

# Exponential smoothing : a simple example

Exponential smoothing is a time series forecasting technique that produces predictions by assigning exponentially decreasing weights to past observations.

Let  $S_t$  be the "smoothed" value (forecast) at time  $t$ , and  $x_t$  the observed value at time  $t$ . The exponential smoothing formula is given by :

$$S_t = \alpha x_t + (1-\alpha) S_{t-1}, \quad \alpha \in [0,1] \text{ smoothing parameter}$$

Example :  $x = (10, 12, 15, 18)$  observation vector  
 $\alpha = 0.5$ ,  $S_0 = 10$

$S_1, S_2, S_3$  and  $S_4$  can be computed iteratively as such:

$$S_1 = \alpha x_1 + (1-\alpha) S_0 = 10$$

$$S_2 = \alpha x_2 + (1-\alpha) S_1 = 11$$

$$S_3 = \alpha x_3 + (1-\alpha) S_2 = 13$$

$$S_4 = \alpha x_4 + (1-\alpha) S_3 = 15.5$$

Smoothed values  $\tilde{x} = (10, 11, 13, 15.5)$   
and the forecast at  $t=5$  is

$$\underline{S_4 = 15.5.}$$