Systeem- en Regeltechniek
EE2S21

Bond Graphs + Block Diagrams

Lecture 3

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DIAM / DCSC

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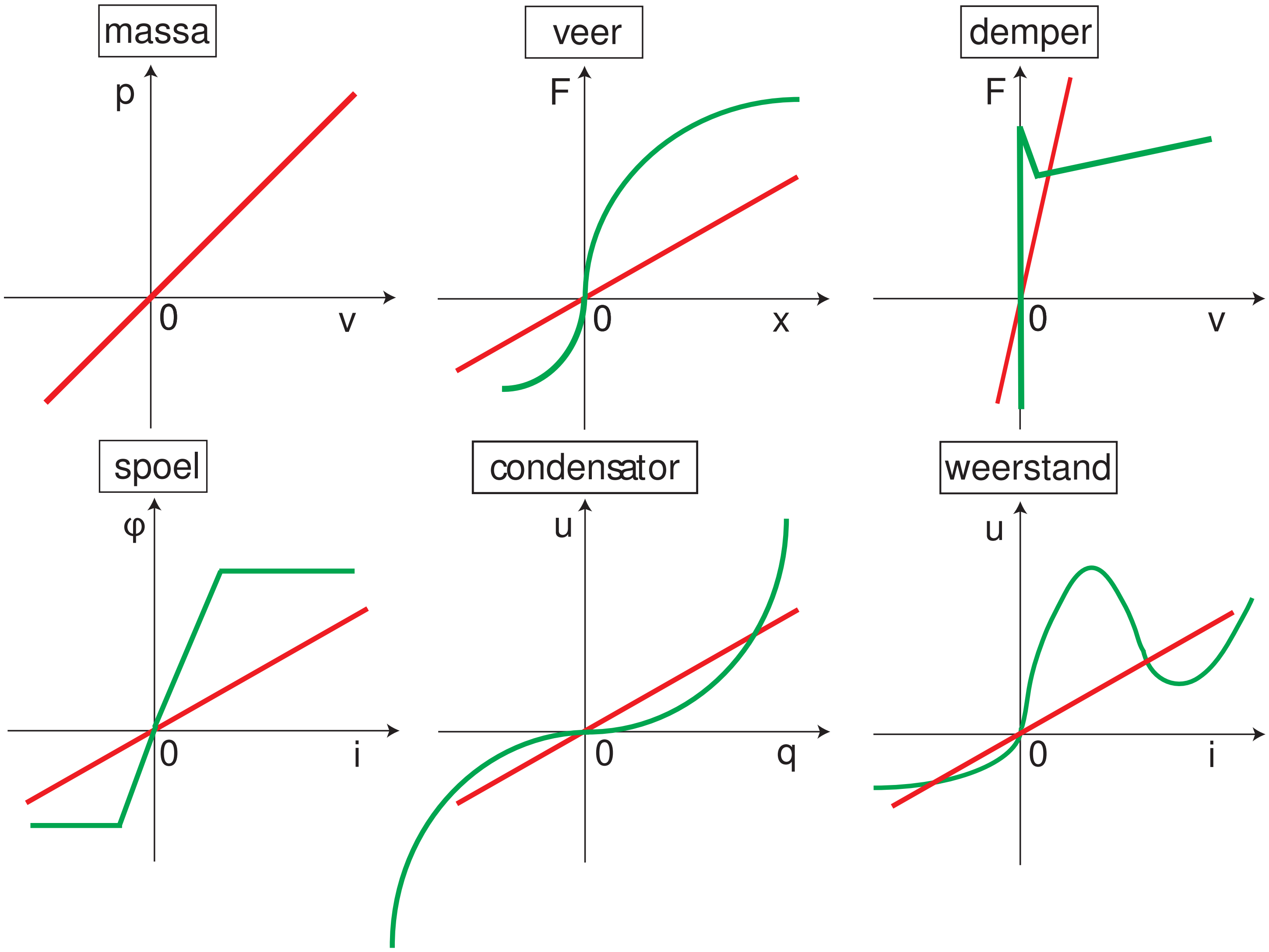
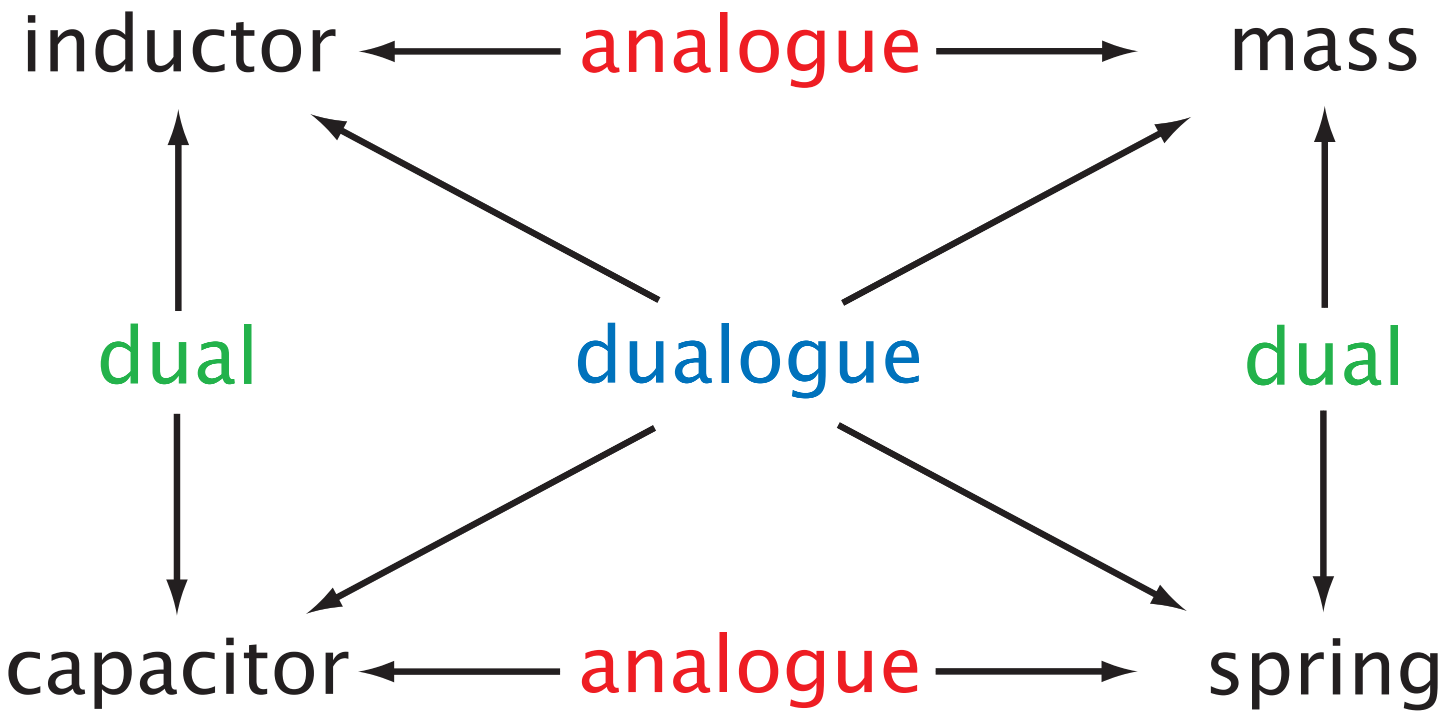
Constitutive relationships



Classical force-voltage or mass-inductance analogy:



	Effort	Flow	Gen. Position	Gen. Momentum
	e	f	q	p
Electric	voltage u [V]	current i [A]	charge q [C]	flux ϕ [Vs]
Translation	force F [N]	velocity v [m/s]	displ. x [m]	mom. p [Ns]
Rotation	torque τ [Nm]	angular vel. ω [rad/s]	angl. displ. θ [rad]	rot. mom. L [Nms]
Hydraulic	pressure p [N/m^2]	vol. flow Q [m^3/s]	volume V [m^3]	press. mom. Γ [Ns/m^2]
Thermo-dynamic	temp. T [K]	entropy flow f_T [WK^{-1}]	entropy S [J/K]	-

Generalized Elements:

- "**I**" elements: $f = \hat{f}(p)$ or $p = \hat{p}(f)$

  $\Rightarrow$ masses, inductors, etc.

- "**C**" elements: $e = \hat{e}(q)$ or $q = \hat{q}(e)$

  $\Rightarrow$ springs, capacitors, etc.

- "**R**" elements: $e = \hat{e}(f)$ or $f = \hat{f}(e)$

  $\Rightarrow$ dampers, resistors, etc.

- "**S**" elements: $e = \hat{e}(f)$ or $f = \hat{f}(e)$

  $\Rightarrow$ supplied forces, voltage source, etc.

- "**TF**" and "**GY**" elements: $e_1 = \hat{e}_1(f_1)$ and $f_2 = \hat{f}_2(e_2)$

  $\Rightarrow$ transformers and gyrators.

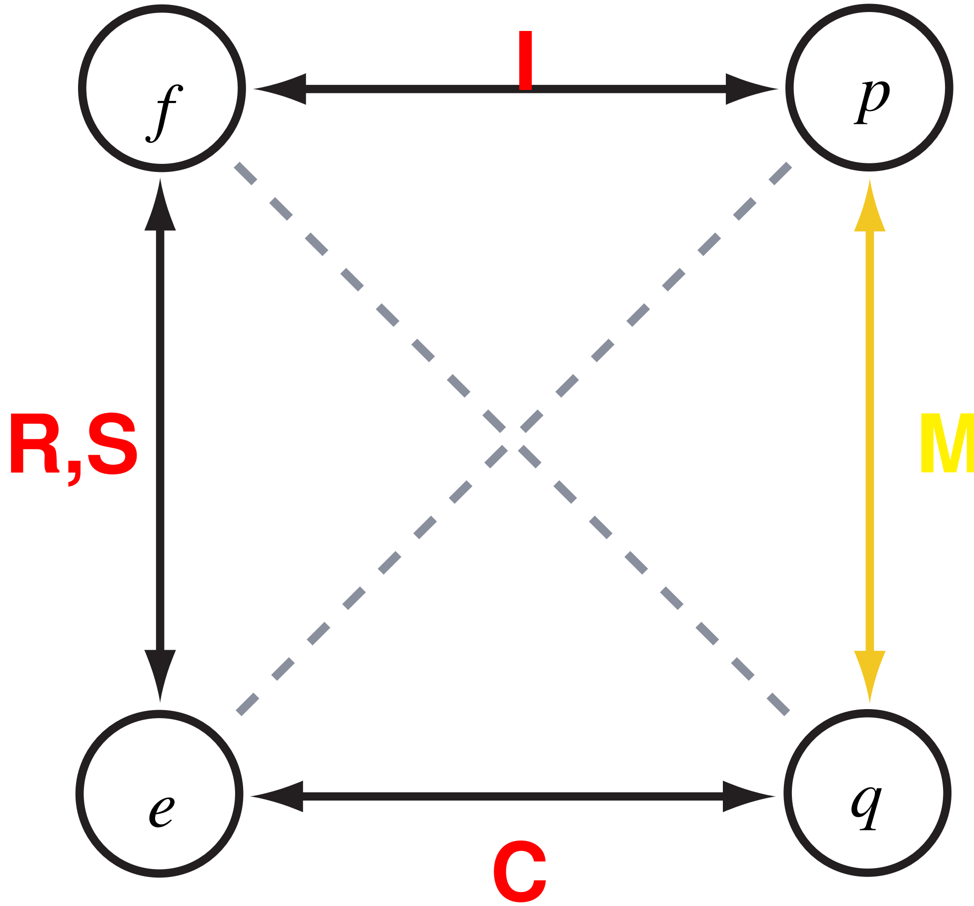
Generalized Dynamical Relationships:

- $f(t) = \dfrac{dq(t)}{dt}$, or $q(t) = q(t_0) + \int_{t_0}^{t} f(\tau)d\tau$

- $e(t) = \dfrac{dp(t)}{dt}$, or $p(t) = p(t_0) + \int_{t_0}^{t} e(\tau)d\tau$

Note: Generalized component relationships follow in similar way. Generalized interconnective relationships are called **junctions** (treated later with **bond graphs**).

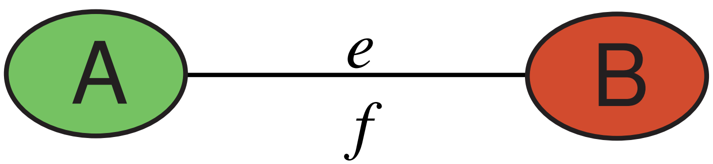
Four Element Quadrangle:



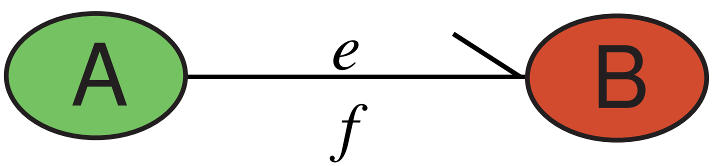
"M" stands for generalized memristor: $p = \hat{p}(q)$ or $q = \hat{q}(p)$.

Bond graphs I

- Language for physical modeling that explicitly shows the **interconnection** of the physical elements and the **energy** that is exchanged between them $\Rightarrow$ **power flow**.
- Power $=$ effort $\times$ flow. (voltage $\times$ current or force $\times$ velocity)
- In terms of **effort** variable e, **flow** variable f



bond: exchange of energy



positive direction of power flow

- Very powerful to **connect** different engineering domains.

Bond graphs II

- Linear **I**-element: $\xrightarrow[f]{e} \; I : I_i$

$$f(t) = f(0) + \frac{1}{I_i}\int_0^t e(\tau)d\tau$$

- Linear **C**-element: $\xrightarrow[f]{e} \; C : C_i$

$$e(t) = e(0) + \frac{1}{C_i}\int_0^t f(\tau)d\tau$$

- Linear **R**-element: $\xrightarrow[f]{e} \; R : R_i$

$$e(t) = R_i f(t), \text{ or } f(t) = \frac{1}{R_i}e(t)$$

Bond graphs III

- **S**-elements:
  - Effort source: $S_e : e_{S_i} \overset{e}{\underset{f}{\rightharpoonup}}$
  - Flow source: $S_f : f_{S_i} \overset{e}{\underset{f}{\rightharpoonup}}$

Next: **interconnection structure...**

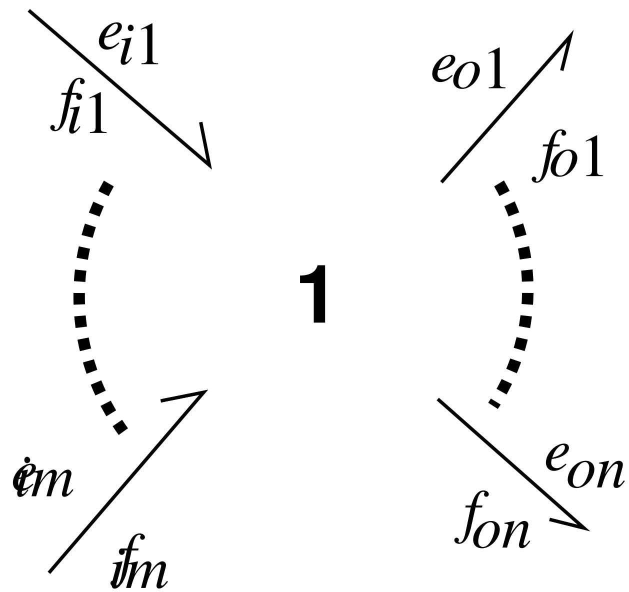
Bond graphs IV

Interconnection in bond graphs $\Rightarrow$ **Junction structure**

1-Junction (or flow junction):

$$\sum_{k=1}^{m} e_{ik} = \sum_{k=1}^{n} e_{ok}$$

$$f_{i1} = \cdots = f_{im} = f_{o1} = \cdots = f_{on}$$

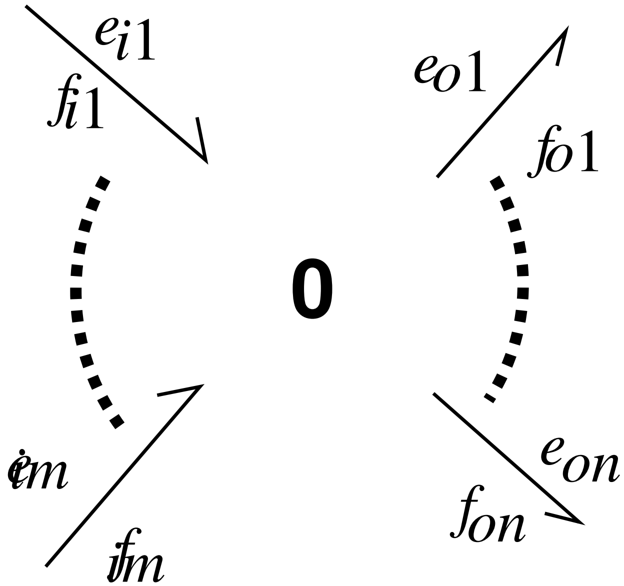


$\Rightarrow$ e.g., Kirchhoff's voltage law (KVL)

Bond graphs V

0-Junction (or effort junction):

$$\sum_{k=1}^{m} f_{ik} = \sum_{k=1}^{n} f_{ok}$$



$$e_{i1} = \cdots = e_{im} = e_{o1} = \cdots = e_{on}$$

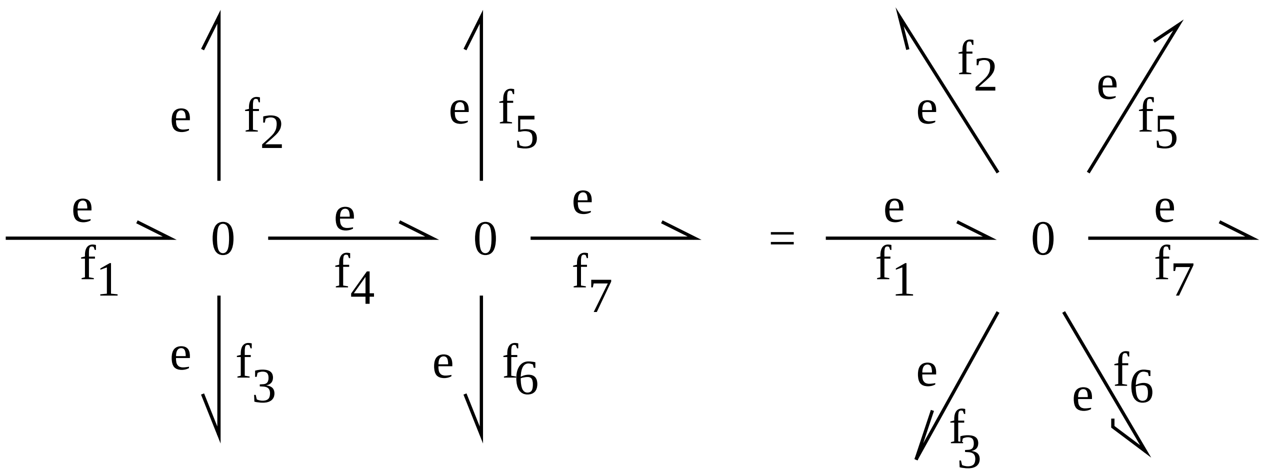
$\Rightarrow$ e.g., Kirchhoff's current law (KCL)

Note: **Power continuity** for both junction structures (verify!)

Simplifications of the Bond Graphs

$$\xrightarrow[f_1]{e} \; 0 \; \xrightarrow[f_2]{e} \quad = \quad \xrightarrow[f_1 = f_2]{e}$$

$$\xrightarrow[f]{e_1} \; 1 \; \xrightarrow[f]{e_2} \quad = \quad \xrightarrow[f]{e_1 = e_2}$$



Similar

for 1

junction

Systematic procedures I

Electrical domain:

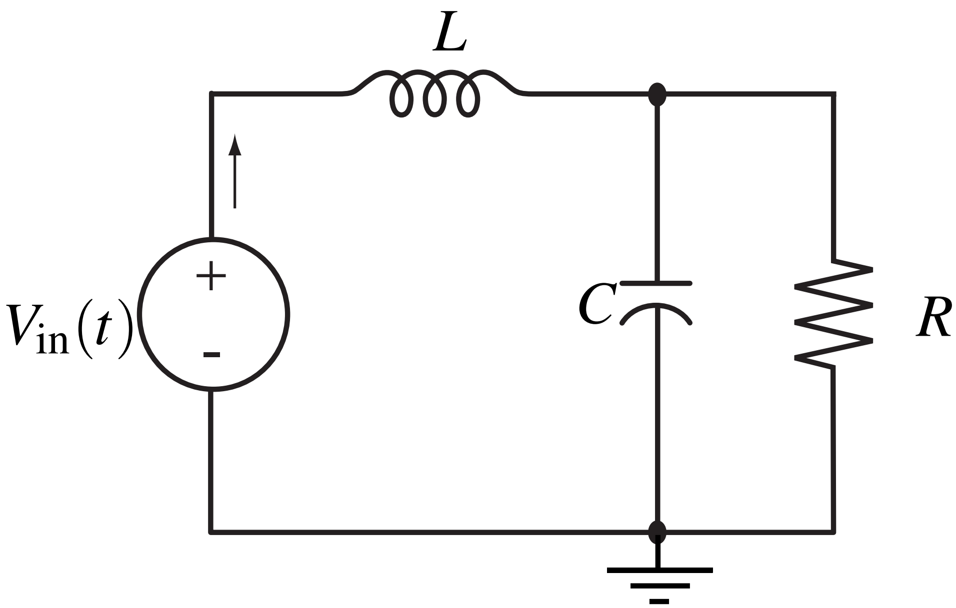
- 0-junction at every well-defined potential
- 1-junction with every **I**, **C**, **R**, or **S** element
- use grounded points that have zero-voltage to remove bonds
- use simplification rules.

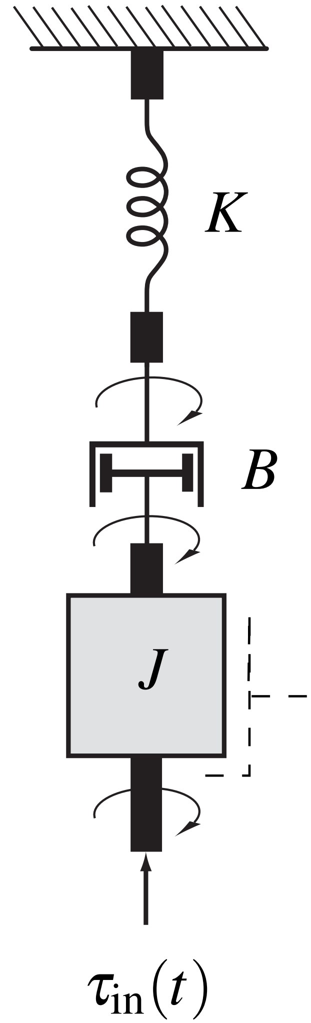
Systematic procedures II

Mechanical domain:

- 1-junction at every “fixed” speed
- 0-junction to make speed difference, and additionally 1-junction to use the speed difference as a “fixed” speed
- introduce the elements
- use simplification rules
- use zero velocity points to remove bonds.

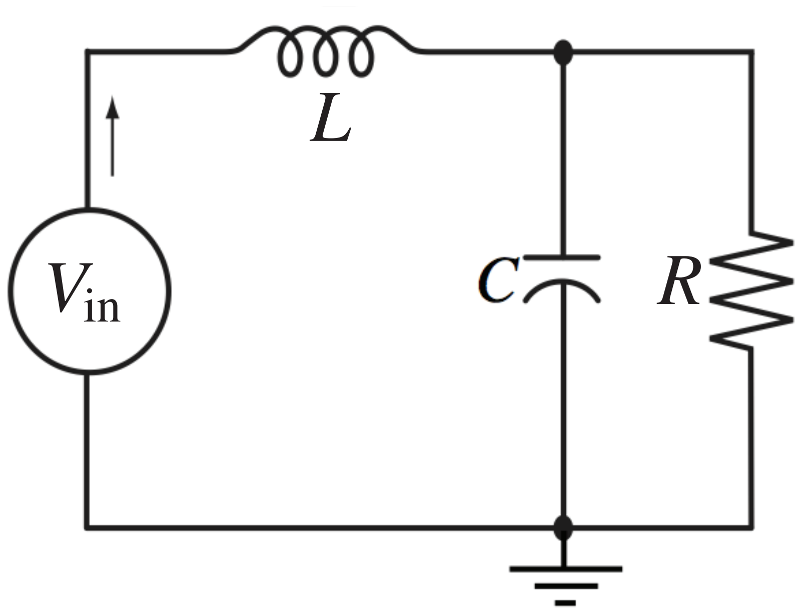
Example: Electrical and Mechanical System



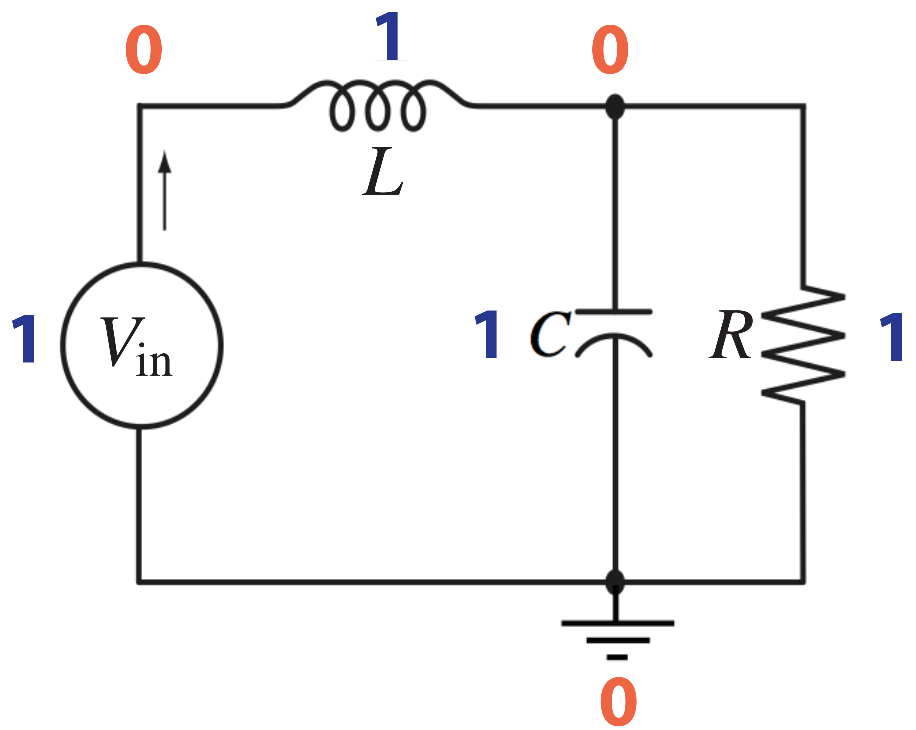


Draw the bond graphs...

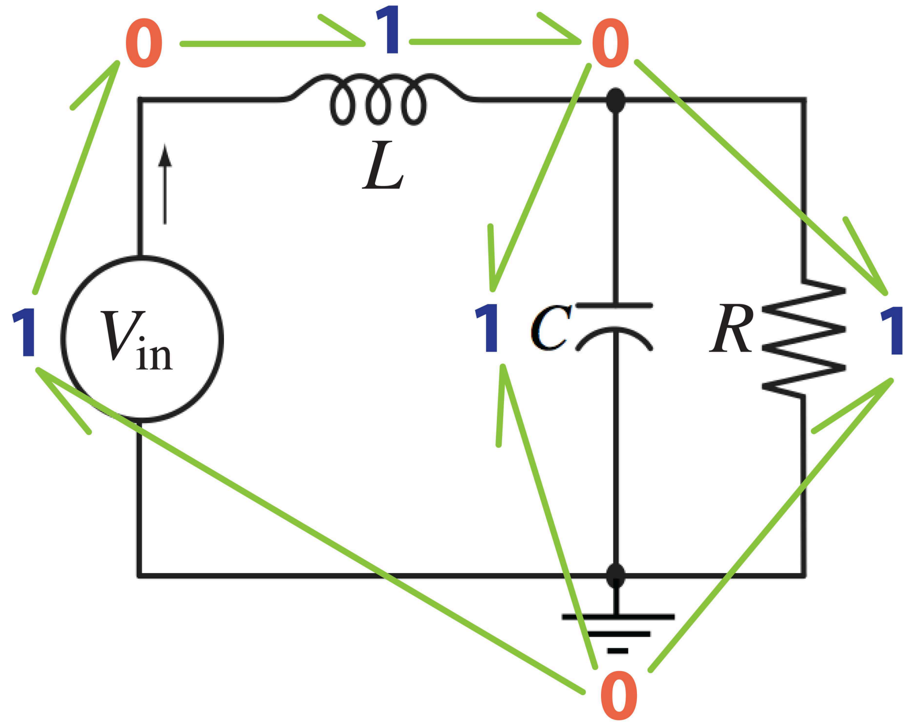
Example: Electrical System



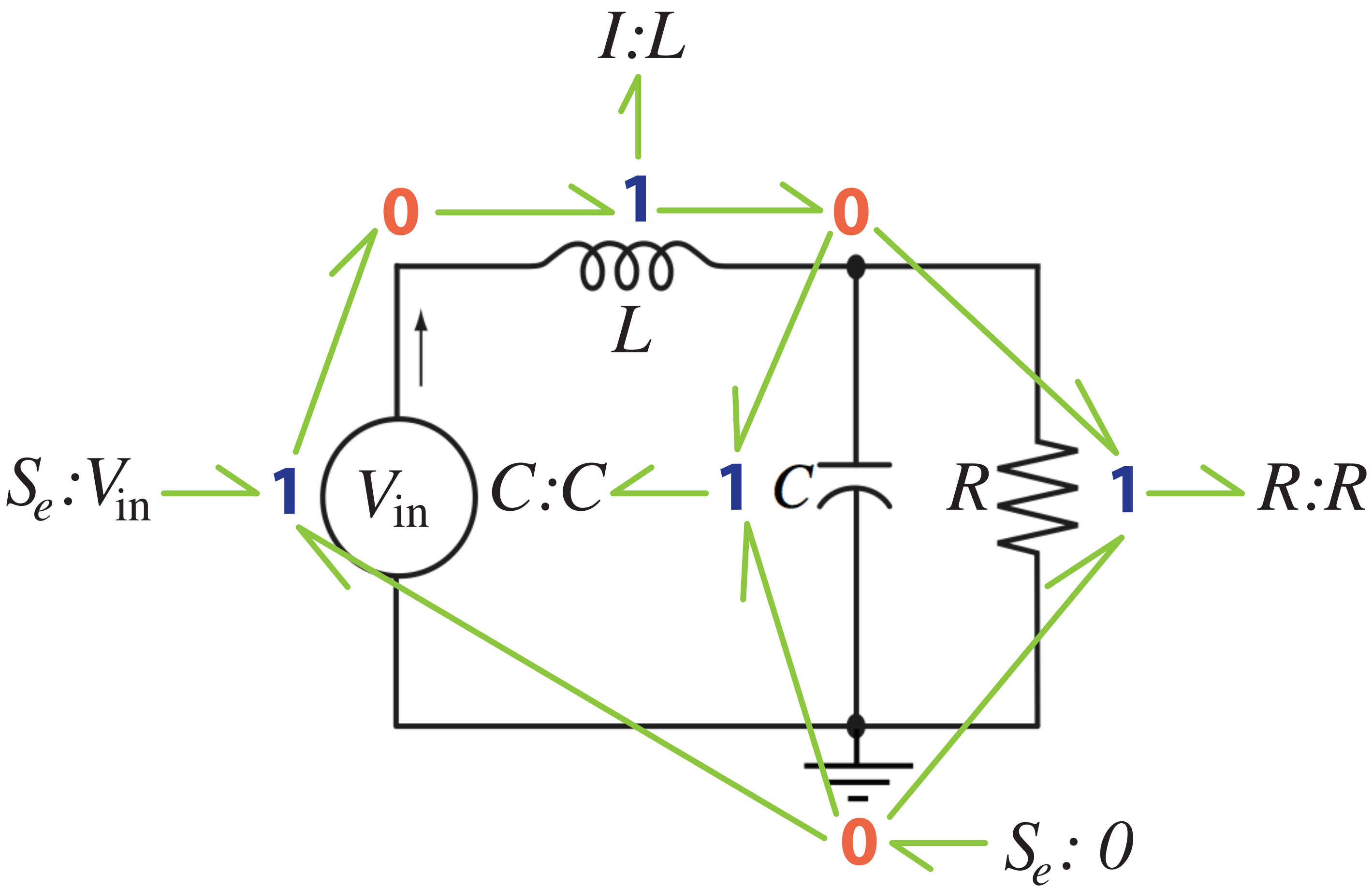
Example: Electrical System



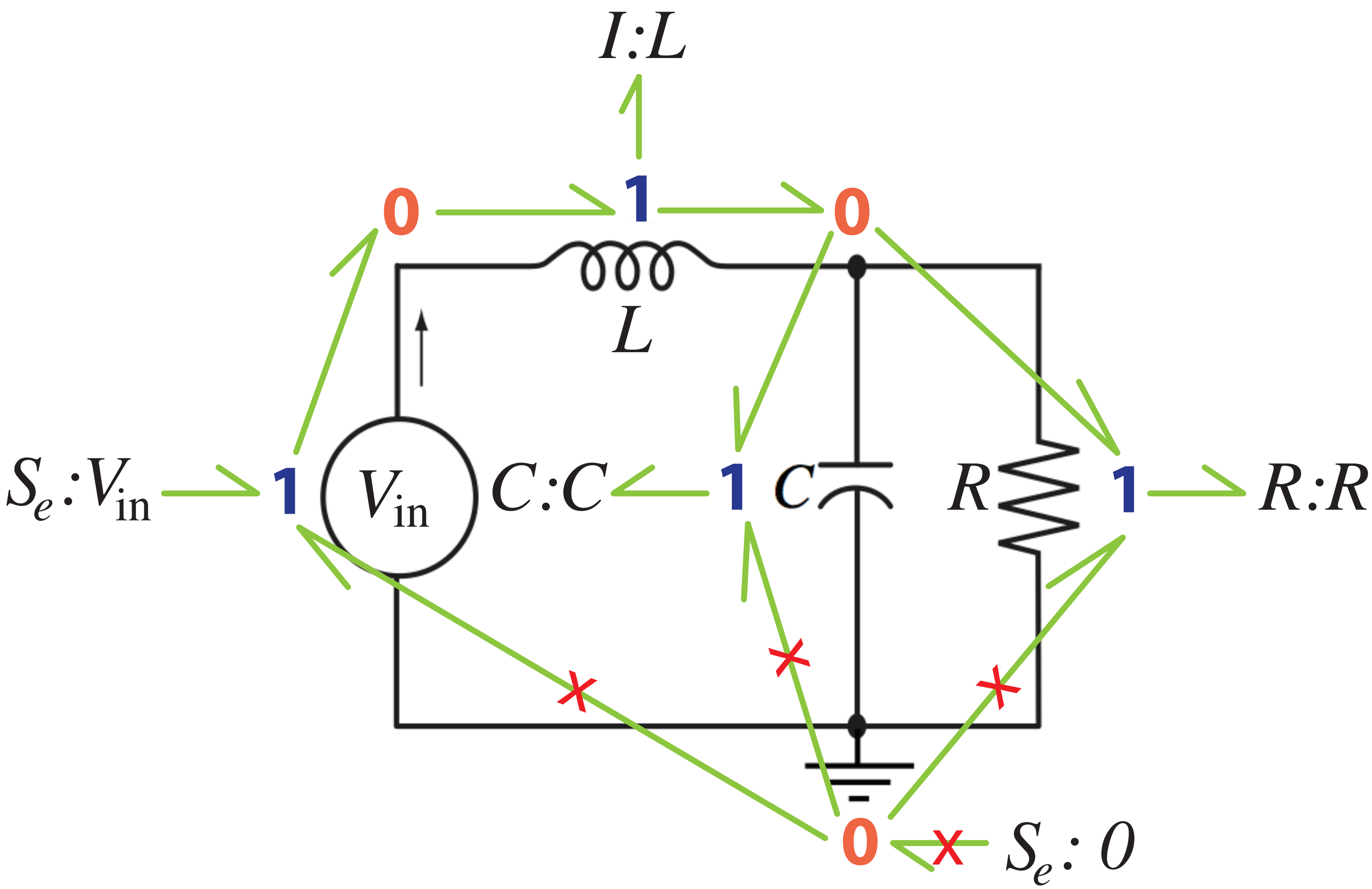
Example: Electrical System



Example: Electrical System



Example: Electrical System



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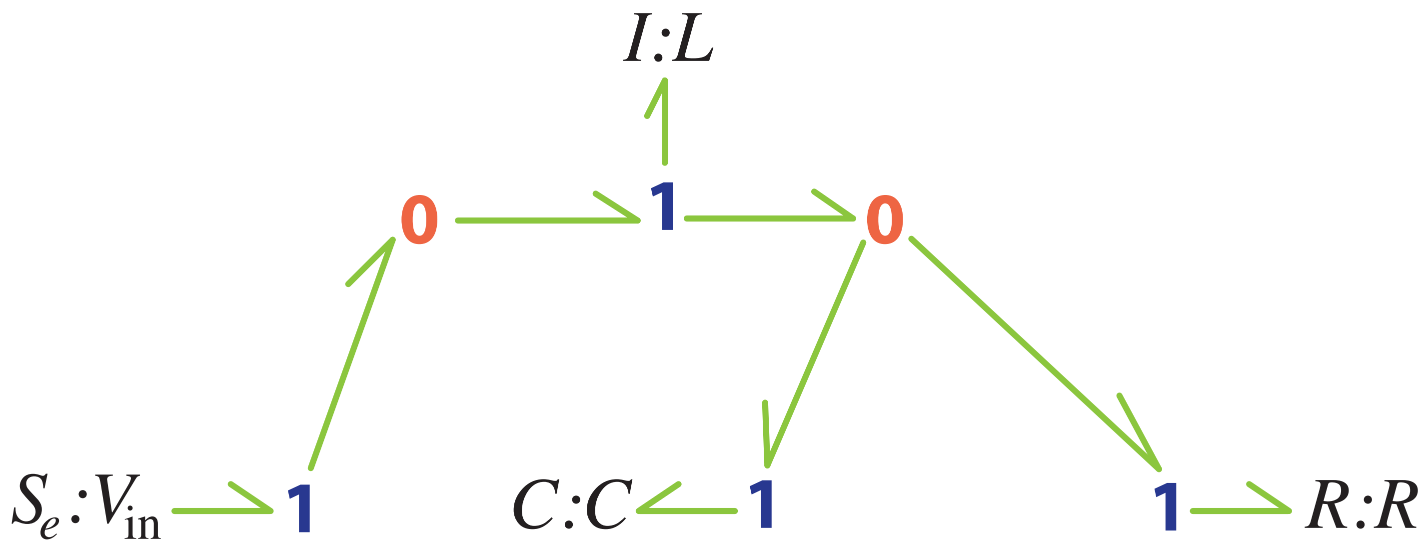
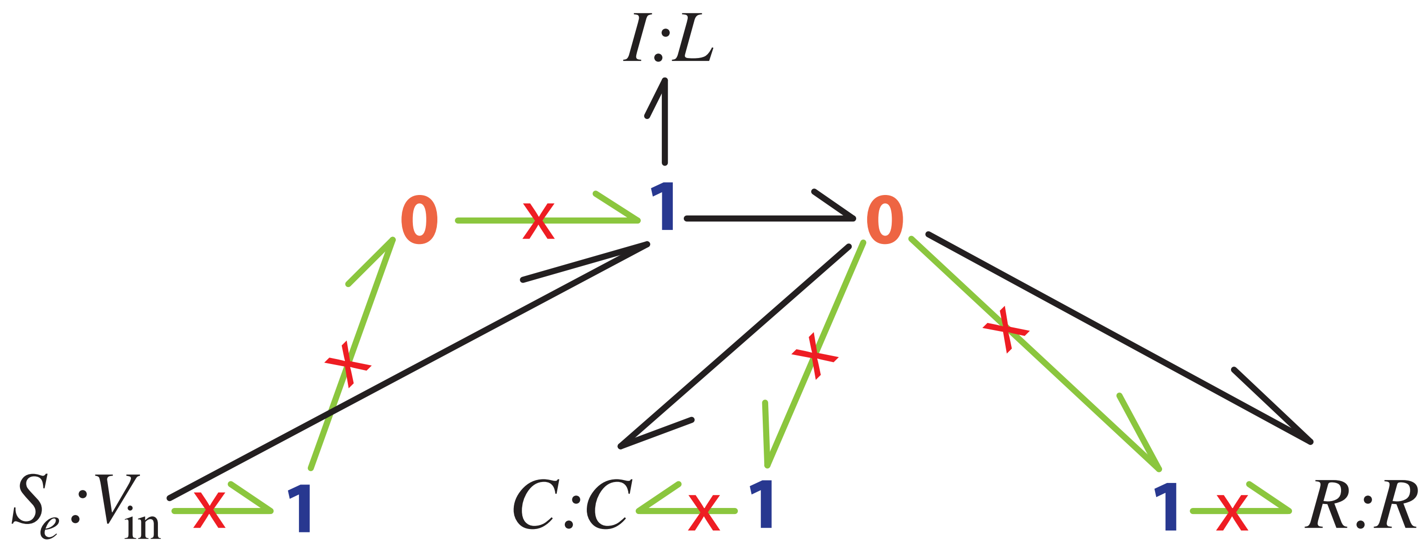
Example: Electrical System



Example: Electrical System



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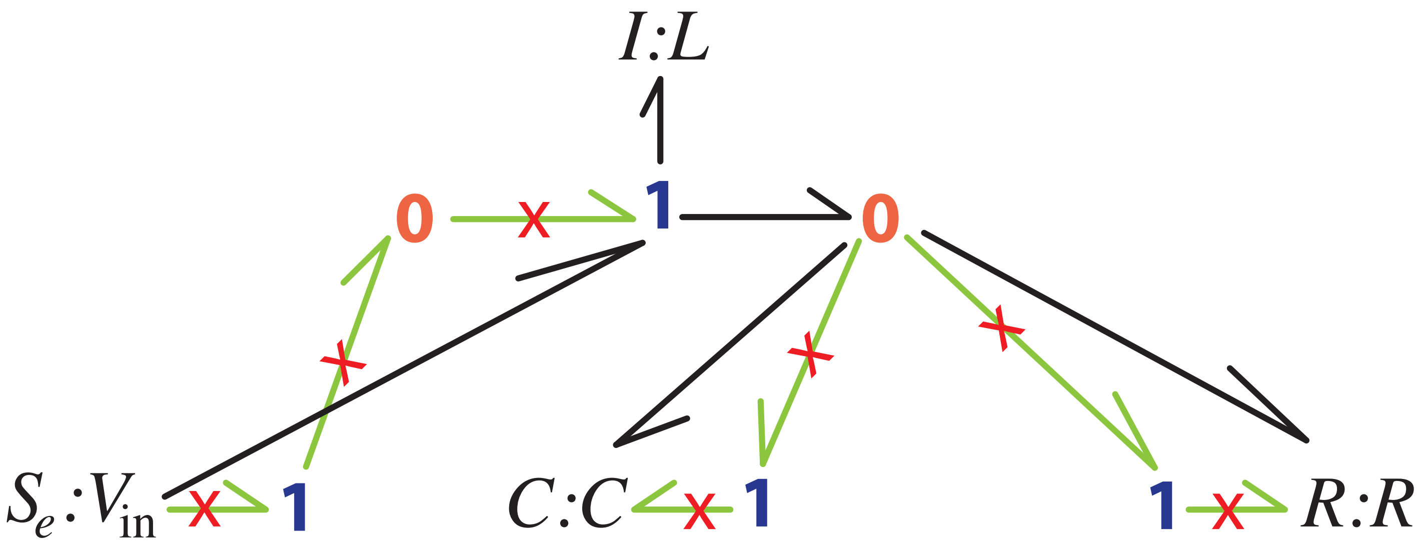
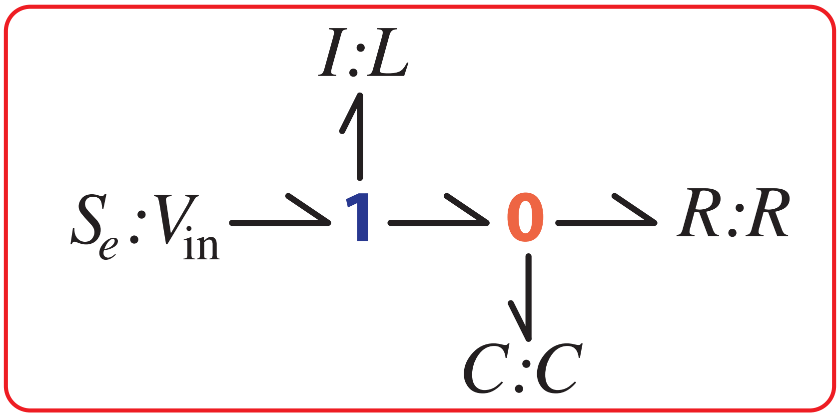
Example: Electrical System





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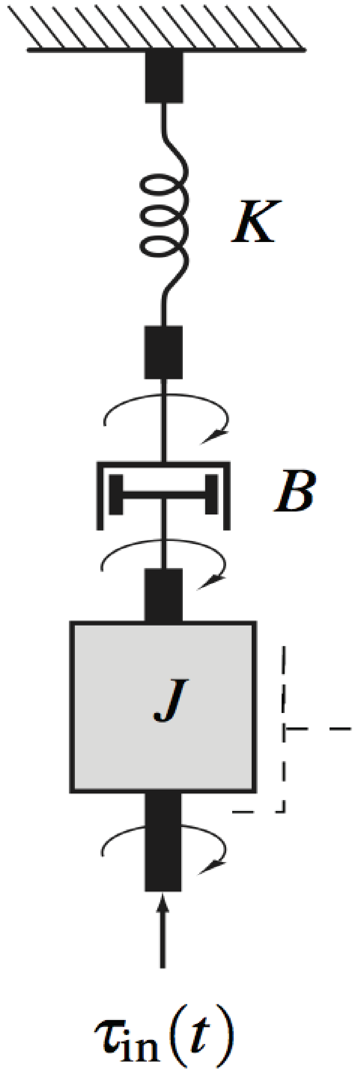
Example: Mechanical System



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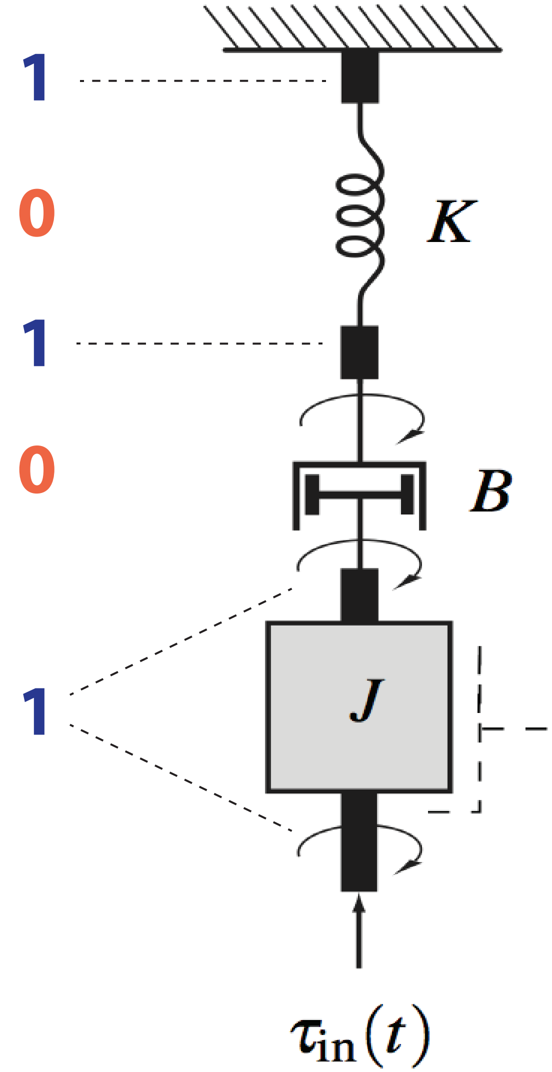
Systematic procedures (Recall)

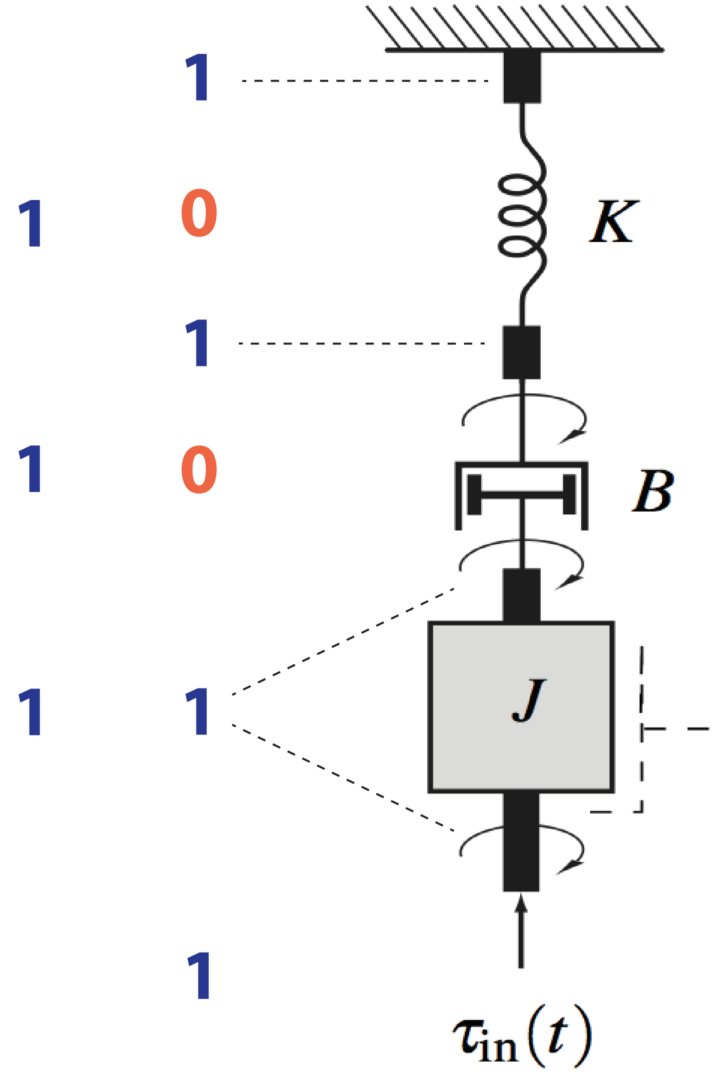
Mechanical domain:

- 1-junction at every “fixed” speed
- 0-junction to make speed difference, and additionally 1-junction to use the speed difference as a “fixed” speed
- introduce the elements
- use simplification rules
- use zero velocity points to remove bonds.

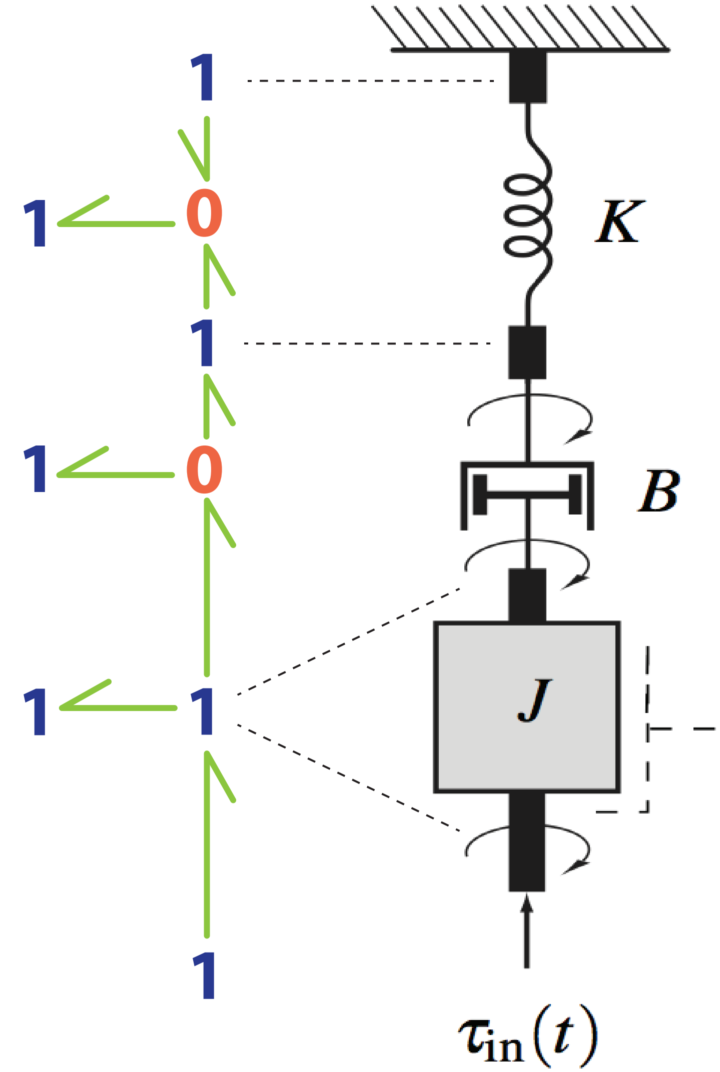
Example: Mechanical System



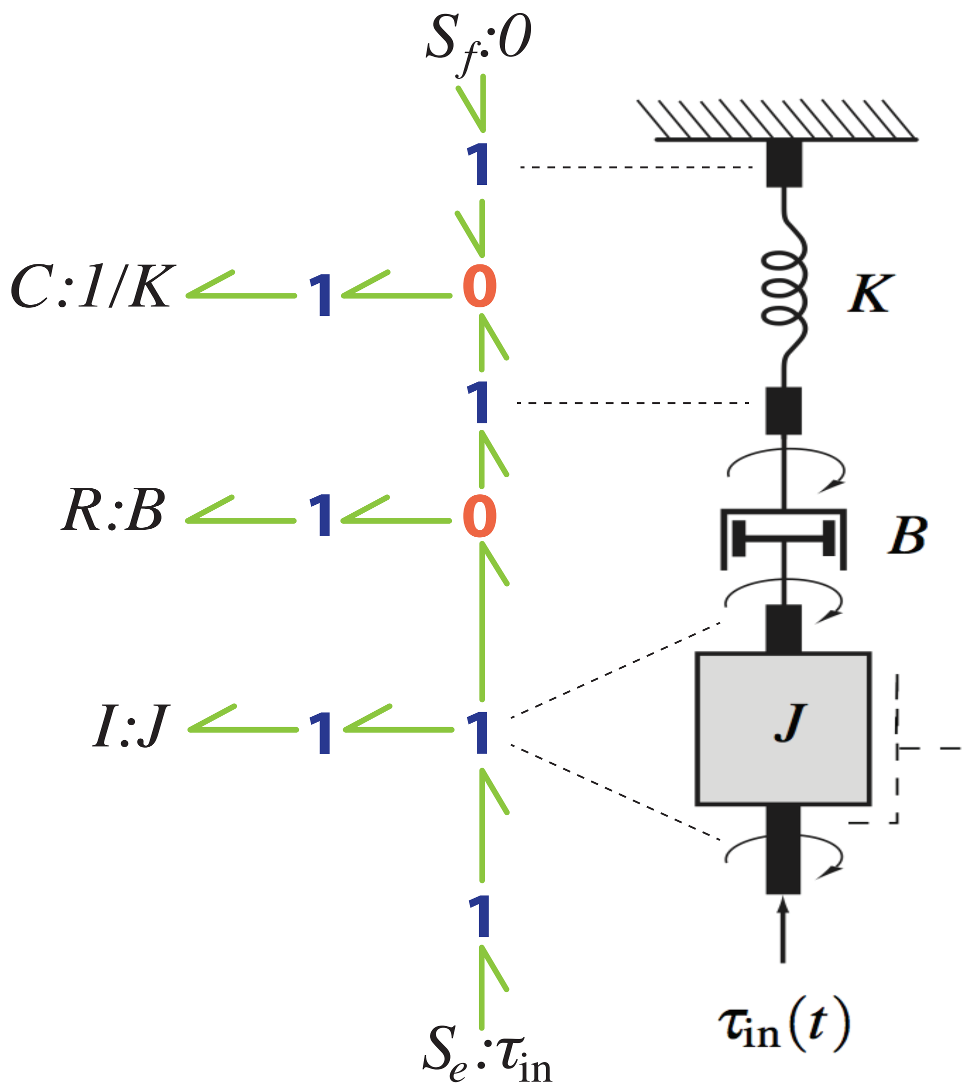
Example: Mechanical System



Example: Mechanical System



Example: Mechanical System



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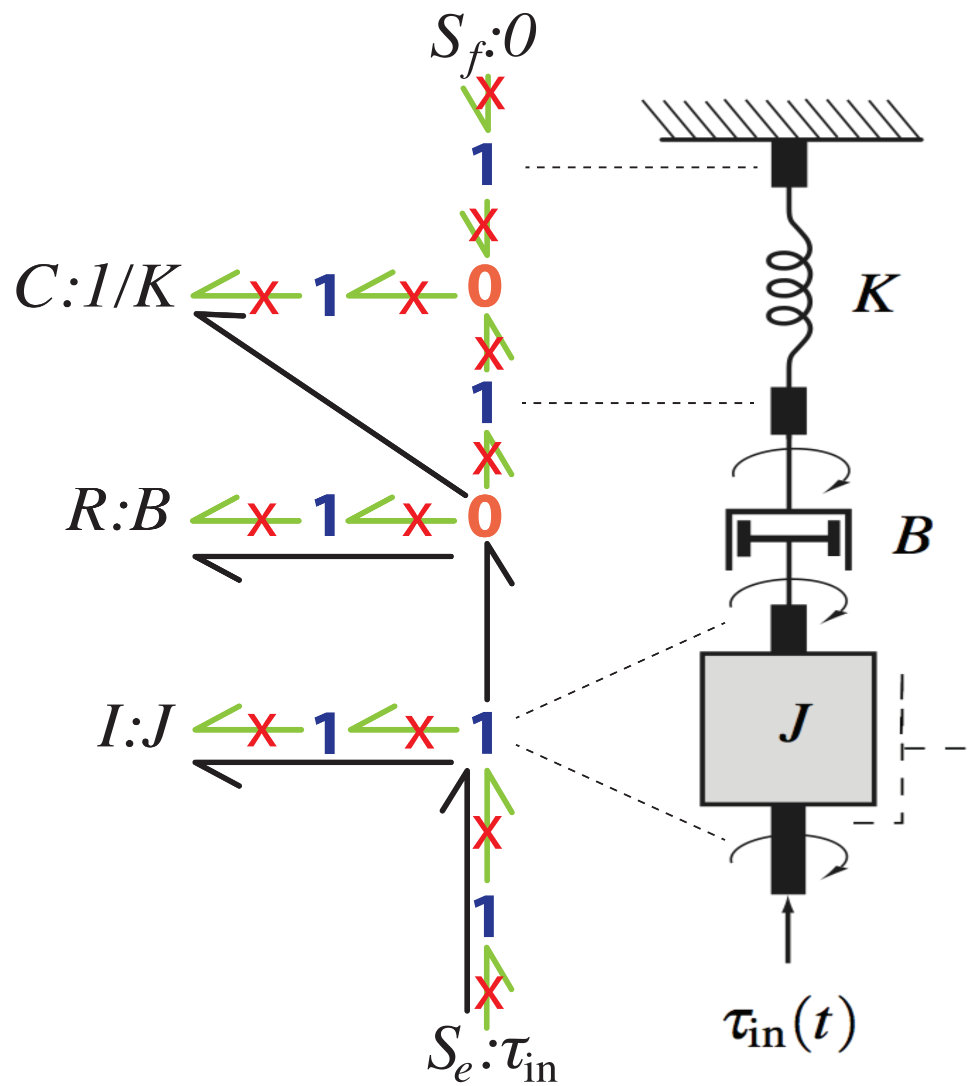
Example: Mechanical System



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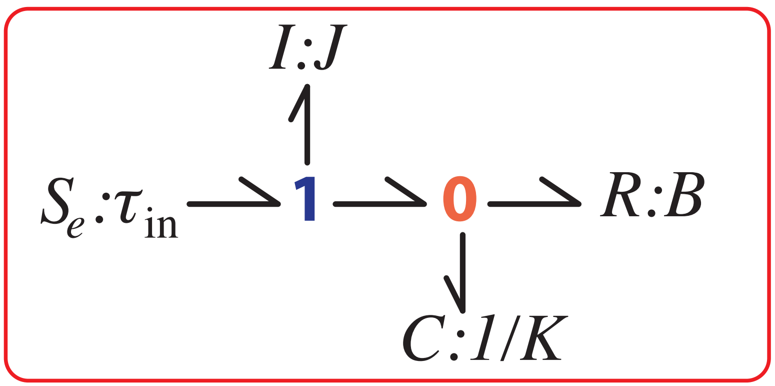
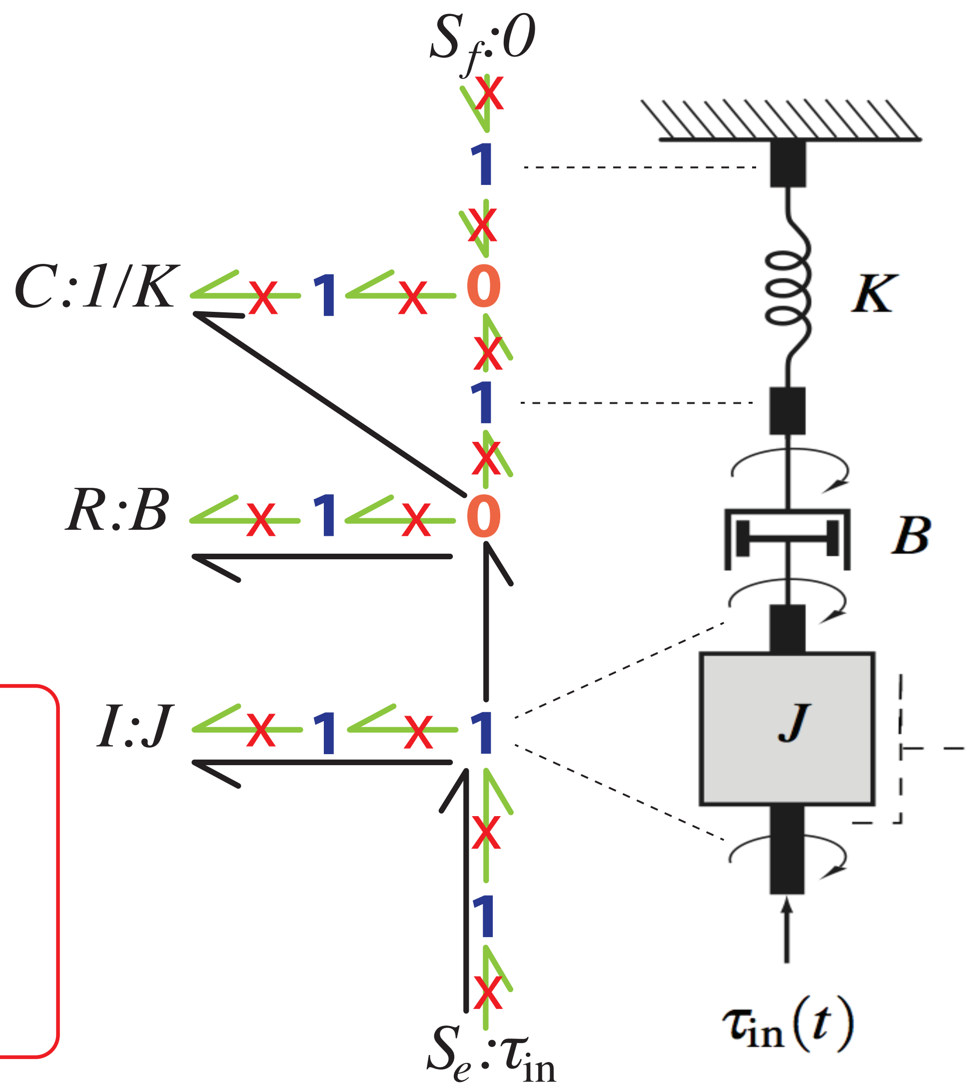
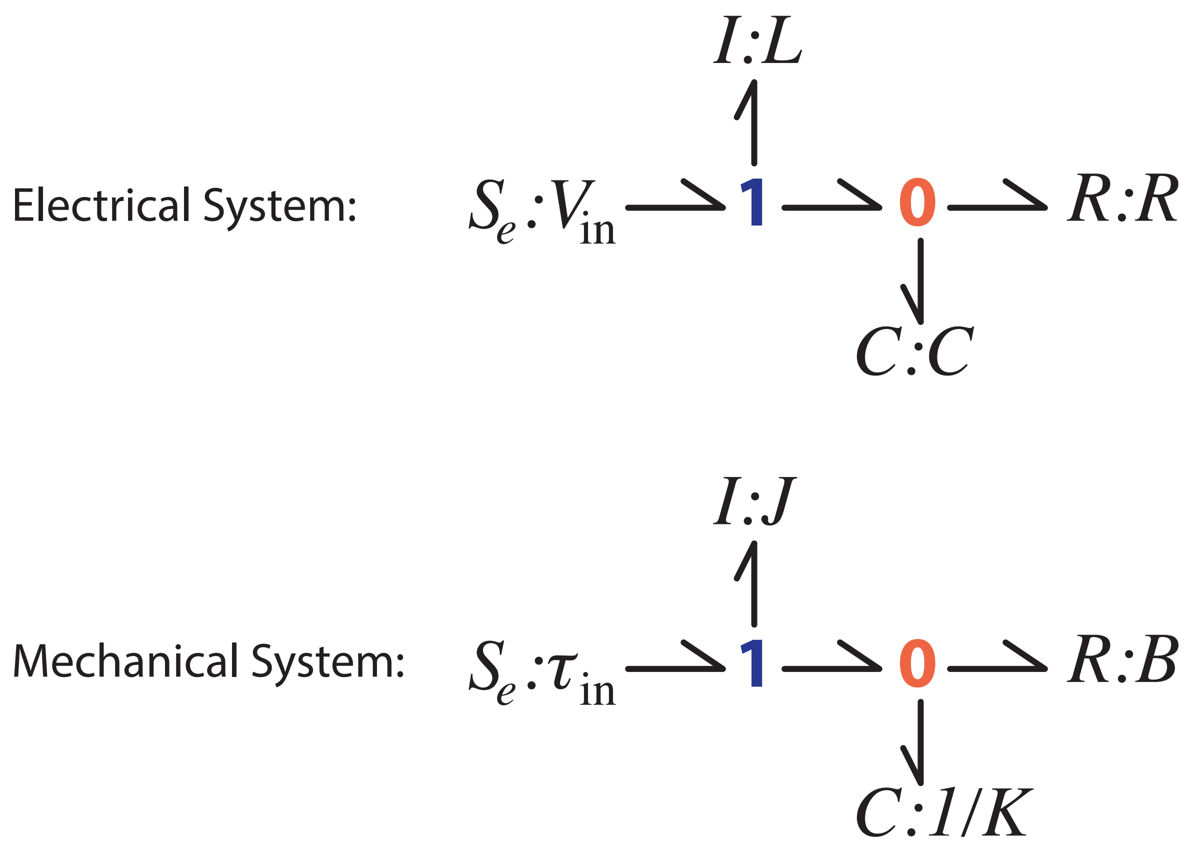




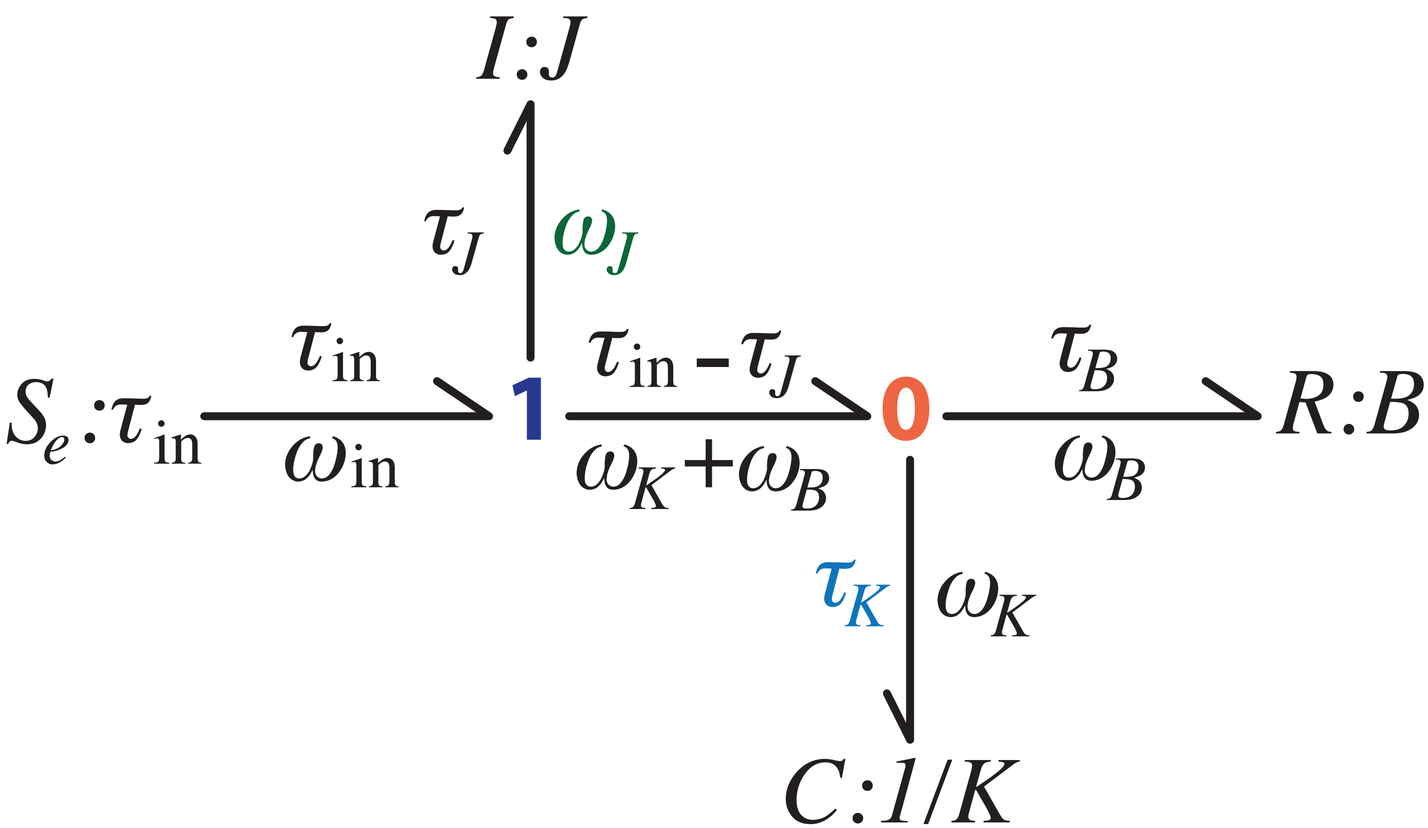
Hey, Wait a Minute...



$\Rightarrow$ When two systems from a different domain possess the same bond graph structure, they are analogues.

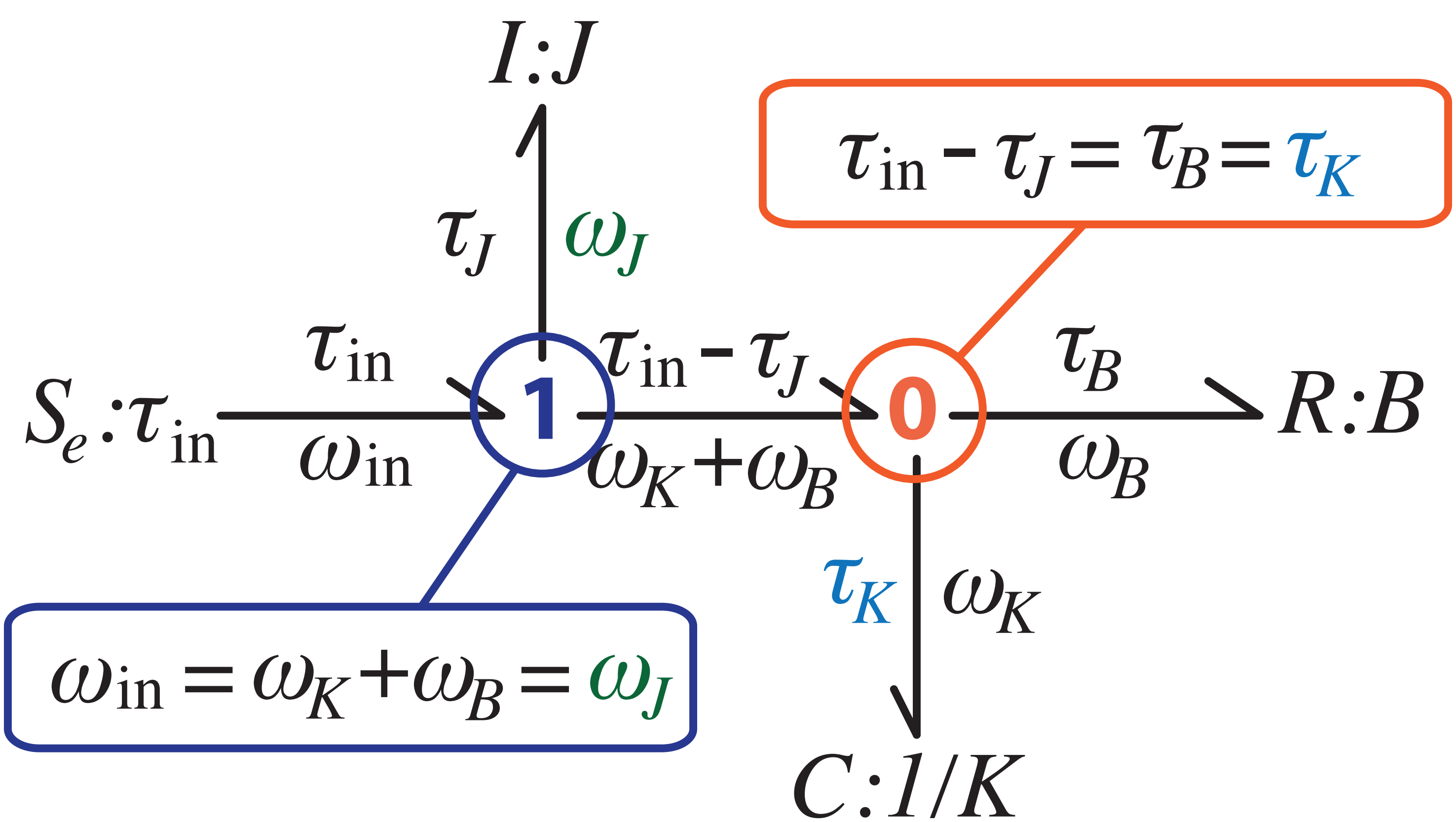
State Equations

Step 1: Add all the effort and flow variables to the bond graph:

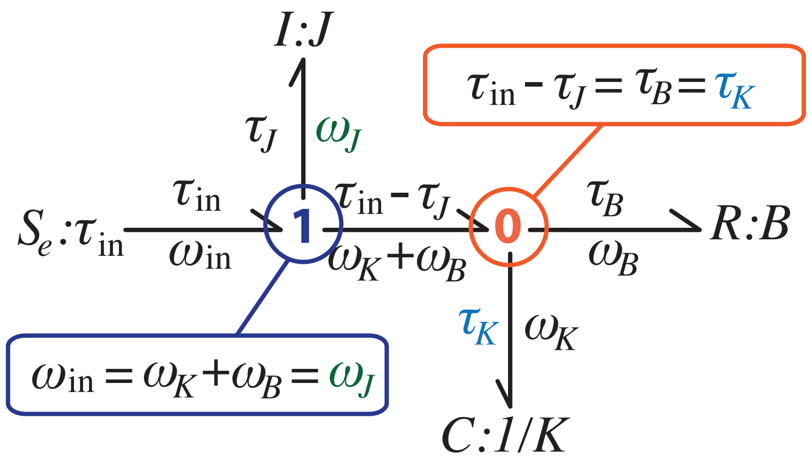


State Equations

Step 2: Write the relations associated to the junctions:



State Equations



Step 3: Choose the state variables:

$$\tau_J = J\dot{\omega}_J \Rightarrow \boxed{\omega_J}$$

$$\omega_K = \frac{\dot{\tau}_K}{K} \Rightarrow \boxed{\tau_K}$$

Step 4: Write the remaining constitutive relationships:

$$\tau_B = B\omega_B \text{ or } \omega_B = \frac{\tau_B}{B}$$

Step 5: Combine the previous steps:

$$\omega_K + \omega_B = \omega_J \Rightarrow \omega_K = \frac{\dot{\tau}_K}{K} = \omega_J - \omega_B = \omega_J - \frac{\tau_B}{B} = \omega_J - \frac{\tau_K}{B}$$

$$\tau_{\text{in}} - \tau_J = \tau_K \Rightarrow \tau_J = J\dot{\omega}_J = \tau_{\text{in}} - \tau_K$$

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State Equations

Step 6: Write the equations in the form $\dot{x} = f(x,u)$:

$$J\dot{\omega}_J = \tau_{\text{in}} - \tau_K \Rightarrow \dot{\omega}_J = \frac{\tau_{\text{in}} - \tau_K}{J} = f_1(\tau_K, \tau_{\text{in}})$$

$$\frac{\dot{\tau}_K}{K} = \omega_J - \frac{\tau_K}{B} \Rightarrow \dot{\tau}_K = K\left(\omega_J - \frac{\tau_K}{B}\right) = f_2(\omega_J, \tau_K)$$

or, in case the system is LTI:

$$\begin{bmatrix} \dot{\omega}_J \\ \dot{\tau}_K \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{J} \\ K & -\frac{K}{B} \end{bmatrix} \begin{bmatrix} \omega_J \\ \tau_K \end{bmatrix} + \begin{bmatrix} \frac{1}{J} \\ 0 \end{bmatrix} \tau_{\text{in}}$$

which is in the form $\dot{x} = Ax + Bu$.

Transformers I

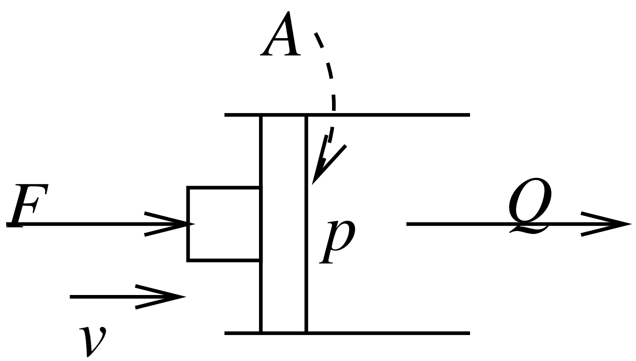
Ideal transformer “TF”: $e_2 = ne_1$ and $f_2 = \frac{1}{n} f_1$

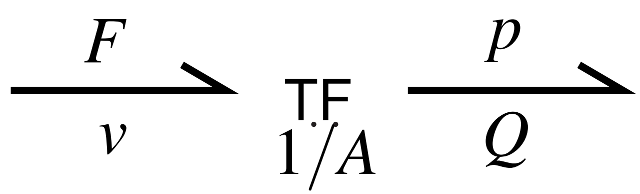
$$\underset{f_1}{\overset{e_1}{\rightharpoonup}} \quad \underset{n}{\mathrm{TF}} \quad \underset{f_2}{\overset{e_2}{\rightharpoonup}}$$

with $e_1 f_1 = e_2 f_2$.

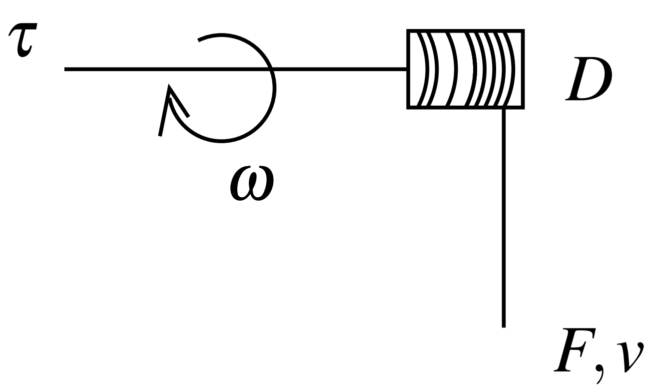
Transformers II

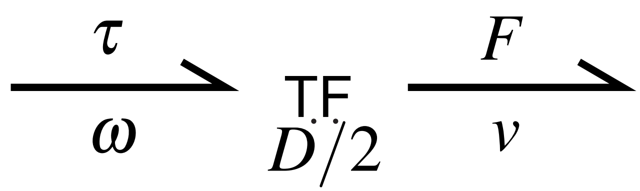
One domain: e.g., ideal el. transformer, or **different domains:** e.g., a transformer from mech. to hydr. domain:





or from mech. translation to rotation domain:





Gyrator I

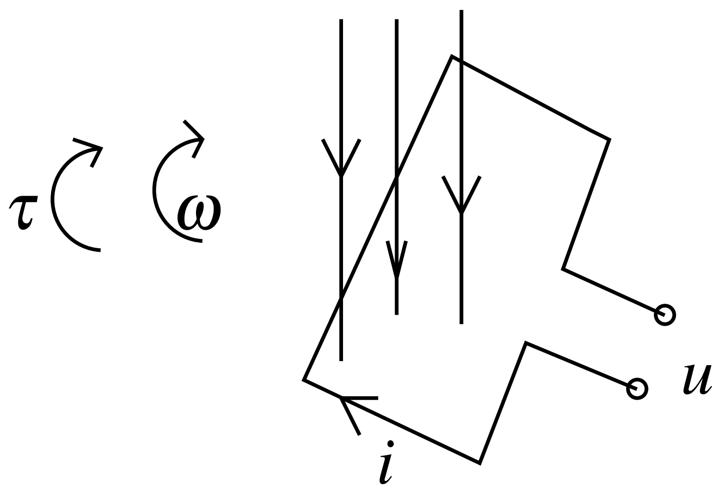
Ideal Gyrator "GY": $e_2 = r f_1$ and $f_2 = \frac{1}{r} e_1$

$$\xrightarrow[f_1]{e_1} \ \underset{r}{\text{GY}} \ \xrightarrow[f_2]{e_2}$$

with $e_1 f_1 = e_2 f_2$.

Gyrator II

Between **different domains**, e.g., a gyrator from electrical domain to mechanical rotation domain (motor, or, if the domains are reversed: a generator):

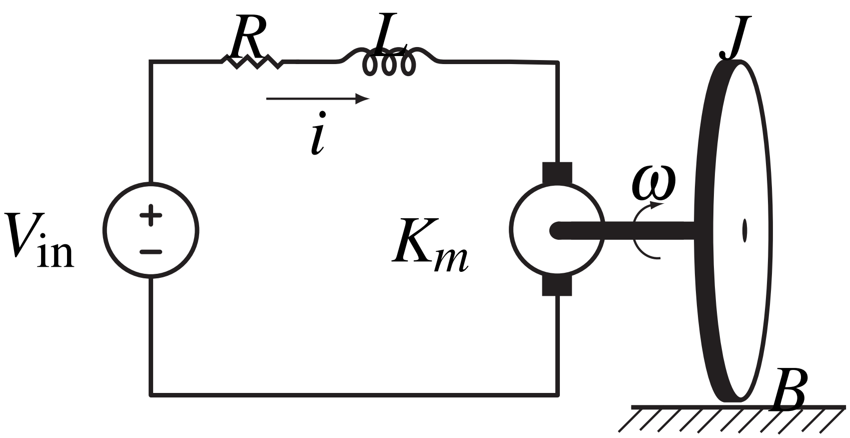


$$\xrightarrow[i]{u} \underset{\dot{K}_m}{\text{GY}} \xrightarrow[\omega]{\tau}$$

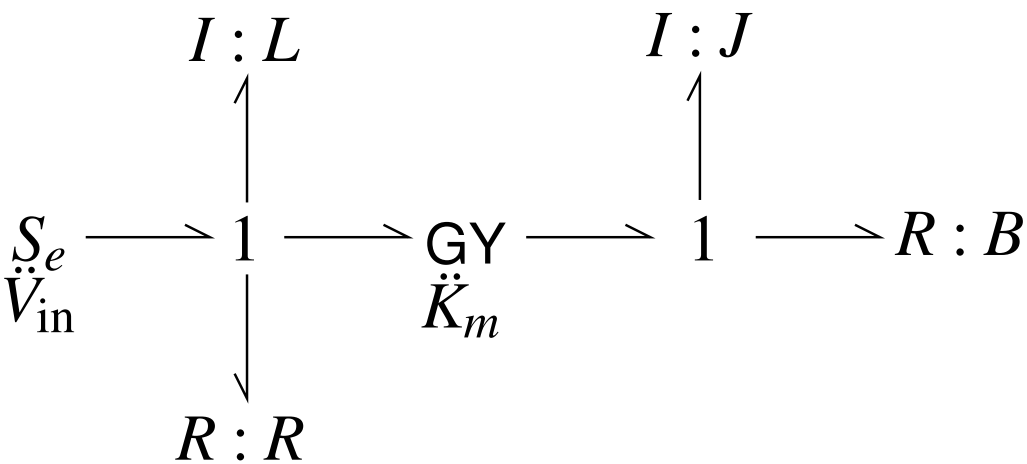
From mech. rot. to hydr. domain: pump. Reverse: turbine.

Note: the power continuity for both TF and GY!

Example: DC motor



- Two states corresponding to storage elements L and J $\Rightarrow$ order 2
- Two dissipative elements
- One gyrator
- One source



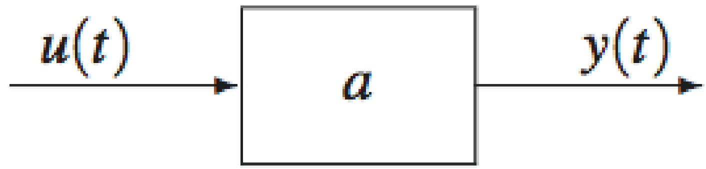
How to implement this all in a computer???

Towards Simulation

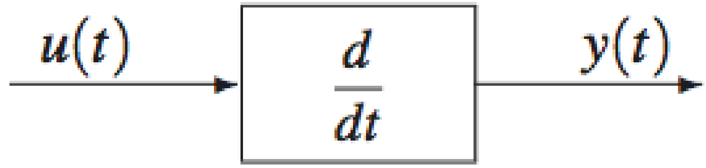
- Bond graph simulation tools: 20sim, Dimola, etc.
- However, we use Matlab
- Matlab contains a nice package called Simulink
- Block oriented simulation package
- Graphical implementation of DV's
- You don't have to worry about the sequence/iterations
- States are determined in an 'instantaneous' manner
- Build your model using basic blocks, like scaling, integration, differentiation, summation, subtraction, products, etc.

Primitive Operators

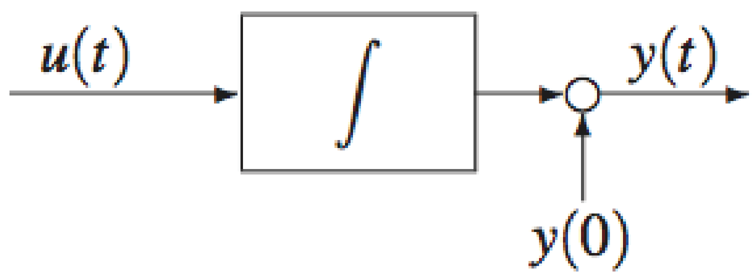
scaling operator



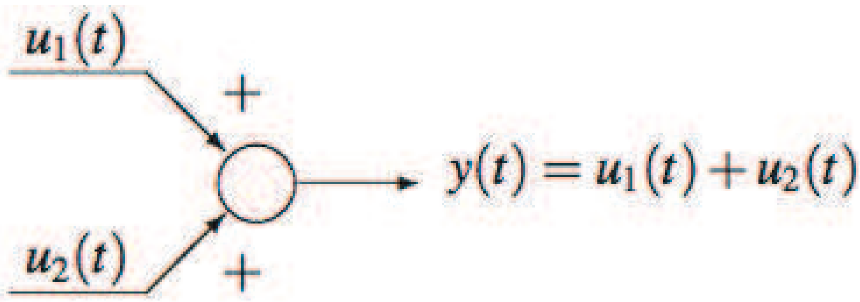
differential operator



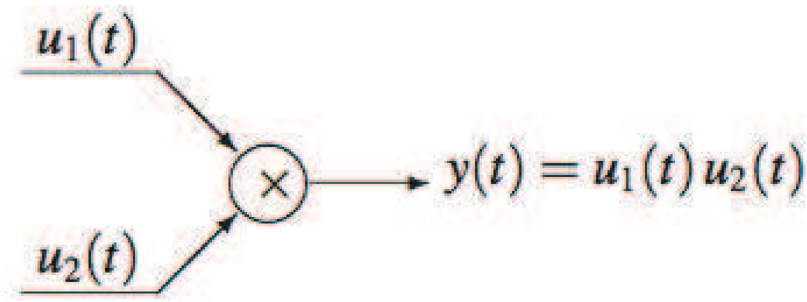
integral operator



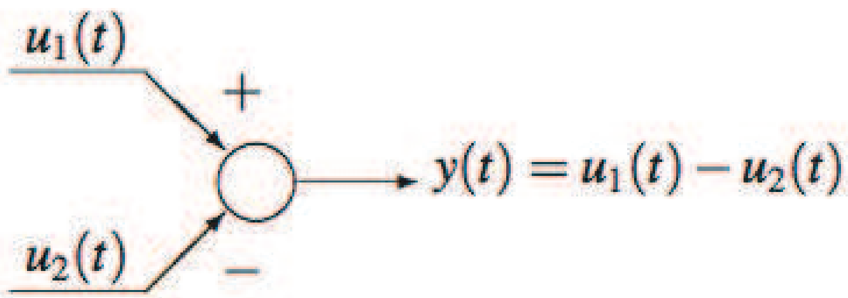
Primitive Operators



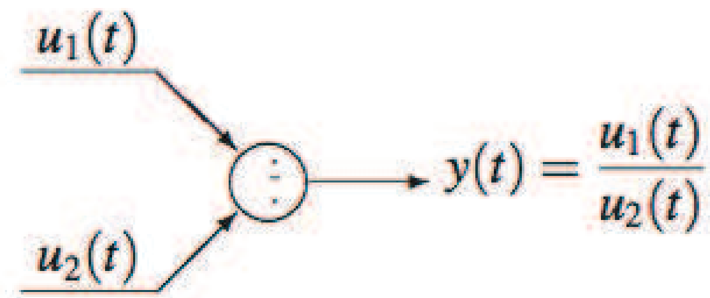
a) addition



c) multiplication



b) subtraction



d) division

Example

For the mechanical rotational system we found using the bond graph:

- the element relationships:

$$\omega_J(t) = \frac{1}{J}\int_0^t \tau_J(s)ds + \omega_J(0),$$

$$\omega_B(t) = \frac{1}{B}\tau_B(t),$$

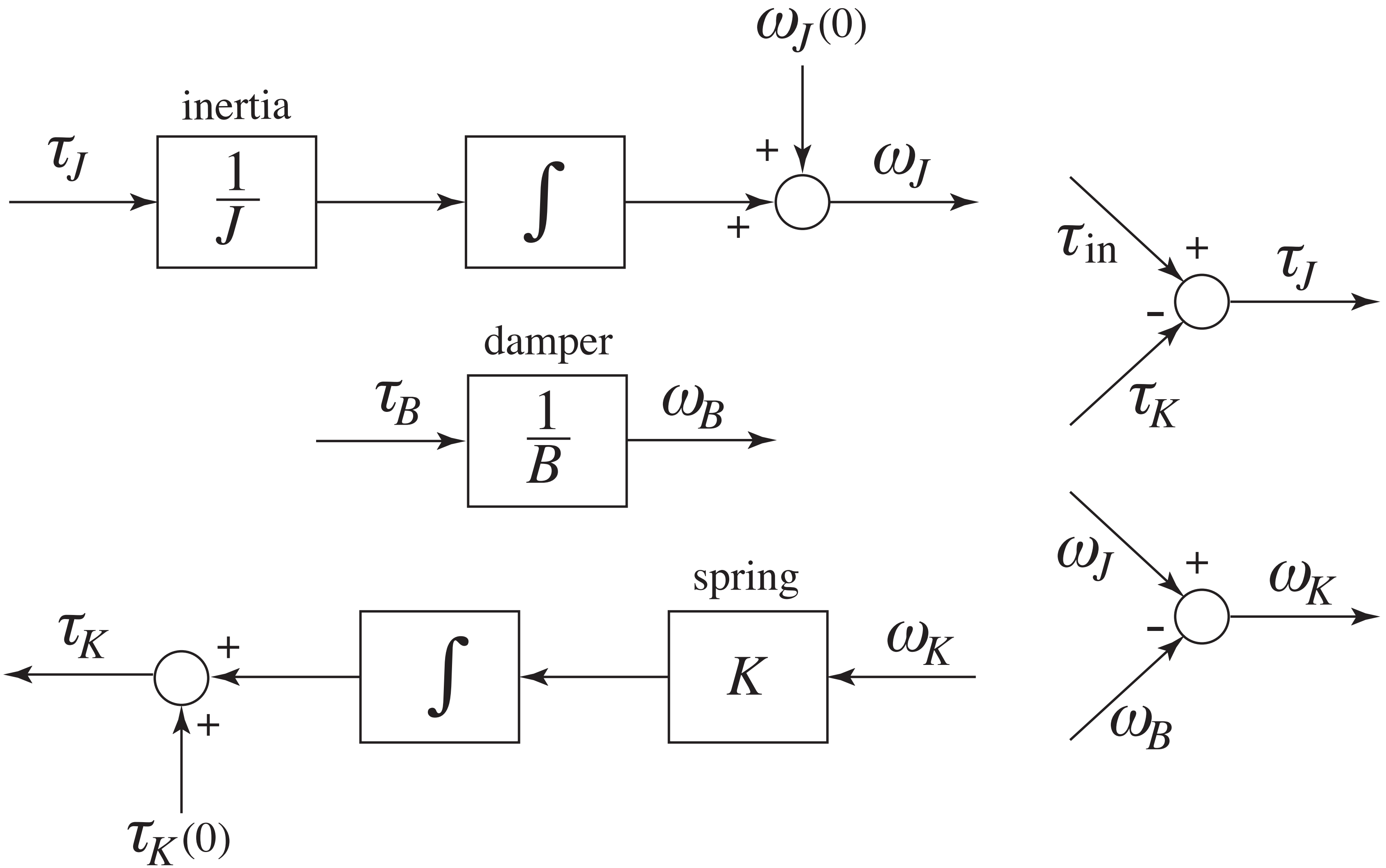
$$\tau_K(t) = K\int_0^t \omega_K(s)ds + \tau_K(0),$$

- and the interconnection structure

$$\tau_J(t) = \tau_{\text{in}}(t) - \tau_K(t)$$

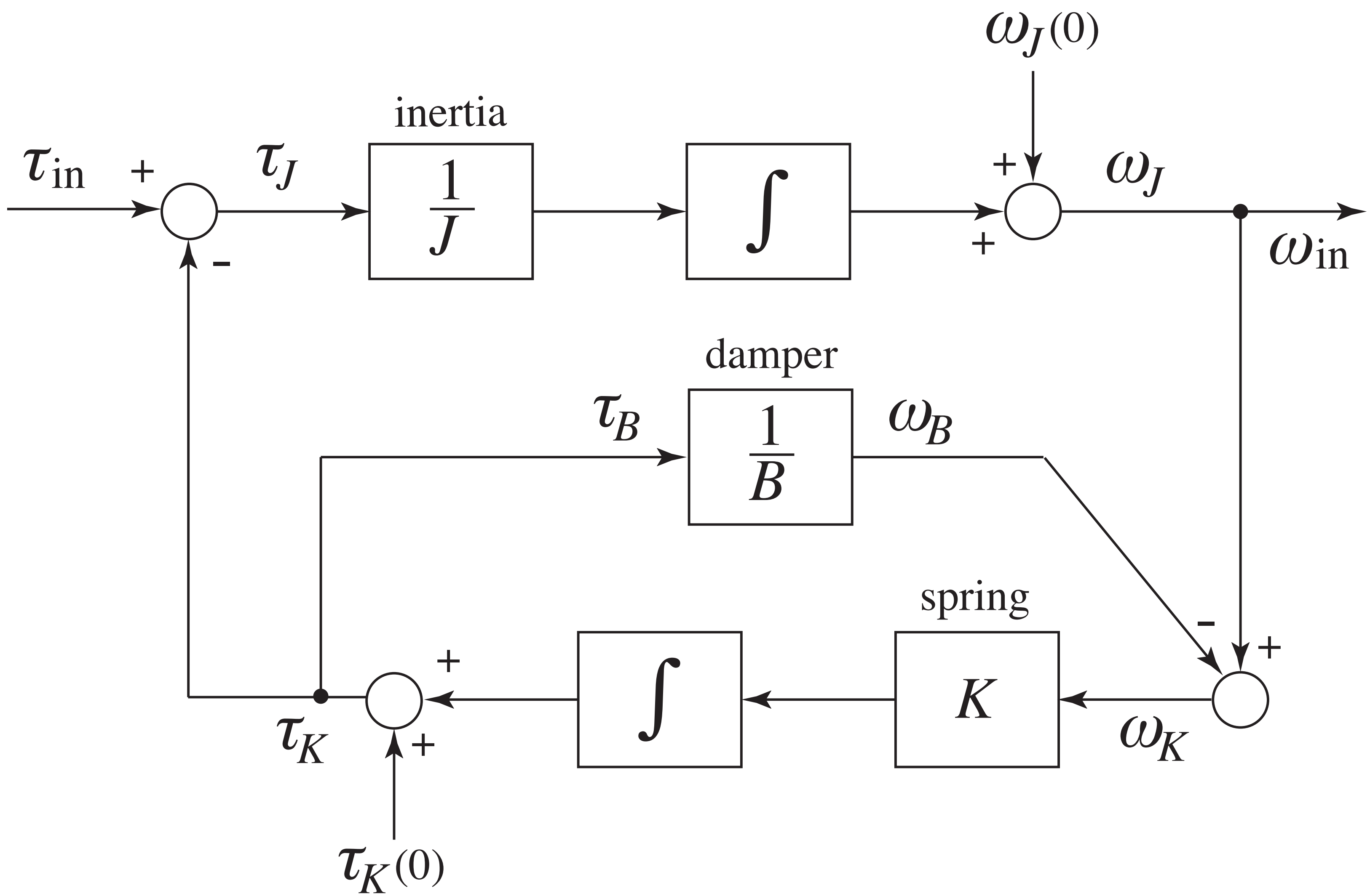
$$\omega_K(t) = \omega_J(t) - \omega_B(t)$$

Example



$\omega_J(0)$
inertia
τ_J
$\frac{1}{J}$
$\int$
+
+
ω_J
τ_{in}
+
τ_J
-
τ_K
damper
τ_B
$\frac{1}{B}$
ω_B
ω_J
+
ω_K
-
ω_B
spring
τ_K
+
$\int$
K
ω_K
+
$\tau_K(0)$

Example



$\omega_J(0)$
inertia
τ_{in}
+
τ_J
$\frac{1}{J}$
$\int$
+
ω_J
+
-
ω_{in}
damper
τ_B
$\frac{1}{B}$
ω_B
spring
-
+
+
$\int$
K
τ_K
+
ω_K
$\tau_K(0)$