Continuous Uniform distribution

Expectation

Let $X \sim U_{[a,b]}$.

PDF:
$$f_{a,b}(x) = \begin{cases} \frac{1}{b-a} & \text{if } x \in [a,b], \\ 0 & \text{otherwise} \end{cases}$$

Then we have that $E[X] = \frac{a+b}{2}$. Proof.

$$\begin{aligned} E[X] &= \int_{-\infty}^{+\infty} x f_{a,b}(x)\, dx \\ &= \frac{1}{b-a} \int_a^b x\, dx \\ &= \frac{1}{b-a} \left. \frac{x^2}{2} \right|_a^b \\ &= \frac{1}{b-a} \frac{b^2 - a^2}{2} \\ &= \frac{1}{b-a} \frac{(b-a)(b+a)}{2} \\ &= \frac{a+b}{2} \quad \blacksquare \end{aligned}$$

Reference: Any basic probability textbook.