Fisher Information for a Binomial model

Suppose that we observe a sample $x_1, \dots, x_n \sim Bin(n, \pi)$

The Fisher information of the sample is defined as

$E\left[\left(\frac{\partial \ell(x|\pi)}{\partial \pi}\right)^2\right]$ or equivalently $-E\left[\frac{\partial^2 \ell(x|\pi)}{\partial^2 \pi}\right]$.

So we have

$$L(x|\pi) = \prod_{i=1}^{n} \binom{n}{x_i} \pi^{x_i} (1-\pi)^{n-x_i}$$

$$\ell(x|\pi) = \sum_{i=1}^{n} \ln\binom{n}{x_i} + \sum_{i=1}^{n} x_i \ln(\pi) + \left(n - \sum_{i=1}^{n} x_i\right)(1-\pi)$$

$$\frac{\partial \ell(x|\pi)}{\partial \pi} = \sum_{i=1}^{n} x_i / \pi - \left(n - \sum_{i=1}^{n} x_i\right) / (1-\pi)$$

$$\frac{\partial^2 \ell(x|\pi)}{\partial^2 \pi} = -\sum_{i=1}^{n} x_i / \pi^2 - \left(n - \sum_{i=1}^{n} x_i\right) / (1-\pi)^2$$

and we note that $E\left[\sum_{i=1}^{n} x_i\right] = n\pi$, so we have that

$$-E\left[\frac{\partial^2 \ell(x|\pi)}{\partial^2 \pi}\right] = -\frac{n\pi}{\pi^2} - \frac{(n - n\pi)}{(1-\pi)^2}$$

$$= \frac{-n\pi(1-\pi)^2 - (n - n\pi)\pi^2}{\pi^2(1-\pi)^2}$$

$$= n\pi(1-\pi) / \pi^2(1-\pi)^2$$

$$= \frac{n}{\pi(1-\pi)}$$

Fisher information for one observation : $1/\pi(1-\pi)$