Gamma distribution as part of the Exponential Family

If $y \sim Ga(\alpha, \beta)$, then its density is

$$f(y,\theta,\phi) = \frac{\beta}{\Gamma(\alpha)}(\beta y)^{\alpha-1}\exp(-\beta y) \quad , y \geq 0$$

and $$\log(f(y,\theta,\phi)) = \log\left(\frac{\beta}{\Gamma(\alpha)}\right) + (\alpha-1)\log(\beta y) - \beta y \quad (i)$$

$$\boxed{\text{mean: } \frac{\alpha}{\beta} = E[Y] = \mu \iff \beta = \frac{\alpha}{\mu}}$$

So (i) is rewritten as

$$\underbrace{\log\left(\frac{\alpha}{\mu\,\Gamma(\alpha)}\right) + (\alpha-1)\log\left(\frac{\alpha}{\mu}y\right)}_{\text{term I}} \underbrace{- \frac{\alpha}{\mu}y}_{\text{term II}}$$

$$\text{term I} = -\log(\mu) + \underline{\log\left(\frac{\alpha}{\Gamma(\alpha)}\right) + (\alpha-1)\log(\alpha y)} - (\alpha-1)\log(\mu)$$

$$= -\alpha\log(\mu) + \log(\mu) - \log(\mu) = \underbrace{-\alpha\log(\mu)}_{\text{term III}}$$

$$\text{term IV} = \log\left(\frac{\alpha}{\Gamma(\alpha)}\right) + (\alpha-1)\log(\alpha y)$$

So now for $f(y,\theta,\phi) = \exp(\text{term II} + \text{term III} + \text{term IV})$

$\theta = -\frac{1}{\mu}$, $\phi = 1/\alpha$, $a(\phi) = \phi$ (term II)

$b(\theta) = -\log(\mu)$, $a(\phi) = -1/\theta$ (term III $= b(\theta)/a(\phi)$)

term IV $= c(y,\phi)$