Mean and Variance of Exponential r.v.

PDF $p_\lambda(x) = \lambda e^{-\lambda x} \mathbb{1}_{R+}(x)$

So if $X \sim \mathcal{E}xp(\lambda)$ we have,

$$E[X] = \int_0^\infty x\lambda e^{-\lambda x} dx$$

$$= \int_0^\infty u e^{-u} \frac{du}{\lambda} \qquad \left(u = \lambda x,\ x = \frac{u}{\lambda},\ dx = \frac{du}{\lambda}\right)$$

$$= \frac{1}{\lambda} \underbrace{\int_0^\infty u e^{-u} du}_{\Gamma(2) = 1\Gamma(1) = 1} = \underline{1/\lambda}$$

$$E[X^2] = \int_0^\infty x^2 \lambda e^{-\lambda x} dx = \frac{1}{\lambda}\int_0^\infty u^2 e^{-u} \frac{du}{\lambda}$$

$$= \frac{1}{\lambda^2} \underbrace{\int_0^\infty u^2 e^{-u} du}_{\Gamma(3) = 2} = 2\frac{1}{\lambda^2} = \underline{2/\lambda^2}$$

$$\mathrm{var}(X) = E[X^2] - (E[X])^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$$

$$= \underline{1/\lambda^2}$$

Reference: Introduction to Mathematical Statistics, Hogg, McKean, Craig ; 2019