Poisson distribution as member of the Exponential family

density $f(y;\theta,\phi) = \mu^y e^{-\mu}/y!$

that we can rewrite as

$$\exp(y\log(\mu) - \mu - \log(y!))$$

So we have $\mathbb{E}[Y] = \text{var}(Y) = \mu$

$\theta = \log(\mu)$, $\phi = 1$

$a(\phi) = 1$, $b(\theta) = \exp(\theta)$,

$c(y,\phi) = -\log(y!)$

natural parameter: $\eta(\mu) = \theta = \log(\mu)$

sufficient statistic: $T(y) = y$ is sufficient for θ.

So we conclude that the Poisson distribution is a member of the Exponential family.