Elementary Statistical Inference

Binomial distribution;

Let $X = \begin{cases} 1 & \text{success}, \\ 0 & \text{otherwise} \end{cases}$ $X \sim Bin(n, \theta)$, $\theta \in [0,1]$

Method of Moments estimation:

$\boxed{\text{Let } X_1, \dots, X_n \text{ be a sample from a distribution with parameter } \theta. \text{ A Method of Moments estimator for } \theta \text{ is given by } E_\theta[X^J]. \text{ The } J^{th} \text{ sample moment is then } \overline{X^J} = n^{-1}\sum_{i=1}^{n} X_i^J}$

For Binomial r.v., we have $E[X] = n\theta$ and $var(X) = n\theta(1-\theta)$. We want to estimate both n and θ.

$$E[X] = \bar{X} = n\theta \iff \frac{1}{n}\sum_{i=1}^{n} x_i = n\theta$$

$$\iff \hat{\theta}_{mom} = \hat{n}_{mom}^{-2}\sum_{i=1}^{n} X_i \neq \hat{\theta}_{mle} = n^{-1}\sum_{i=1}^{n} X_i$$

for $\hat{n}_{mom}$, we find $\hat{n}_{mom} = \dfrac{\bar{X}^2}{\bar{X} - \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}$

if we define $var(X) = E[X^2] - (E[X])^2$

$$n\frac{\bar{X}}{n}\left(1-\frac{\bar{X}}{n}\right) = \bar{X} - \frac{\bar{X}}{n}, \text{ with } \frac{\bar{X}}{n} = n^{-2}\sum_{i=1}^{n} X_i$$

Reference: An introduction to mathematical statistics; Bijma; Jonker; van der Vaart, 2016