Maximum Likelihood Estimation

Exponential model

$$L(\lambda|X) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i} = \lambda^n e^{-\lambda \sum_{i=1}^{n} x_i}$$

log likelihood :

$$\ell(\lambda|X) = n \ln(\lambda) - \lambda \sum_{i=1}^{n} x_i$$

$$\frac{\partial \ell(\lambda|X)}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i \overset{\text{set}}{\Longrightarrow} 0$$

$$\frac{n}{\lambda} = \sum_{i=1}^{n} x_i \iff \hat{\lambda}_{mle} = \frac{n}{\sum_{i=1}^{n} x_i} = \frac{1}{\bar{x}}$$

Reference: Any good mathemalical Statistics textbook.