Fisher Information for a Poisson model

Suppose that we observe a sample $x_1, \dots, x_n \sim Poi(\lambda)$

The Fisher information of the sample is defined as

$E\left[\left(\frac{\partial \ell(x|\lambda)}{\partial \lambda}\right)^2\right]$ or equivalently $-E\left[\frac{\partial^2 \ell(x|\lambda)}{\partial^2 \lambda}\right]$.

So we have

$$L(x|\ ) = \prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} = \frac{\lambda^{\sum_{i=1}^{n} x_i} e^{-n\lambda}}{\prod_{i=1}^{n} x_i!}$$

$$\ell(x|\ ) = \sum_{i=1}^{n} x_i \ln(\lambda) - n\lambda - \sum_{i=1}^{n} \ln(x_i!)$$

$$\frac{\partial \ell(x|\lambda)}{\partial \lambda} = \frac{\sum_{i=1}^{n} x_i}{\lambda} - n$$

$$\frac{\partial^2 \ell(x|\lambda)}{\partial^2 \lambda} = -\sum_{i=1}^{n} x_i / \lambda^2$$

and we note that $E\left[\sum_{i=1}^{n} x_i\right] = n\lambda$, so we have that

$$-E\left[\frac{\partial^2 \ell(x|\lambda)}{\partial^2 \lambda}\right] = -E\left[-\sum_{i=1}^{n} x_i\right] / \lambda^2$$

$$= - - n\lambda / \lambda^2$$

$$= n / \lambda$$

■

The Fisher information of one observation is $1/\lambda$.