Hidden Markov Models

Hidden Markov Model is a family of statistical models (for sequential or time series data) in which the variable of interest Y_t depends on an unobserved state X_t that evolves over time according to a Markov chain (Markov process).

Given $Y_{1:T} = (Y_1, \dots, Y_T)$, an observed sequence and $X_{1:T} = (X_1, \dots, X_T)$, a latent process where each $X_t \in \{1, \dots, M\}$ is a hidden state, and for

$$P(X_1 = i) = \pi_i \quad , i = 1, \dots, M \qquad \text{(initial state)}$$

$$P(X_t = j \mid X_{t-1} = i) = q_{ij} \,, \; Q = [q_{ij}] \qquad (Q, \text{ some transition matrix})$$

The model:

$$Y_t \mid X_t = m \sim f_m(Y_t \mid \theta_m)$$

The PDF (or PMF) of Y_t when the system is in state m. θ_m are parameters specific to state m.

See: Monte Carlo Statistical Methods; Robert & Casella