The Yule-Walker equations : example

Rationale : Used to find the parameters of an AR(p) model in time series analysis.

Consider an AR(p) model

$$X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t \quad , \varepsilon_t \text{ are White Noise}$$

The **Yule-Walker equations** are given by

$$\gamma_k = \sum_{i=1}^{p} \phi_i \gamma_{k-i}$$

where γ_k are the autocovariance at lag k and ϕ_i are the AR estimates.

Numerical example : let's assume that we have $\gamma_0 = 1$, $\gamma_1 = 0.8$ and $\gamma_2 = 0.5$. We have :

$$\gamma_1 = \phi_1 \gamma_0 + \phi_2 \gamma_1 \quad \text{and} \quad \gamma_2 = \phi_1 \gamma_1 + \phi_2 \gamma_0$$

$$\begin{cases} 0.8 = \phi_1 + 0.8 \gamma_2 \\ 0.5 = 0.8 \phi_1 + \phi_2 \end{cases} \iff \phi_1 = 0.8 - 0.8 \phi_2$$

$$\Rightarrow \quad 0.5 = 0.8 (0.8 - 0.8 \phi_2) + \phi_2$$

$$\Rightarrow \quad 0.5 = 0.64 + 0.36 \phi_2 \iff \underline{\phi_2 = -0.3\overline{8}}$$

$$\iff 0.5 = 0.8 \phi_1 - 0.3\overline{8} \iff \underline{\phi_1 = 1,\overline{1}}$$