Normal distribution as part of the Exponential Family

Density : $f(y,\theta,\phi) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right)$

That we can rewrite as

$$\exp\left[\frac{y\mu - \mu^2/2}{\sigma^2} - \frac{1}{2}\left(\frac{y^2}{\sigma^2} + \log(2\pi\sigma)\right)\right]$$

Then we have $E[Y] = \mu$, $var(Y) = \sigma^2$

$\theta = \mu$, $\phi = \sigma^2$, $a(\phi) = \phi$, $b(\theta) = \theta^2/2$

and $c(y,\phi) = -(y^2/\phi + \log(2\pi\phi))/2$

So the Normal distribution is a member of the Exponential Family.

For Generalized Linear Models, the link function is in general <u>the identity link</u>, i.e. $g(\mu) = \mu$ that links the mean of the distribution of the variable of interest Y to the linear predictor.