The Delta method

The Delta method is a technique used to approximate the variance of a nonlinear function of an estimator by using a first order Taylor expansion. Given a function $g(\theta)$ and an estimator $\hat{\theta}$, the approximate variance of $g(\hat{\theta})$ is:

$$var\left(g(\hat{\theta})\right) \approx \left(\frac{dg(\theta)}{d\theta}\right)^2 var(\hat{\theta}).$$

example 1 : $\hat{\theta}$ is an estimate of the mean θ. We are interested in $g(\theta) = \log(\theta)$.

By the Delta method, $var\left(\log(\hat{\theta})\right) \approx \left(\frac{1}{\hat{\theta}}\right)^2 var(\hat{\theta})$

If $\hat{\theta} = 4$ and $var(\hat{\theta}) = 0.2$, then $var\left(\log(\hat{\theta})\right) = 0.0125$.

Example 2 : Let $\hat{\theta}$ be an estimate of θ and we are interested in the variance of the squared estimate $g(\theta) = \theta^2$. Using the Delta method, we have :

$$var\left(\hat{\theta}^2\right) \approx \left(2\hat{\theta}\right)^2 var(\hat{\theta})$$

If $\hat{\theta} = 3$ and $var(\hat{\theta}) = 0.05$, the variance of $\hat{\theta}^2$ is $(2 \cdot 3)^2 \cdot 0.05 = 0.45$.