

The strong law of Large Numbers

Let X_1, X_2, \dots be a sequence of iid random variables, each having finite mean μ . Then

$$P\left(\lim_{n \rightarrow \infty} \bar{X}_n = \mu\right) = 1$$

$$\text{with } \bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i, \quad \mu = E[X_i]$$

That is that the sample averages converge with probability 1 (or converge almost surely) to the common mean μ . ($\bar{X}_n \xrightarrow{\text{a.s.}} \mu$)

Note: Convergence with probability 1 implies convergence in probability.

Reference: Probability and Statistics, 2nd edition, Michael J. Evans and Jeffrey S. Rosenthal, 2009