Contingency tables and odds ratios

Gender \ Belief in afterlife	Yes	No	Total
Females	509 (n_{11})	116 (n_{12})	625 ($n_{1.}$)
Males	398 (n_{21})	104 (n_{22})	502 ($n_{2.}$)
Total	907 ($n_{.1}$)	220 ($n_{.2}$)	1127 (n)

$\pi = \frac{n_{.1}}{n}$ $\quad \pi_1 = \frac{n_{11}}{n_{1.}}$

relative risk (RR) $= \frac{\pi_1}{\pi_2} = \frac{509/625}{398/502} = \underline{1.03}$

odds (O) $= \frac{\pi}{(1-\pi)} = \frac{907/1127}{220/1127} \approx \underline{4.123}$ (412 yes for 100 no)

$\pi = \frac{O}{(O+1)} \approx \frac{4.123}{5.123} = \underline{0.805}$

odds ratio (OR) $= \frac{O_1}{O_2} = \frac{\pi_1(1-\pi_1)}{\pi_2(1-\pi_2)}$ (if > 1, Female more likely than π_1)

The sample $\widehat{OR} = \frac{509/116}{398/104} = \frac{4.388}{3.827} = \underline{1.147}$

CI_{95} for $\log(\widehat{OR}) = \log(\widehat{OR}) \pm z_{\alpha/2}\, se(\widehat{OR})$

with $se(\widehat{OR}) = \left(\frac{1}{n_{11}} + \frac{1}{n_{12}} + \frac{1}{n_{21}} + \frac{1}{n_{22}}\right)^{1/2} = \underline{0.1507}$

So $CI_{95}^{\log(OR)} = [-0.159,\ 0.432]$; $\log(\widehat{OR}) = \underline{0.137}$

$CI_{95}\ OR = [0.853,\ 1.541]$; $\widehat{OR} = \underline{1.147}$

This interval contains 1. It seems there is no difference for females or males (low RR).

Reference: An Introduction to Categorical Data Analysis, A. Agresti, 2007