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Proving that the Power Set of a Countably Infinite Language is Uncountable

Cantor invented the theory of diagonalization to prove that certain sets are countably infinite, such as the set of all rational numbers () is countably infinite by proving that there was a possible binary mapping to represent every number in **Q.** Specifically**,** an infinite set was countably infinite if you could prove that it had a bijective binary mapping to the set of natural numbers.

We can begin a rebuttal by first presuming that the power set of any arbitrary set of languages is countably infinite. Imagine we have a set of languages represented as and we want to represent , or all sets of the sets of . We would start by creating a cantor diagonalization mapping of all possibilities.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | {φ} | {a} | {a,a} | {a,b} | {b,a} | {b,b} | … |
| L1 | 1 | 0 | 0 | 0 | 0 | 0 |  |
| L2 | 0 | 1 | 0 | 0 | 0 | 0 |  |
| L3 | 1 | 1 | 0 | 0 | 0 | 0 |  |
| L4 | 0 | 0 | 1 | 0 | 0 | 0 |  |
| L5 | 1 | 0 | 1 | 0 | 0 | 0 |  |
| L6 | 0 | 1 | 1 | 0 | 0 | 0 |  |
| … |  |  |  |  |  |  |  |

Simply looking at this graph we could assume that it is indeed one-to-one and that every language is mapped. However we must take into account that each language is also **countably** **infinite**. We take this into account imagine each language as a series of sets denoted , and e ∈ , where i represents the row and j represents the column. We can rebuild this graph beginning with the second language up to the seventh.

|  |  |
| --- | --- |
| L2(010) |  |
| L3(011) |  |
| L4(100) |  |
| L5(101) |  |
| L6(110) |  |
| L7(111) |  |
| L8(1000) |  |

Now what this graph does is represent each subset of the set that was previously being counted with their individual binary mapping in parenthesis.

Now Imagine another language that comes from the same set called where . This means that this new set has a representation of a set that exists in , but is not represented in . We could stop at that and say that we have a contradiction because is a but is not a subset of any L so there is no one-to-one bijection from L to , but to further explain we could create a new set that represents a new set and proof by diagonalization.

Imagine we have a new set with new elements. We will grab each element where i=j and then add an additional element represented as + where ∈ . We now have this new language, we will represent as is equal to . If we try to find this new language in the original mapping we will notice that each element is different than one other element in each Language. We could attempt to say it is L2 but is different than . If we try to compare the language to L3 we notice that is different than . We can repeat this comparison with each one of the languages and we will get the same reasoning that this new language ⊄ , but it is a possible language that could be. This leads to a contradiction, can and cannot be in. This shows that is not true and leads to the conclusion that is uncountably infinite.

There is one more simple example that we can bring up that is very closely related to this problem, natural numbers are countably infinite and mapping them is very simple if we just map or Now imagine we wanted to find the power set of denoted . We could create a similar table and map the power sets beginning with and it would seem to work, but you would once you started to arrive at sets way down the line like {1,2,3,4,5,6,7,8,9,10} for example and had them mapped. You could perform a diagonalization of the sets prior and after and create some arbitrary set {1,3,4,7,2,4,8,9,10} and find that you haven’t mapped out a natural number for that set yet but have already passed the opportunity. Is uncountably infinite.