Quantum Optimization for the Knapsack Problem

Qingtian Miao
Texas A&M University

The Knapsack Problem

- **What is the Knapsack Problem?**
A classic optimization challenge focused on making the best choice with limited resources.

- **The Goal:** Select a combination of items—each with a specific cost and value—to achieve the **maximum possible value** without exceeding a total budget or capacity.

- **Real-World Application:** Allocating a fixed budget across non-profit projects to maximize social impact.

Capacity = Total Budget
Items = Projects
Cost = Project Funding
Value = Social Impact Score

All images were generated with Canva AI.

The Scaling Challenge: Why We Need a Smarter Approach

QUBO

$$\min_{x} x^T Q x + c + \lambda \sum_{i=1}^{m} [\max(0, b_i - (Ax)_i)]^2$$

Trying every possible combination is not feasible. Assuming we can check one solution every nanosecond (1e^-9 seconds):

•**10 Items** (2^10 cases): ~1 microsecond

•**50 Items** (2^50 cases): **~13 days**

•**100 Items** (2^100 cases): **~40 trillion years** (older than the universe)

37 Items (2^37 cases)

```
Evaluation 720  | Cost (CVaR): -1155.048544 | Global Best Obj: -1251.00
Evaluation 721  | Cost (CVaR): -1173.427184 | Global Best Obj: -1254.00
Evaluation 722  | Cost (CVaR): -1167.009709 | Global Best Obj: -1254.00
Evaluation 723  | Cost (CVaR): -1144.611650 | Global Best Obj: -1254.00
Evaluation 724  | Cost (CVaR): -1176.242718 | Global Best Obj: -1254.00
Evaluation 725  | Cost (CVaR): -1169.747573 | Global Best Obj: -1254.00
Evaluation 726  | Cost (CVaR): -1139.970874 | Global Best Obj: -1254.00

✅ Optimization Finished in 218.01s!
```

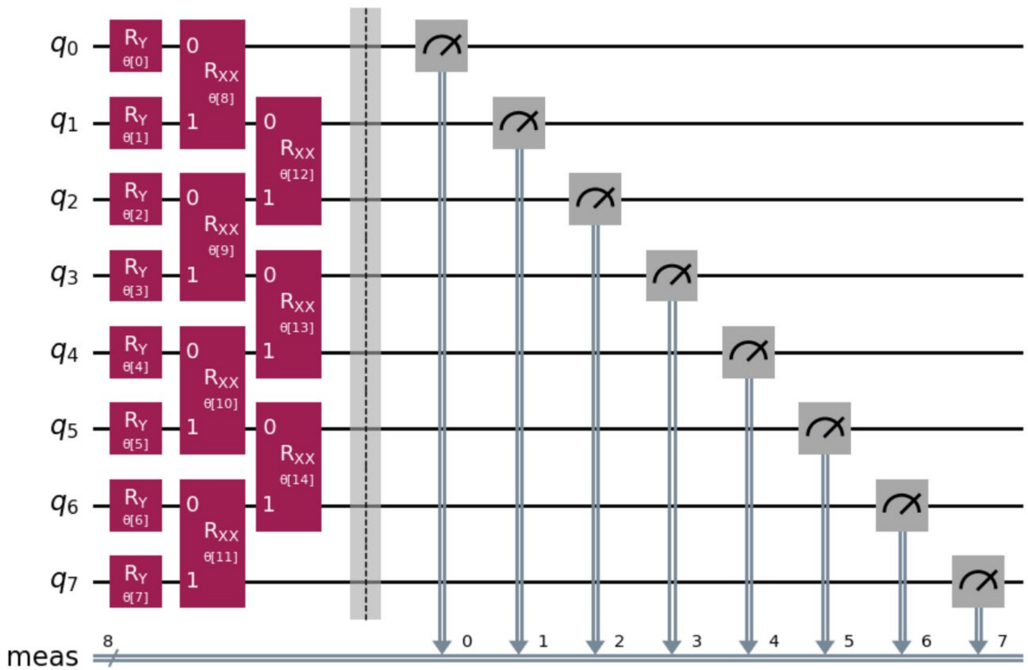
All images were generated with Canva AI.

Our Solution: Circuit, Optimizer, and Validation

•Quantum Circuit (Ansatz):

We designed a TwoLocal variational circuit using RY rotation gates and RXX entangling gates.
A custom **bilinear entanglement** pattern was used to ensure effective exploration of the solution space.

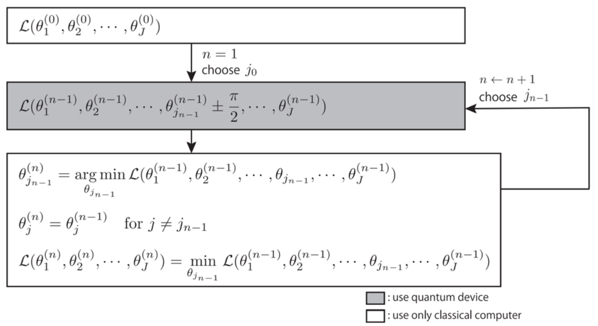
For the case of 8 qubits, the ansatz is depicted below.



In general, for n qubits, the circuit contains 2n−1 gates and 2n−1 tunable parameters to be optimized to minimize objective function by the optimizer.

•Optimizer:

We used a lightweight **NFT**.

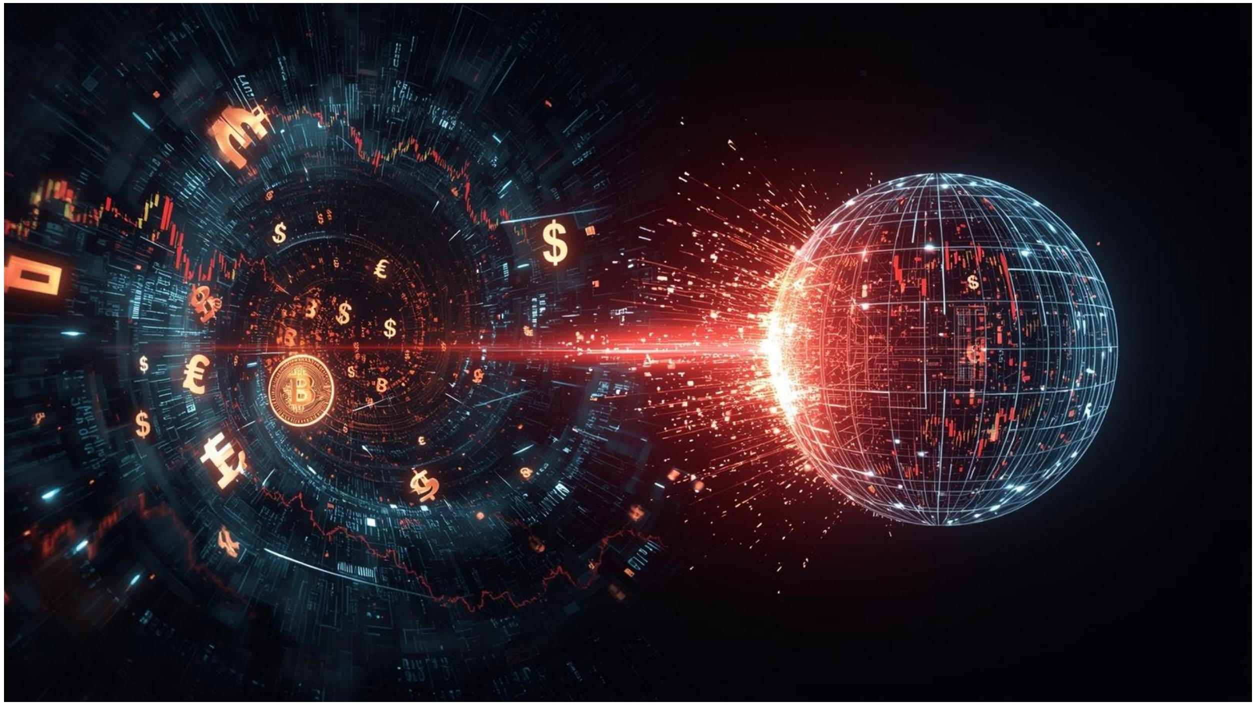


K. M. Nakanishi, K. Fujii, and S. Todo,
Phys. Rev. Research, vol. 2, p. 043158, 2020

37 Items (37 qubits)

```
quantum solution:
[0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, …
classical solution:
[0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, …
```

Building Defenses: Mitigating the Risks of Quantum Misuse



•Access Control & Security Protocols:

As rare, cloud-based resources, quantum computers will require **strict access control and security protocols**. Platform providers must ensure only vetted users can run tasks to prevent misuse.

•Industry Standards & Certification:

Industry bodies should develop **standards and certification** for quantum applications.
For example, quantum models for finance must undergo **mandatory risk assessments** to prevent systemic instability.

All images were generated with Canva AI.

Thank you!