Hannan - Rissanen algorithm

It is an iterative procedure to estimate the parameters of an Autoregressive model (AR(p)). The model is first fitted by Least Squares then the estimates are refined based on the residuals.

Let $\{Y_t\}_{t=1}^{n}$ be a time series for which we fit an AR(p) model

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + \varepsilon_t$$

with ε_t being the residuals and ϕ's are the parameters. We first write the model as a linear regression problem:

$$\underline{Y} = \underline{X}\underline{\Phi} + \underline{\varepsilon} \quad \text{with } \underline{X} \text{ being a matrix of lagged values}$$

and use least squares to obtain $\hat{\underline{\Phi}}$ as

$$\hat{\underline{\Phi}} = (\underline{X}^T\underline{X})^{-1}\underline{X}^T\underline{Y}$$

Then residuals $\hat{\varepsilon}_t$ are obtained as:

$$\hat{\varepsilon}_t = Y_t - \hat{\phi}_1 Y_{t-1} - \hat{\phi}_2 Y_{t-2} - \dots - \hat{\phi}_p Y_{t-p}$$

We then refined the estimates $\hat{\phi}$'s using the current residuals iteratively until convergence, i.e. until

$$\| \hat{\underline{\Phi}}_{K-1} - \hat{\underline{\Phi}}_K \| \leq \alpha \quad \text{where } \alpha \text{ is a specific threshold.}$$