

Random Walks

Let e_1, e_2, \dots be a sequence of iid r.v. with mean 0 and variance σ_e^2 . Then the series

$\{Y_t : t = 1, 2, \dots\}$ is constructed as follows:

$$Y_1 = e_1$$

$$Y_2 = e_1 + e_2$$

\vdots

$$Y_t = e_1 + e_2 + \dots + e_t$$

That we can rewrite as

$$\boxed{Y_t = Y_{t-1} + e_t}$$

$$\text{Then } \underline{E[Y_t]} = E[e_1] + E[e_2] + \dots + E[e_t]$$
$$= 0 + 0 + \dots + 0 = \underline{0}$$

$$\text{and } \underline{\text{var}(Y_t)} = \text{var}(e_1 + e_2 + \dots + e_t)$$
$$= \text{var}(e_1) + \text{var}(e_2) + \dots + \text{var}(e_t)$$
$$= \sigma_e^2 + \sigma_e^2 + \dots + \sigma_e^2$$
$$= \underline{t \sigma_e^2}$$

Reference : Time Series Analysis With Applications in R ; Cryer, Chan ; 2008.