The innovations algorithm

The innovations algorithm is a recursive method for forecasting moving average MA(q) models.

Let $X_t = Z_t + \sum_{k=1}^{q} \theta_k Z_{t-k}$ be a MA(q) process where $\{Z_t\} \sim WN(0, \sigma^2)$.

algorithm:

1. Compute one-step-ahead forecast as

$$\hat{X}_{t+1|t} = \sum_{j=1}^{q} \theta_j e_{t+1-j}$$

2. Update e_t as:

$$e_t = X_t - \hat{X}_{t|t-1}$$

3. For $k > 1$ steps ahead:

$$\hat{X}_{t+k|t} = \sum_{j=1}^{q} \theta_j \hat{e}_{t+k-j}$$

where $\hat{e}_{t+k-j}$ are updated errors.

Example: $X_t = Z_t + 0.5 Z_{t-1}$, $Z_t \sim WN(0,1)$ and $\{X_1, X_2, X_3\} = (1, 0.5, 1.5)$

One-step-ahead forecast:

$t=2$ $\quad \hat{X}_{2|1} = 0.5 \cdot 1 = 0.5 \quad e_2 = X_2 - \hat{X}_{2|1} = 0$

$t=3$ $\quad \hat{X}_{3|2} = 0.5 \cdot 0 = 0 \quad e_3 = X_3 - \hat{X}_{3|2} = 1.5$

$$\hat{X}_{4|3} = 0.5\, e_3 = 0.5\,(1.5) = \underline{0.75}$$