Bayesian Inference : estimating the probability of a female birth

probability of a female birth θ.

model for births : $y \sim Bin(n, \theta)$

y : number of female birth

n : total number of births

θ : unknown probability of a female birth

prior for θ : $\theta \sim Beta(\alpha, \beta)$

our posterior : $\theta | y \sim Beta(\alpha + y, \beta + n - y)$

Let us assume a weakly informative prior

$\alpha = \beta = 2$, so $\theta \sim Beta(2, 2)$

and we observed $y = 13$ out of $n = 20$

Getting posterior quantities is straightforward with R or Python as we know the posterior to be $\theta | y \sim Beta(15, 9)$

with posterior mean : 0.625, expected probability of a female birth

posterior variance : $\approx$ 0.0094

95% credible interval : [0.427, 0.803]

Example inspired from "Bayesian Data Analysis" Gelman et al.