Example of a two-states discrete Markov Chain

We need a state space and a transition matrix

State space : $S = \{1, 2\}$ 1 : sunny
2 : rainy

Transition matrix :

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} = \begin{array}{c|c} & \begin{array}{cc} s & r \end{array} \\ \begin{array}{c} s \\ r \end{array} & \begin{pmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{pmatrix} \end{array}$$

eg. $P_{11} = P(\text{Sunny} \rightarrow \text{Sunny}) = 0.8$

$P_{12} = P(\text{Sunny} \rightarrow \text{Rainy}) = 0.2$

What are the forecasts 2 and 10 days ahead?

2 days ahead : $\pi_2 = \pi_0 P^2$ with initial state sunny : $\pi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

so

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{pmatrix}^2 = \begin{pmatrix} 0.72 \\ 0.28 \end{pmatrix}$$

Probability of the weather being 'sunny' in two days is 72% VS 28% rainy.

10 days ahead : $\pi_{10} = \pi_0 P^{10}$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{pmatrix}^{10} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

The chain has converged to its stationary distribution

50% 'sunny' VS 50% 'rainy'.