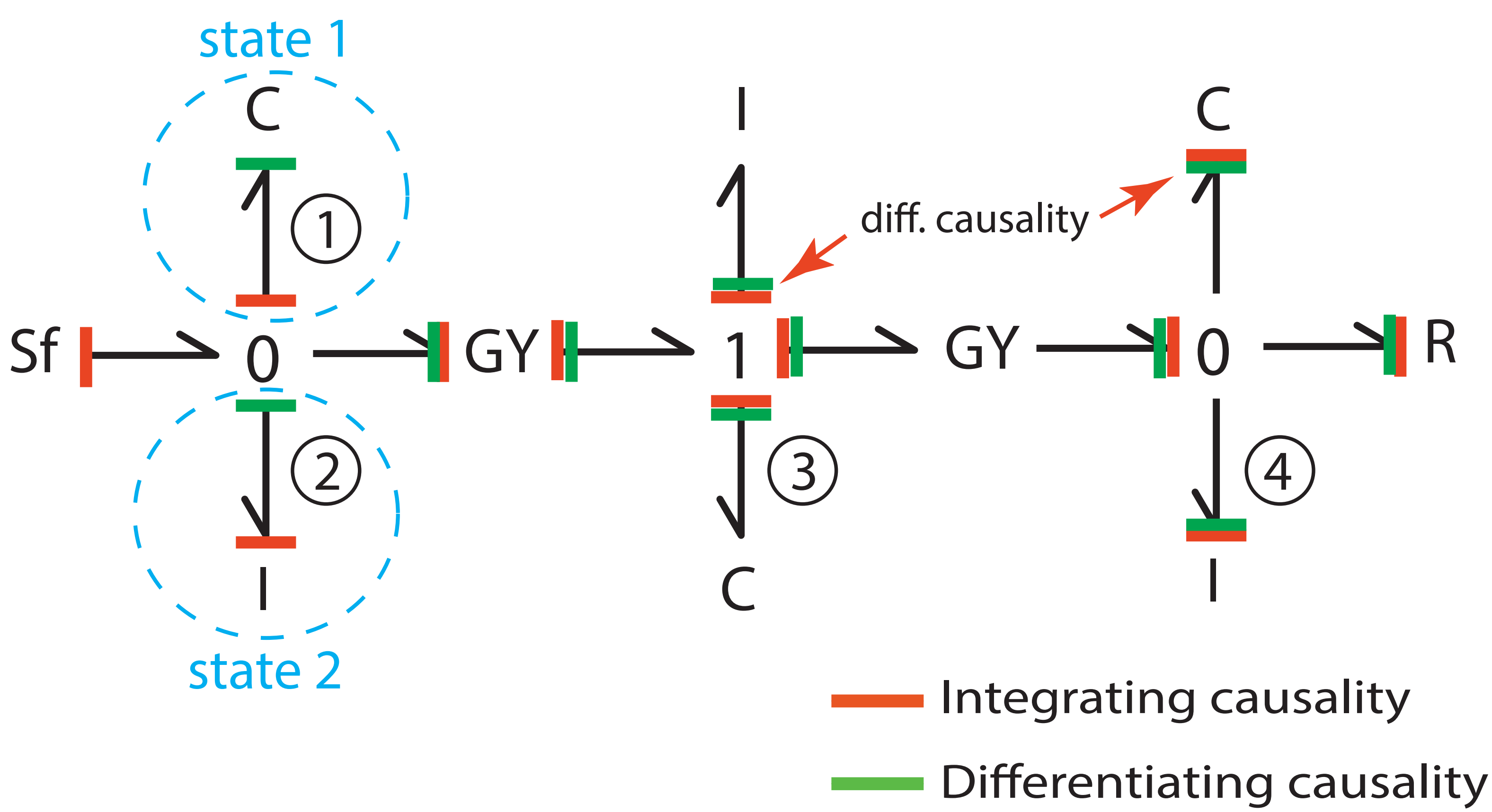
Example 3



C
I
C
1
2
3
4
diff. causality
Sf
0
GY
1
GY
0
R
I
C
I
Integrating causality
Differentiating causality

Example 3



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Summary on bond graphs I

- Bond graph modeling similar for different engineering domains.
- Based on **power flow** (change in energy).
- Easy to connect different domains with each other.
- Furthermore, it provides **physical insight**!
- Bond graphs are just a representation tool, **not** a goal in itself!
- Graphical topology (based on electrical domain).
- Effort e and flow f variables are basic variables ($P = ef$).
- Causality analysis and obtaining state-space equations from bond graphs possible.

$\vdots$

Summary on bond graphs II

⋮

- Simulation tools: 20sim, Dimola, etc.
- Note that we have treated **linear** systems.
- Nonlinear extensions possible.
- Signal flows without energy exchange possible (so-called **controlled elements**).
- Multi-bond graph extensions possible.
- Etc....

Shortcomings of block-diagram modeling I

State-space form not always possible.

Example: RC circuit with nonlinear resistor

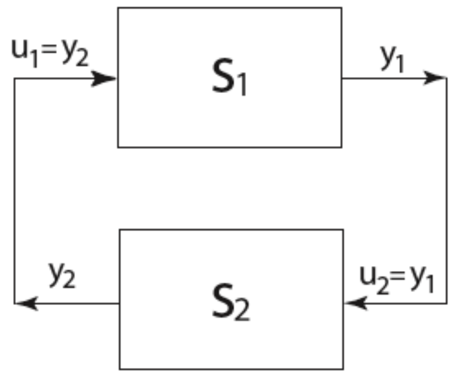
$$
\begin{aligned}
C\dot{u}_C &= i \\
u_R &= Ri^2 \\
u_S - u_C - u_R &= 0.
\end{aligned}
$$

No simple tranformation into state-space form!

Shortcomings of block-diagram modeling II

The algebraic loop problem

Interconnection of state-space models not always state-space model



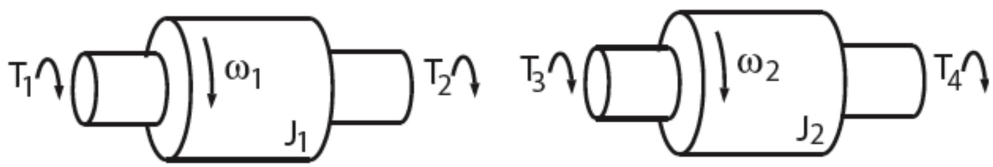
$$S_1 : \begin{cases} \dot{x}_1 &= f_1(x_1, u_1) \\ y_1 &= h_1(x_1, u_1) \end{cases} \qquad S_2 : \begin{cases} \dot{x}_2 &= f_2(x_2, u_2) \\ y_2 &= h_2(x_2, u_2) \end{cases}$$

Need to solve $u_1 = h_2(x_2, h_1(x_1, u_1))$ to find consistent states

Shortcomings of block-diagram modeling III

Insufficient interconnection structure

Interconnections often introduce algebraic constraints on states



Interconnection gives

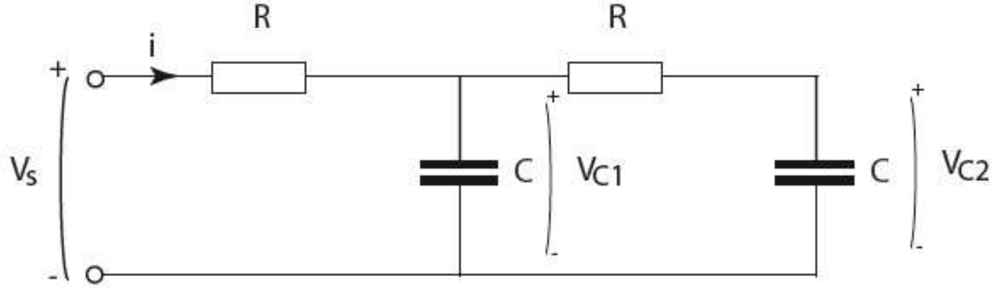
$$\omega_1 = \omega_2 \qquad\qquad T_2 = -T_3$$

Cannot be reflected in standard block diagrams!

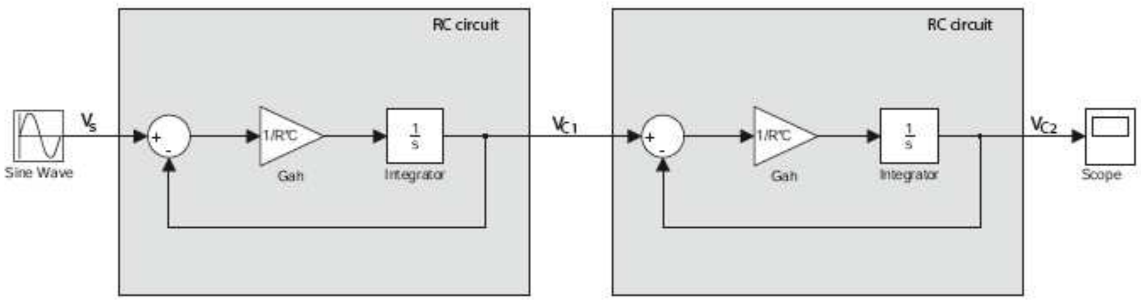
Shortcomings of block-diagram modeling IV

Hard to build model libraries

When modeling an RC filter bank,



we may be tempted to reuse our block-diagram model of the RC filter



However, this *is not correct* and leads to an erraneous model!

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