The Yule-Walker equations (matrix form)

Rationale: Used to find the parameters of an AR(p) model in time series analysis.

Consider an AR(p) model

$$X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t \quad , \quad \varepsilon_t \text{ are White Noise}$$

The Yule-Walker equations (in matrix form) are given by:

$$\begin{pmatrix} \gamma_0 & \gamma_1 & \cdots & \gamma_{p-1} \\ \gamma_1 & \gamma_0 & \cdots & \gamma_{p-2} \\ \vdots & \vdots & & \\ \gamma_{p-1} & \gamma_{p-2} & \cdots & \gamma_0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_p \end{pmatrix} = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_p \end{pmatrix}$$

Numerical example: Let us consider the values $\gamma_0 = 1$, $\gamma_1 = 0.8$ and $\gamma_2 = 0.5$. We then have:

$$\begin{pmatrix} 1 & 0.8 \\ 0.8 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0.8 \\ 0.8 & 1 \end{pmatrix}^{-1} = \frac{1}{1-0.8^2} \begin{pmatrix} 1 & -0.8 \\ -0.8 & 1 \end{pmatrix} = \begin{pmatrix} 2.\overline{7} & -2.\overline{2} \\ -2.\overline{2} & 2.\overline{7} \end{pmatrix}$$

Now $\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} 2.\overline{7} & -2.\overline{2} \\ -2.\overline{2} & 2.\overline{7} \end{pmatrix} \begin{pmatrix} 0.8 \\ 0.5 \end{pmatrix}$

$\iff$ $\phi_1 = 1.\overline{1}$ and $\phi_2 = 0.3\overline{8}$