Aperiodicity of Markov Chains

A Markov Chain $\{X_t\}_{t \geq 0}$ on a state space $\mathcal{X}$ and with transition kernel $P(x, A)$ is aperiodic if there exists no integer $d \geq 1$ such that for every state $x \in \mathcal{X}$ the return probability occurs only at multiples of d:

$$d = \gcd \{ t \geq 1 \mid P^t(x,x) > 0 \} = 1 \ , \ \forall x \in \mathcal{X}$$

In other words, the greatest common divisor (gcd) at which the chain can return to any state after t steps is 1. The chain does not get "trapped" in a "strict" cycle.

Let us consider a simple random walk on integers with transitions probabilities

$$P(X_{t+1} = x+1 \mid X_t = x) = 0.5 \ , \ P(X_{t+1} = x-1 \mid X_t = x) = 0.5$$

This is an example of an aperiodic chain as it can return to previous states or step to new states freely.

It is important for MCMC methods that chains are aperiodic.