

Binomial distribution as part of the Exponential family

density: $p(y, \theta, \phi) = \binom{n}{y} p^y (1-p)^{n-y}$
we can rewrite the density as

$$\exp \left[y \log \left(\frac{p}{1-p} \right) + n \log(1-p) + \log \left(\binom{n}{y} \right) \right]$$

then $E[Y] = np$, $\text{var}(Y) = np(1-p)$,

$\theta = \log \left(\frac{p}{1-p} \right)$ is the natural parameter

and $b(\theta) = -\log(1-p) = \log(1 + \exp(\theta))$

$\phi = 1/n$, $a(\phi) = \phi$ and $c(y, \phi) = \binom{n}{y}$

The natural parameter $\theta = \log \left(\frac{p}{1-p} \right)$ is called the log-odds or logit of p and is used as the canonical link function in Generalized Linear Models.

The Binomial distribution is thus a member of the Exponential family.