Durbin - Levinson algorithm

It is a recursive algorithm for solving fast and efficiently the Yule - Walker equations used to estimate the parameters of an AR(p) model by iteratively computing the coefficients and minimizing the prediction errors.

Let $\{Y_t\}_{t=1}^{n}$ be a time series for which we want to solve the Yule - Walker equations:

$$\gamma_k = \sum_{i=1}^{p} \phi_i \gamma_{k-i}, \quad k = 1, 2, \dots, p$$

γ_k are autocovariances and ϕ_i are autoregressive param.

Algorithm:

(1) compute autocovariances: $\gamma_k = \frac{1}{n} \sum_{t=k+1}^{n} (Y_t - \bar{y})(Y_{t-k} - \bar{y})$

(2) initialization: $\phi_0 = 1$ and $\phi_1 = \gamma_1 / \gamma_0$

(3) compute coefficients: $\alpha_k = \dfrac{\gamma_k - \sum_{i=1}^{k-1} \phi_i \gamma_{k-i}}{\gamma_0 - \sum_{i=1}^{k-1} \phi_i \gamma_{k-i}}$

where $\phi_k = \phi_{k-1} + \alpha_k \phi_{k-1}$ for $k = 2, 3, \dots, p$

(4) By recursion, we compute

$$\phi_1, \phi_2, \dots, \phi_p$$