Inference for a Binomial proportion

The sampling distribution of $\hat{\theta}_{mle}$ has mean

$E[\hat{\theta}_{mle}] = \theta$ (unbiased), and

$$var(\hat{\theta}_{mle}) = \left(\frac{\theta(1-\theta)}{n}\right)$$

The sampling distribution is approximately normal for large "n":

For $H_0: \theta = \theta_0$, the test statistic is

$$z = \frac{\hat{\theta}_{mle} - \theta_0}{\left(\frac{\theta_0(1-\theta_0)}{n}\right)^{1/2}}$$

<u>Example</u>: Do a majority or minority of U.S. adults believe that a pregnant woman should be able to obtain an abortion?

Let θ: proportion of "yes"; we test $H_0: \theta = 0.5$ vs

$H_1: \theta \neq 0.5$

Results from 2002 survey:

yes: 400, no: 493, $\hat{\theta}_{mle} = 400/893 = \underline{0.448}$

we find $z = \underline{-3.1}$, $p\text{-val} = 2 \cdot P(z \geq z_{1-\alpha/2} \mid H_0)$

$\approx \underline{0.002}$

Strong evidence that a minority favored legal abortion (2002).

<u>Reference</u>: An Introduction to Categorical Data Analysis, A. Agresti, 2016