Control Systems EE2S21 – Lecture 1

Bart De Schutter

DCSC

Today:

Organizational details

History of feedback control

Mathematical basics

Instructional Staff

Lecturer:

- Bart De Schutter
  b.deschutter@tudelft.nl

Teaching assistant:

- Tomas Pippia
  t.m.pippia@tudelft.nl

Preliminaries I

Announcements/important info/downloads via Brightspace (please enroll!):

- Slides of lectures
- Assignments (practicum and homework)
- Matlab files
- Old exams, etc.

Preliminaries II

Course Material:

Feedback Control of Dynamic Systems

G.F. Franklin,
J.D. Powell, and
A. Emami-Naeini

Seventh Edition, Prentice Hall, 2014.

[Available from student organization (ETV)]

Teaching objectives

- Getting familiar with basic systems and control concepts
- Getting insight in wide range of multi-disciplinary applications of control
- Being able to compute dynamic responses and analyze stability
- Being able to design stabilizing controllers
- Being able to design PID controllers
- Acquiring basic knowledge required for advanced control courses and/or interaction with control system designers

Schedule

- Lectures
  - Mondays 15.30–17.30, room Ampere
  - Thursdays 10.30–12.30, room Aula B
  - **No lectures** on March 19 and 22
- Instructions/extra lectures:
  - Feb. 27, 15.30-17.30, room Ampere
  - Mar. 6, 15.30-17.30, room Ampere
  - Mar. 27, 15.30-17.30, room Ampere
  - Apr. 3, 15.30-17.30, room Ampere
- Simulink practicum (only for “real” EE2S21 students)
  - Mar. 8, Apr. 5: group A
  - Mar. 9, Apr. 6: group B
- Exam: Tuesday, Apr. 18, 13.30-16.30

Simulink practicum

- For "real" EE2S21 students, not for WB2230 students / "schakelstudenten WB" (see Brightspace page of WB2230 for details & options)
- Practicum is done in teams of two
- Full-out assignment report to be handed in at end of practicum or to be sent in after weekend
- Pass or fail
- Participating in and passing *both* Simulink practicums is **obligatory** before you can start EPO-4
- Even if you passed the Simulink practicums in a previous year, you will have to redo them.

Homework assignments

- 4 homeworks in total
- Individual assignments
- *Voluntary* but highly recommended
- Up to 1 bonus point can be earned (0.25 per homework)
- Bonus point is valid for 1 year, for exam and for resit

- Assignment will be published on Brightspace
- 1 week to complete
- Fill out by hand, hand in hardcopy $\rightarrow$ hard constraint!
- Hand in *before* start of lecture $\rightarrow$ hard constraint!

Overview of the course

- Feb. 12: Introduction
- Feb. 15, 19, 22: Dynamical models
- Feb. 26: Systems response and transfer functions
- Feb. 27: Linearization, stability, controllability
- Mar. 1: Observability, realization
- Mar. 5: Pole placement
- Mar. 6: Feedback control
- Mar. 12: PID
- Mar. 15: Pole placement and root locus
- Mar. 26: Frequency domain specs – Bode plots
- Mar. 27: Nyquist stability criterion

Overview of the course (continued)

- Mar. 29: Lead compensator via loop shaping
- Apr. 3: Lag compensator & PID
- Apr. 9: Digital control implementation
- Apr. 9, 12: Exercises and questions

Contents of the Lecture

- Preliminaries
- **History of Control**
  - To introduce a number of generic concepts of feedback control
  - To lift a tip of the veil of the scope of applications
  - To indicate that we are standing on the shoulders of giants
- Mathematical background knowledge
- Summary

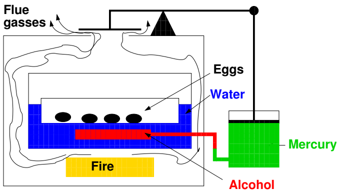
Can you ride/control this one?

Rear-wheel steering of bike

Problem is in the "Dynamics"

How to design and evaluate a Dynamic control problem?

Historical Landmarks in Feedback Control

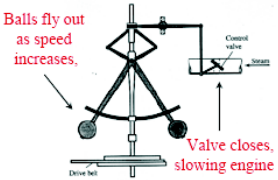


Pre-1700

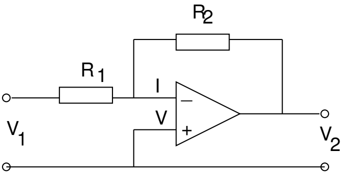
- Water clock ($\sim$ 300 BC, Alexandria), float valves
- Egg incubator (Drebbel, 1624) - tempurature control

Watt Governor (1788)

- Regulate speed of steam engines
- Reduce effects of variations in load (disturbance rejection)
- Stability analysis of unstable engine due to lubrication (J.C. Maxwell, 1868).



Historical Landmarks in Feedback Control



Emergence of Control (1920-1945)

- Black's use of negative feedback to reduce uncertainy (robustness)
- Mathematical foundation (fundamental similarities between different systems were noticed)

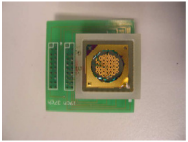
A second Wave (1960s)

- Application driven developments (Sputnik (1957), etc.)
- Use of Digital computers
- Theoretical break-throughs (Kalman (1960), etc.)

Modern Engineering Applications (DCSC)

Micro-systems

- Optical Communication
- Swarms of Nano-satellites
- Smart Structures

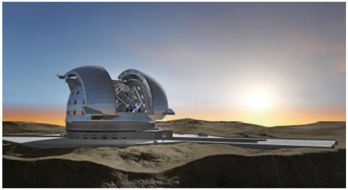


Robotics

- Home Robotics
- Humanoids
- Rescue Robots

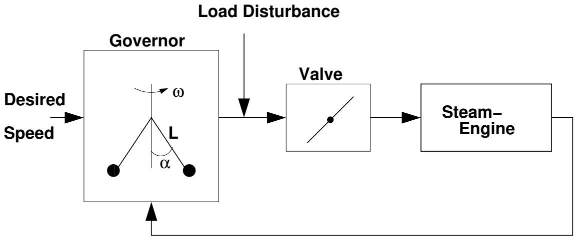
Smart Optics

- Lithography
- Micro-, Nanoscopy
- Astronomy



Magic of Feedback (1)

Schematic Representation of Watt's Governor

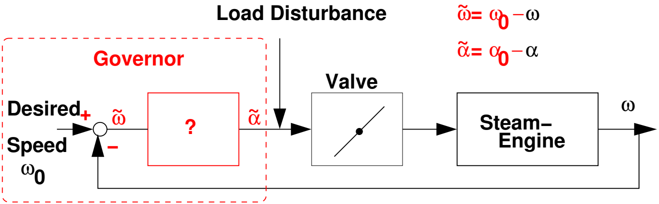


Key Components

- Sensor: measuring the speed of the engine (ω)
- Actuator: valve determining the steam input to the engine
- Calculator: relationship between sensor and actuator (ω versus α)

Feedback Law of Watt's Governor

Block Scheme Representation:

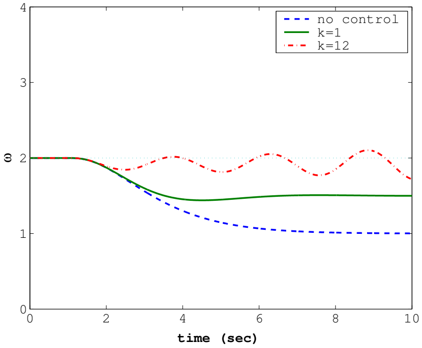


Static Analysis: $\cos(\alpha) = \frac{g}{L\omega^2} \Rightarrow -\sin\alpha_0 \partial\alpha = -\frac{2g}{\omega_0^3 L}\partial\omega$

Watt's Governor is Proportional (Negative) Feedback

- Negative Feedback: when steam engine runs harder, the valves reduces the steam input (inversion mechanism required)
- Proportional Feedback: $? \quad \approx \quad \tilde{\alpha} = \mathbf{k}\tilde{\omega}$
- Tuning controller: The size of the controller gain **k** by changing the rod length L.

Simulation results with Watt's Governor



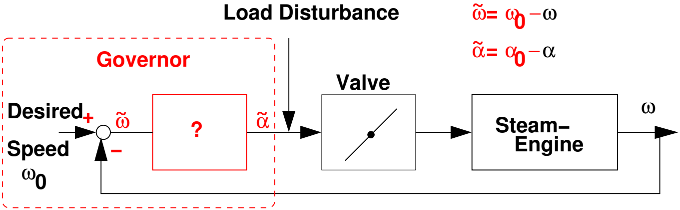
Summary Observations:

- (Proportional) Feedback influences system dynamics
- Increasing the proportional gain can:
  - reduce the steady state error
  - ⚠ destabilize the system.

Feedback Design is Making Trade-offs

Siemens modification of Watt's governor

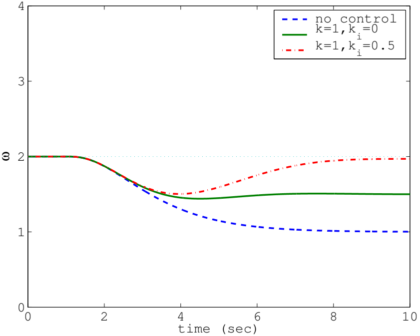
Block Scheme Representation of Watt's Governor:



Siemens modification of Watt's Governor is Proportional *plus Integral* Feedback.

- $\tilde{\alpha} = \mathbf{k}\tilde{\omega} + \mathbf{k_i} \int_0^t \tilde{\omega}(\tau) d\tau$
- What does it do?

Simulation results with Watt's Governor



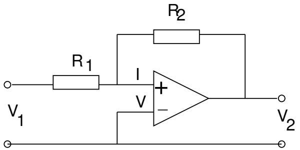
Summary Observations:

- Integral action can remove steady-state errors.

$\boxed{\text{Feedback Design is more than tuning a gain}}$

Magic of Feedback (2)

The Armstrong Amplifier (1915):



Model ($i \approx 0$): $\frac{V_1 - V}{R_1} = \frac{V - V_2}{R_2}$

Let open-loop gain amplifier be

$$V_2 = GV$$

we obtain the input-output model:

$$\frac{V_2}{V_1} = G\frac{R_2}{R_1 + R_2 - GR_1}$$

Sensitivity of gain $\frac{V_2}{V_1}$ with respect to changes in the passive components

Example: Consider

$G = 5, R_1 = 24, 24.5, 25\ \mathrm{k}\Omega,\ \ R_2 = 100\ \mathrm{k}\Omega$

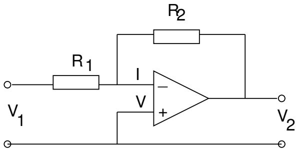
This changes the amplifier gain $\frac{V_2}{V_1}$ from $25G, 50G$ to ∞.

Explanation:

$$\frac{\partial \frac{V_2}{V_1}}{\partial R_1} = -G\frac{R_2(1-G)}{(R_1 + R_2 - GR_1)^2}$$

The Modern Break-through of Feedback

The Black Amplifier (1927):



Model ($i \approx 0$): $\dfrac{V_1 - V}{R_1} = \dfrac{V - V_2}{R_2}$

Let open-loop gain amplifier be

$$V_2 = -GV$$

we obtain the input-output model:

$$\frac{V_2}{V_1} = -\frac{R_2}{R_1} \frac{1}{1 + \frac{1}{G}\left(1 + \frac{R_2}{R_1}\right)}$$

Sensitivity of gain $\frac{V_2}{V_1}$ with respect to changes in the active component G, which typically vary from 10^5 to 10^8 can be neglected

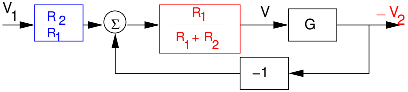
Explanation:

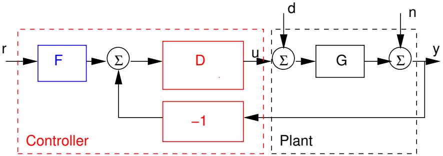
$$\frac{\partial \frac{V_2}{V_1}}{\partial G} = -\frac{R_2(R_1 + R_2)}{(GR_1 + (R_1 + R_2))^2}$$

Value and Abstraction of Feedback Amplifier

IEEE Lamme Medal (1957): *"... the entire explosive extension of the area of control, both electrical and mechanical (fluid, heat, ...), grew out of the understanding of the feedback principle ..."*

Standard Feedback Block-scheme

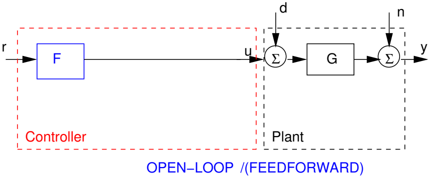


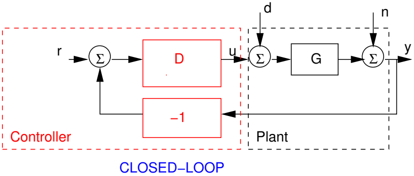


Convention: positive plant gain and negative feedback gain

Generic Controller Architectures

Open- and Closed-loop configuration:



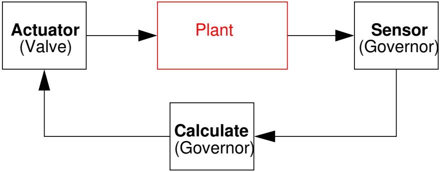


Reference Tracking

Disturbance reduction & robustness

Summary of Magic of Feedback

Feedback consists of a sensing, actuation AND calculation element



Feedback has the potential to

1. reduce the effect of (load) disturbances
2. change the dynamic response
3. ⚠ destabilize the system

> Feedback involves making compromise (trade-off) between different performance criteria (1-2) and stability (3)

Key Steps in Feedback Control Design

- Model the process (using insights in the physics, etc.)
- Analyze the model (stability, response, etc.)
- Design a controller to fulfill given performance criteria

$\rightarrow$ that is what we will do in this course

Required background knowledge

- Roots of quadratic functions and higher-degree polynomials
- Linear differential equations
- Laplace transforms
- Partial fraction expansions
- Complex numbers
- Dynamics of electrical and mechanical systems

Mathematical Background (Recap)

1. Dynamical Models
   - Differential equations
   - Transfer functions
2. Laplace Transform
3. Response to a sinusoid
4. Final Value Theorem (FVT)

Basic System Analysis

$\boxed{\text{Differential equation}}$

$$\ddot{y}(t) + a_1\dot{y}(t) + a_2 y(t) = bu(t)$$

with Laplace transform: $Y(s) = \int_{0^-}^{\infty} y(t)e^{-st}dt \; \left(y(t) \xrightarrow{\mathcal{L}_-} Y(s)\right)$

$\boxed{\text{Transfer function (TF):}}$

$$H(s) := \frac{Y(s)}{U(s)} = \frac{b}{s^2 + a_1 s + a_2}$$

while $y(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau$ with $h(\tau) \xleftarrow{\mathcal{L}_-^{-1}} H(s)$ being the impulse response of the system.

Input-output (i/o) relation through transfer function

For

$$u(t) = e^{\lambda t} \qquad \lambda \in \mathbb{C}$$

then

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} h(\tau) u(t-\tau) d\tau \\ &= \underbrace{\int_{-\infty}^{\infty} h(\tau) e^{-\lambda\tau} d\tau}_{H(s)|_{s=\lambda}} \cdot e^{\lambda t} \end{aligned}$$

so that

$$y(t) = H(s)|_{s=\lambda} \cdot u(t)$$

Response of a system to a sinusoidal input

Let

$$u(t) = A\cos(\omega t) = \frac{A}{2}\left[e^{j\omega t} + e^{-j\omega t}\right]$$

then

$$y(t) = \frac{A}{2}\left[H(j\omega)e^{j\omega t} + H(-j\omega)e^{-j\omega t}\right]$$

If $H(j\omega) = |H(j\omega)|e^{j\angle H(j\omega)} = M(\omega)e^{j\phi(\omega)}$ then

$$y(t) = A \cdot M \cdot \cos(\omega t + \phi)$$

$H(j\omega)$ is the frequency response function (FRF) (complex valued)

Final value theorem

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s)$$

Consequence:

If u is a unit step, $U(s) = \frac{1}{s}$,

$$\begin{aligned} \lim_{t \to \infty} y(t) &= \lim_{s \to 0} s \cdot G(s) \cdot \frac{1}{s} \\ &= \lim_{s \to 0} G(s) \end{aligned}$$

is DC-gain of G

Example FVT

Consider the transfer function of a laser positioning system in a copy machine:

$$Y(s) = \frac{5(s+100)}{s^2+60s+500} R(s)$$

What is the final value of the output $y(t)$ ($\lim_{t\to\infty} y(t)$) when the input $r(t)$ is a unit step?

$$\begin{aligned}\lim_{t\to\infty} y(t) &= \lim_{s\to 0} sY(s) \\ &= \lim_{s\to 0} s\frac{5(s+100)}{s^2+60s+500}\frac{1}{s} \\ &= \boxed{1}\end{aligned}$$

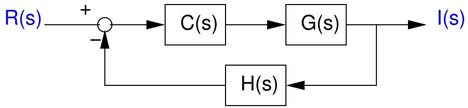
⚠ What is the final value of $y(t)$ when it is the unit step response of the system with transfer function:

$$G(s) = \frac{K}{s-1}$$

Is it $-K$?

Calculation Transfer functions of a closed-loop system

Consider a block scheme representation of controlling a lamp's intensity by an opto-transistor feedback loop:



Determine the transfer function $\frac{I(s)}{R(s)}$?

$$
\begin{aligned}
I(s) &= G(s)C(s)\Big(R(s) - H(s)I(s)\Big) \\
\Big(I + G(s)C(s)H(s)\Big)I(s) &= G(s)C(s)R(s) \\
\frac{I(s)}{R(s)} &= \Big(1 + \boxed{G(s)C(s)H(s)}\Big)^{-1}\boxed{G(s)C(s)}
\end{aligned}
$$

Rule for Negative Feedback SISO systems: The direct-through path over one plus the loop gain (counterclockwise!)