

# The weak law of Large Numbers

Let  $X_1, X_2, \dots$  be a sequence of independent r.v., each having the same mean  $\mu$  and having finite variance. Then

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$$

$\forall \varepsilon > 0$ , we have that

$$\lim_{n \rightarrow \infty} P(|\bar{X}_n - \mu| \geq \varepsilon) = 0 \quad \mu \in [X_i]$$

That is that the sample averages converge in probability to the common (population) mean  $\mu$ .

Proof: using Chebychev's inequality.

Convergence in probability is "weaker" than almost sure convergence (strong LLN).

Not all  $X_i$  are "almost sure" to converge but the sample mean, for " $n$ " large.

Reference: Probability and Statistics, 2nd ed.,  
Michael J. Evans and Jeffrey S. Rosenthal,  
2009.