Mean and Variance of Exponential Family models (2/3)

Given a model with location parameter θ and scale parameter ϕ, with log likelihood function $\ell(\theta, \phi, y)$ and having density or mass function

$$f(y;\theta,\phi) = \exp\left(\frac{y\theta - b(\theta)}{a(\phi)} + c(y,\theta)\right)$$

wher $a(\cdot)$, $b(\cdot)$ and $c(\cdot)$ are specific functions.

We can prove that $E_y\left[\frac{\partial \ell}{\partial \theta}\right] = 0$, indeed:

$$\frac{\partial \ell}{\partial \theta} = \frac{\partial \ln f(y;\theta,\phi)}{\partial \theta}$$

$$= \partial\left(\frac{y\theta - b(\theta)}{a(\phi)} + c(y,\phi)\right) / d\theta$$

$$= \frac{y}{a(\phi)} - \frac{b'(\theta)}{a(\phi)} \qquad \text{since } E[y] = b'(\theta)$$

So $$E_y\left[\frac{\partial \ell}{\partial \theta}\right] = \frac{b'(\theta)}{a(\phi)} - \frac{b'(\theta)}{a(\phi)}$$

$$= 0$$

$$\boxed{\text{The expectation with respect to the variable of interest } y \text{ of the log likelihood function (derivative of the likelihood function with respect to the location parameter } \theta\text{), is 0 for all exp. family models}}$$