C → storage of flow f (current i, velocity v)

$$q(t) = f(t_0) + \int_{t_0}^{t} f(\tau)\, d\tau \qquad \underline{\text{or}} \qquad \dot{q}(t) = f(t)$$

$$e = e(q)$$

<u>examples</u> capacitor: $\dot{q} = i$ $\qquad u = \frac{1}{C}\, q$

spring: $\dot{x} = v$ $\qquad F = \frac{1}{1/k}\, x$

$\vdash\!\rightharpoonup$ C : C

preferred causality

State: generalized position q (q, x)

<u>or</u> for linear element: effort (u, F)

I $\rightarrow$ storage of effort e (voltage u, force F) (2)

$$p(t) = e(t_0) + \int_{t_0}^{t} e(\tau)\, d\tau \quad \text{or} \quad \dot{p}(t) = e(t)$$

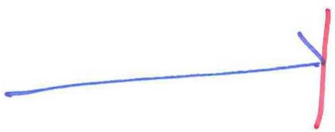
$$f = f(p)$$

examples: inductor $\dot{\phi} = u$ $\quad i = \frac{1}{L}\phi$

mass $\dot{p} = F$ $\quad v = \frac{1}{m} p$

state: generalized momentum p (ϕ, p)

or for linear elements: flow (i, v)



I : L

preferred causality

$R \rightarrow$ dissipate energy

$e = r(f)$

<u>examples</u> resistor $u = Ri$

damper $F = \beta v$

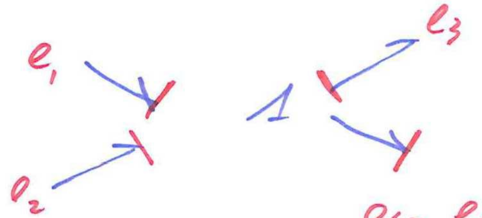
$\longrightarrow R : R$

1 - junction → same flow (i, v)

$\sim$ series

same f

$\sum e_{in} = \sum e_{out}$



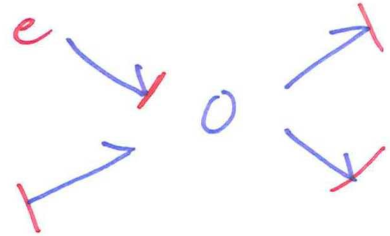
$e_4 = e_1 + e_2 - e_3$

all causal stroke at 1 junction except 1

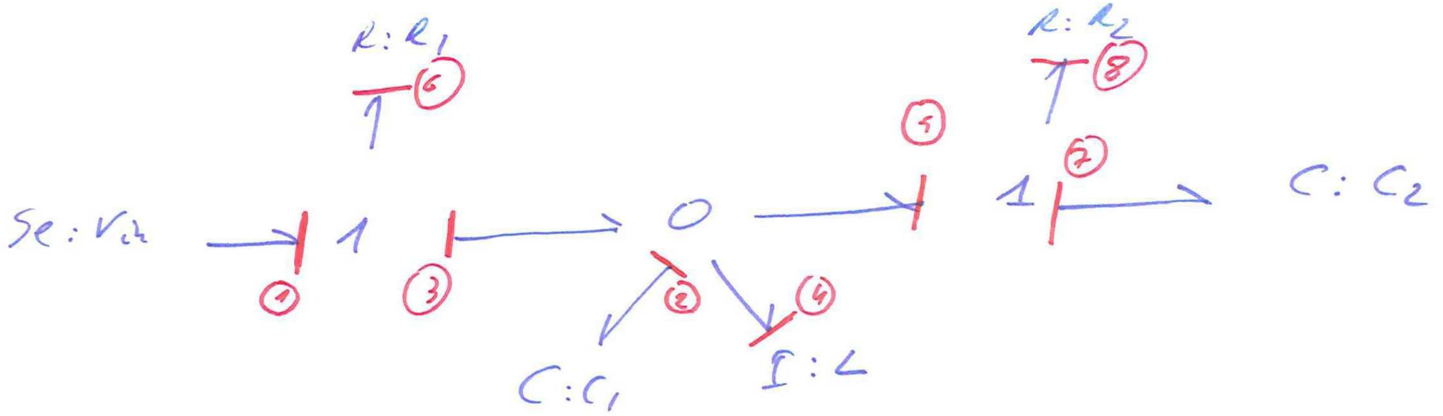
0 - junction → same effort (u, F) $\sim$ parallel

same e

$\sum f_{in} = \sum f_{out}$



exactly 1 causal stroke at 0 junction



1 : Se

2 : preferred causality for C-element

3, 4, 5 : 0-junction

6 : 1-junction

7 : preferred causality for C-element

8 : 1-junction

→ causal, well defined

→ all C/I elements have preferred causality

→ 3 states

⑥

C : linear : u_C $\qquad \dot{u}_C = \frac{1}{C} i_C$

I : linear : i_L $\qquad \dot{i}_L = \frac{1}{L} \cdot u_L$

$u_{in} - u_{C1}$

$$
\begin{tikzcd}
 & R:R_1 & & R:R_2 & \\
Se:V_{in} \arrow[r, "u_{in}"] & 1 \arrow[u, "u_{in}-u_{C1}"] \arrow[r, "u_{C1}"] & 0 \arrow[dl, "u_{C1}"'] \arrow[dr, "u_{C1}"] \arrow[r, "u_{C1}"] & 1 \arrow[u, "u_{C1}-u_{C2}"] \arrow[r, "u_{C2}"] & C:C_2 \\
 & C:C_1 & & I:L &
\end{tikzcd}
$$

$\Rightarrow$ efforts u

$$
\begin{tikzcd}
 & R:R_1 & & R:R_2 & \\
Se:V_{in} \arrow[r, "i_\square"'] & 1 \arrow[u, "i_\square"] \arrow[r, "i_\square"'] & 0 \arrow[dl, "i_x"] \arrow[dr, "i_L"] \arrow[r, "i_*"'] & 1 \arrow[u, "i_*"] \arrow[r, "i_*"'] & C:C_2 \\
 & C:C_1 & & I:L &
\end{tikzcd}
$$

$\Rightarrow$ flows i

$u = Ri$

$i_\square = i_x + i_L + i_* \Rightarrow i_x = i_\square - i_L - i_*$

$R_1 : \quad i_{R_1} = \frac{u_{R_1}}{R_1} = \frac{u_{in} - u_{C1}}{R_1} = i_\square$

$R_2 : \quad i_{R_2} = \frac{u_{C1} - u_{C2}}{R_2} = i_*$

$$C_1: \quad \dot{u}_{C1} = \frac{1}{C_1}\left(\frac{u_{in} - u_{C1}}{R_1} - \frac{u_{C1} - u_{C2}}{R_2} - i_L\right)$$

$$C_2: \quad \dot{u}_{C2} = \frac{1}{C_2}\,\frac{u_{C2} - u_{C1}}{R_2}$$

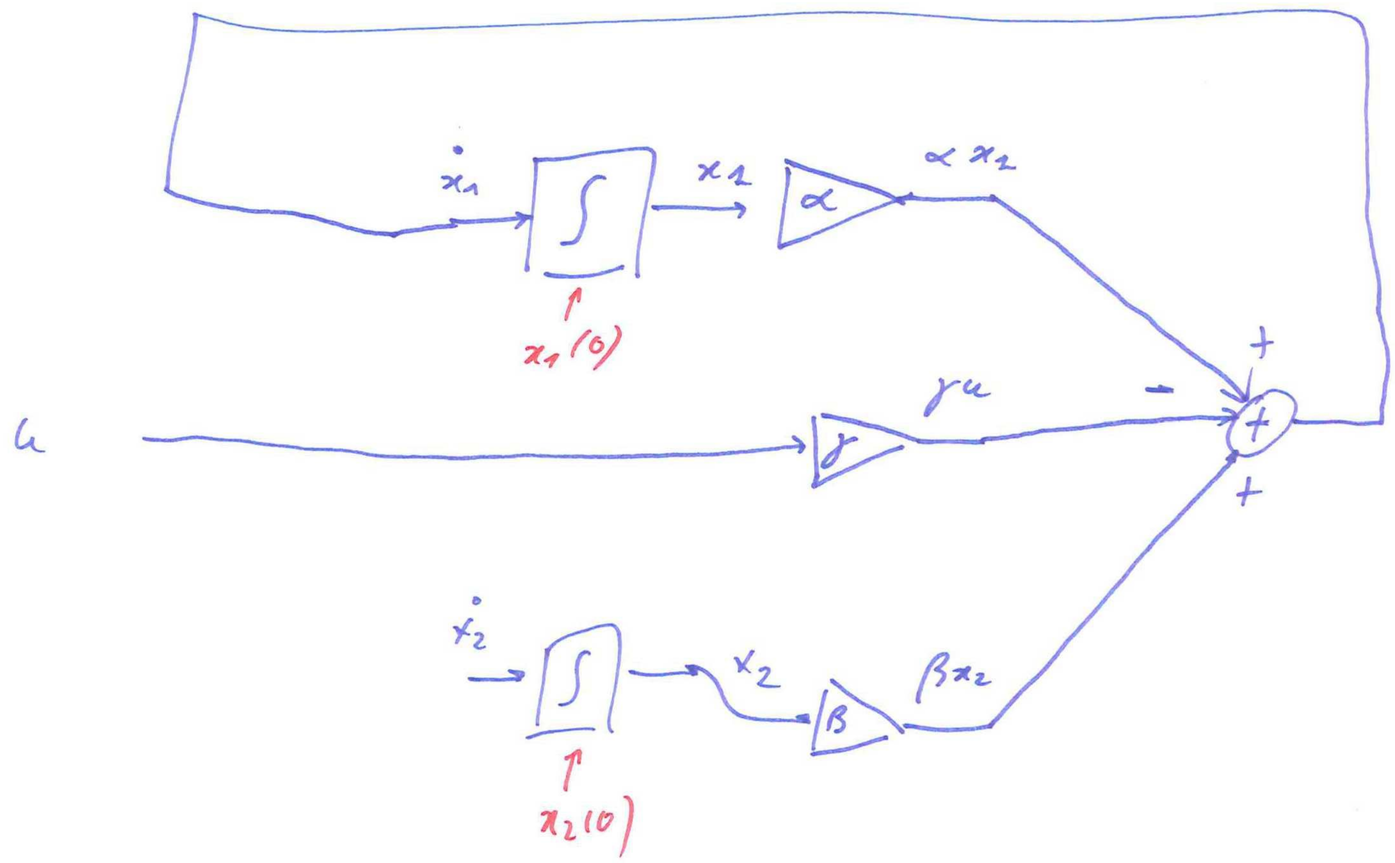
$$L: \quad \dot{i}_L = \frac{1}{L}\,u_{C1}$$

$$\begin{bmatrix} \dot{u}_{C1} \\ \dot{u}_{C2} \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} \frac{1}{C_1}\left(\frac{-1}{R_2} - \frac{1}{R_1}\right) & \frac{1}{R_2 C_1} & \frac{-1}{C_1} \\ \frac{-1}{C_2 R_2} & \frac{1}{R_2 C_2} & 0 \\ \frac{1}{L} & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{C1} \\ u_{C2} \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1 R_1} \\ 0 \\ 0 \end{bmatrix} u_{in}$$

$$\dot{x} = A\,x + B\,u$$

state space model

$$\dot{x}_1 = \alpha x_1 + \beta x_2 - \gamma u$$



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$$\begin{tikzcd} {} \arrow[r, "f_1", "e_1"'] & 0 \arrow[r, "f_2", "e_1"'] \arrow[d, "f_3", "e_1"'] & \overset{n}{TF} \arrow[r, "f_4", "e_2"'] & I : I \\ & I : I & & \end{tikzcd}$$

$$\xrightarrow[e_1]{f_1} I : I_{eq} = \frac{I}{n^2+1}$$

$I_{eq} = ?$

$$\frac{e_1}{I_{eq}} = \dot{f}_1$$

$$\frac{e_1}{I} = \dot{f}_3 \quad (a)$$

$$f_1 = f_2 + f_3$$

$$\frac{e_2}{I} = \dot{f}_4 \quad (5)$$

$$f_2 = n \cdot f_4$$

$$e_2 = n \cdot e_1 \quad (2)$$

$$\dot{f}_2 = n \cdot \dot{f}_4 = \frac{n \cdot e_2}{I} \ (5) = \frac{n^2 \cdot e_1}{I} \ (2) \quad (b)$$

$$\dot{f}_1 = \underbrace{\dot{f}_2}_{(b)} + \underbrace{\dot{f}_3}_{(a)} = \frac{n^2}{I} \cdot e_1 + \frac{e_1}{I} = \frac{n^2+1}{I} \cdot e_1$$

$$I_{eq} = \frac{I}{n^2+1}$$

$$\begin{tikzcd}
S_f \arrow[r] & 0 \arrow[r] \arrow[d] & 1 \arrow[r] \arrow[d] & 0 \arrow[r] \arrow[d] & R \\
& C & I & C &
\end{tikzcd}$$

Causality?

states?

$$\begin{tikzcd}
S_f \arrow[r, "1"] & 0 \arrow[r, "3"] \arrow[d, "2"] & 1 \arrow[r, "5"] \arrow[d, "4"] & 0 \arrow[r, "7"] \arrow[d, "6"] & R \\
& C & I & C &
\end{tikzcd}$$

Causal

C/I → preferred causality

3 states

What if Se instead of Sf?

→ 1 C element will not get its preferred causality