

Linear Model : confidence intervals for β_j

β_j : individual parameters (coefficients) of the model

$$y_i = \beta_0 + \sum_{j=1}^p \beta_j x_{ij} + \varepsilon_i$$

confidence interval :

$$\hat{\beta}_j \pm t_{1-\alpha/2, n-(p+1)} \cdot \underbrace{\hat{\sigma} \sqrt{[(\underline{X}^T \underline{X})^{-1}]_{jj}}}_{\hat{se}(\hat{\beta}_j)}$$

$t_{1-\alpha/2, n-(p+1)}$: quantile of order $1-\alpha/2$ of a Student's t distr. with $n-(p+1)$ degrees of freedom.

$$\hat{\sigma}^2 = \sqrt{\frac{\underline{e}^T \underline{e}}{n-(p+1)}} = \sqrt{\frac{(\underline{y} - \underline{X} \hat{\underline{\beta}})^T (\underline{y} - \underline{X} \hat{\underline{\beta}})}{n-(p+1)}};$$

estimated standard deviation of the model

Because σ^2 is estimated by $\hat{\sigma}^2$, we have that

$$\frac{\hat{\beta}_j - \beta_j}{\hat{\sigma} \sqrt{[(\underline{X}^T \underline{X})^{-1}]_{jj}}} \sim t_{n-(p+1)}$$