Elementary Statistical Inference

Binomial distribution ;

Let $X = \begin{cases} 1 & \text{succes,} \\ 0 & \text{otherwise} \end{cases}$ $X \sim Bin(n, \theta)$, $\theta \in [0,1]$

Bayes estimator (continuous case)

The Bayes estimator w.r.t. the prior density π is the estimator T that minimizes $R(\pi; T)$ over all T.
The Bayes estimate for $g(\theta)$ w.r.t. the posterior density π is given by :

$$T(x) = \frac{\int g(\theta)\, p_\theta(x)\, \pi(\theta)\, d\theta}{\int p_\theta(x)\, \pi(\theta)\, d\theta}$$

n Known ; $\theta \in [0,1]$ to estimate ; Beta prior

$$\pi(\theta) = \frac{1}{B(\alpha,\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1} \mathbb{1}_{[0,1]}(\theta)$$

We find that $\hat{\theta}_{bay} = T_{\alpha,\beta}(X) = \frac{X+\alpha}{n+\alpha+\beta}$ with $X = \sum_{i=1}^{n} X_i$

We find a different Bayes estimator for each combination of parameters (α, β) ; $\alpha > 0$; $\beta > 0$.

Note : $\hat{\theta}_{mle} = X/n$ is not in the class of Bayes estimators. It is the limit case $(\alpha,\beta) \to (0,0)$

Reference : An introduction to mathematical statistics, Bijma ; Jonker ; van der Vaart , 2016