Slutsky Theorem

For two sequences X_n and Y_n. If

$X_n \xrightarrow{d} X$ and $Y_n \xrightarrow{P} c$, then,

we have that $$X_n + Y_n \xrightarrow{d} X + c.$$

Example of the empirical variance (to estimate σ^2 a gaussian variance)

$$S_n^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2 - (\bar{X}_n)^2$$

Since, by the LLN, we have that

$$\frac{1}{n}\sum_{i=1}^{n} X_i \xrightarrow{P} E[X^2] \qquad \text{as } n \to \infty$$

Therefore, by the Slutsky theorem:

$$\frac{1}{n}\sum_{i=1}^{n} X_i^2 - (\bar{X}_n)^2 \xrightarrow{d} E[X^2] - (E[X])^2$$

So S_n^2 is a consistent estimator for σ^2.