Expectation Poisson r.v.

Let $X \sim P(\lambda)$. Then we have $E[X] = \lambda$.

$$
\begin{aligned}
E[X] &= \sum_{x=0}^{\infty} x P(X=x) \\
&= \sum_{x=0}^{\infty} x \frac{\lambda^x e^{-\lambda}}{x!} \\
&= e^{-\lambda} \sum_{x=1}^{\infty} x \frac{\lambda^x}{x!} \\
&= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} \qquad \text{Let } y = x-1 \\
&= \lambda e^{-\lambda} \underbrace{\sum_{y=0}^{\infty} \frac{\lambda^y}{y!}}_{=e^{\lambda}} \qquad \text{Taylor series expansion for } e^{\lambda} \\
&= \lambda e^{-\lambda} e^{\lambda} \\
&= \lambda \qquad \square
\end{aligned}
$$

Similarly, we can prove that
$E[X^2] = \lambda(\lambda+1)$ and $var(X) = \lambda$

Reference: Introduction to Mathematical Statistics; Hogg, McKean, Craig; 2019.