Mean and Variance of Exponential Family models (1/3)

Given a model with location parameter θ and scale parameter ϕ, with log likelihood function $\ell(\theta, \phi, y)$ and having density or mass function

$$f(y; \theta, \phi) = \exp\left(\frac{y\theta - b(\theta)}{a(\phi)} + c(y, \theta)\right)$$

where $a(\cdot)$, $b(\cdot)$ and $c(\cdot)$ are specific functions.

We can prove that

$$E[Y] = b'(\theta) \quad \text{and}$$
$$\text{var}(Y) = b''(\theta)\, a(\phi)$$

So the mean depends only on the location parameter while the variance depends on the location and scale parameter.

Proofs on the two next short notes