Elementary Statistical Inference:

Binomial distribution;

Let $X = \begin{cases} 1 & \text{success,} \\ 0 & \text{otherwise} \end{cases}$ $\quad X \sim Bin(n, \theta), \theta \in [0,1]$

Likelihood estimate based on a sample on X:

$$L(\theta) = \prod_{i=1}^{n} p(x_i ; \theta) = \theta^{\sum_{i=1}^{n} x_i} (1-\theta)^{n - \sum_{i=1}^{n} x_i}$$

logs: $$\ell(\theta) = \sum_{i=1}^{n} x_i \log(\theta) + \left(n - \sum_{i=1}^{n} x_i\right) \log(1-\theta)$$

$$\frac{\partial \ell(\theta)}{\partial \theta} = \frac{\sum_{i=1}^{n} x_i}{\theta} - \frac{n - \sum_{i=1}^{n} x_i}{1-\theta}$$

solving for θ, we get $\hat{\theta}_{ml} = n^{-1} \sum_{i=1}^{n} x_i = \bar{x}$; proportion of successes in "n" indep. Bernoulli trials.

eg. In a study, we find that 18 out of 79 patients are at risk of a disease "d". **Likelihood estimator**:

$$\hat{\theta}_{ml} = \frac{18}{79} \approx \underline{0.228}$$

Reference: Introduction to mathematical statistics
Hogg; McKean; Craig, 2019