Highest Posterior Density Interval (HPDI)

Binomial proportion

Suppose that we flip a biased coin 10 times and obtain 7 heads.

Likelihood: $P(x_{:} \mid \pi) = \binom{10}{7} \pi^{7} (1-\pi)^{3}$

prior: non informative $\text{Beta}(1, 1)$

posterior: $P(\pi \mid x_{:}) = \text{Beta}(\alpha = 8, \beta = 4)$

HPD Interval (see code):

$$HPDI_{95} = [\, 0,44 \; ; \; 0,91 \,]$$

Given the observed data $x_{:}$, a 95% HPD Interval for π is $[0,44; 0,91]$. With a 95% probability, the true proportion of heads lies within this interval, which contains the most credible value of π based on the posterior distribution