

Continuous Mapping Theorem

Let $\hat{\theta}_n$ be an estimator for the parameter θ .
If we have $\hat{\theta}_n \xrightarrow[p]{as\ n \rightarrow \infty} \theta$, then for a continuous function $g(\hat{\theta}_n)$, the convergence in probability is preserved, i.e. we have
(or distribution)

$$g(\hat{\theta}_n) \xrightarrow[p]{as\ n \rightarrow \infty} g(\theta)$$

$g(\hat{\theta}_n)$ will also converge to $g(\theta)$.

$\hat{\theta} = \bar{X}_n$ is an estimator for μ ,

$$g(x) = x^2,$$

So, by the continuous mapping theorem, we have that $(\bar{X}_n)^2 \xrightarrow[p]{as\ n \rightarrow \infty} (E[X])^2$, here

$$(E[X])^2 \xrightarrow[p]{as\ n \rightarrow \infty} \mu^2.$$