Simple Random Sampling (without replacement)

Simple Random Sampling Without Replacement (SRSWOR) is a sampling design with fixed sample size and all the samples have the same probability of being selected, namely

$$p(s) = \begin{cases} \binom{N}{n}^{-1} & \text{if } \#s = n \\ 0 & \text{otherwise} \end{cases}$$

where $n \in \{1, \dots, N\}$ with $\binom{N}{n} = \frac{N!}{n!(N-n)!}$.

The inclusion probabilities are then

$$\underset{\text{(firs order)}}{\pi_k = n/N} \quad \text{and} \quad \underset{\text{(joint)}}{\pi_{k\ell}} = \frac{n(n-1)}{N(N-1)}, \quad \text{and}$$

$$\Delta_{k\ell} = \begin{cases} \pi_{k\ell} - \pi_k \pi_\ell = \frac{n(N-n)}{N^2(N-1)} & \text{if } k \neq \ell \\ \pi_k(1-\pi_k) = \frac{n(N-n)}{N^2} & \text{if } k = \ell \end{cases}$$

Horvitz-Thompson estimator of the mean:

$$\hat{\bar{Y}}_{SRSWOR} = \frac{1}{N} \sum_{k \in s}' \frac{y_k}{\pi_k} = \frac{1}{N} \sum_{k \in s}' y_k \frac{N}{n}$$

Horvitz-Thompson estimator of the total:

$$\hat{Y}_{SRSWOR} = \sum_{k \in s}' \frac{y_k}{\pi_k} = N \hat{\bar{Y}}_{SRSWOR}$$

see: Sampling Methods, Ardilly, Tillé