The irreducibility property of Markov Chains

A Markov Chain $\{X_t\}_{t \geq 0}$ on a state space $\mathcal{X}$ with transition kernel (or transition matrix) $P(x, A)$, (A denotes a general set of a specific state) is said to be **irreducible** if:

$$\forall x, y \in \mathcal{X}, \exists t > 0 \text{ such that } P^t(x, y) > 0$$

That is that for any pair of states x, y there exists a finite number of steps t s.t. the probability of transitioning from x to y in t steps is positive.

In other words, if there are states that cannot be reached in a finite number of steps, the chain is **not irreducible.**

Irreducibility is an important properties that will ensure that a (posterior) stationary distribution is entirely explored in the context of MCMC methods.