

# Moment Generating Function :

## Normal r.v.

Let us recall that  $M_Z(t) = E[e^{tz}] = e^{t^2/2}$ .

Then, for  $X = \mu + \sigma Z \sim N(\mu, \sigma^2)$ , we have :

$$\begin{aligned} M_X(t) &= E[e^{tx}] \\ &= E[e^{t(\mu + \sigma Z)}] \\ &= E[e^{\mu t} e^{t\sigma Z}] \\ &= e^{\mu t} E[e^{t\sigma Z}] \\ &= e^{\mu t} M_Z(t\sigma) \\ &= e^{\mu t} e^{(t\sigma)^2/2} \\ &= e^{\frac{t^2 \sigma^2}{2} + \mu t} \end{aligned}$$

Reference : see for ex. Probability and Statistics; M. Evans and J. Rosenthal, 2009.