

Sum of Normal r.v.

Suppose Y_1, \dots, Y_n are independent and Normally distributed r.v. with distributions

$$Y_i \sim N(\mu_i, \sigma_i^2)$$

Let us define W , a weighted sum s.t.:

$$W = \sum_{i=1}^n a_i Y_i \quad ; \text{ then, }$$

$$W \sim N\left(\sum_{i=1}^n a_i \mu_i, \sum_{i=1}^n a_i^2 \sigma_i^2\right)$$

$$\text{if } Y_i \stackrel{\text{iid}}{\sim} N(0, 1^2) \quad \left(\begin{array}{l} \text{standard} \\ \text{normal} \end{array} \right)$$

Then we have that

$$\sum_{i=1}^n Y_i^2 \sim \chi^2(n) \quad \left(\begin{array}{l} \text{Chi-square} \\ \text{distribution} \end{array} \right)$$

Reference : An Introduction to Generalized Linear Models ; Dobson, Barnett ; 2008.