Moment Generating Function:

Multivariate Normal random vector

$$
\begin{aligned}
M_{\underline{X}}(\underline{t}) &= E\left[e^{\underline{t}^T \underline{X}}\right] \\
&= E\left[e^{\underline{t}^T(\underline{\mu} + \underline{Z}(\Sigma^{1/2})^T}\right] \\
&= e^{\underline{t}^T \underline{\mu}} E\left[e^{\underline{Z}(\underline{t}\Sigma^{1/2})^T}\right] \\
&= e^{\underline{t}^T \underline{\mu}} M_{\underline{Z}}(\underline{t}\Sigma^{1/2}) \\
&= e^{\underline{t}^T \underline{\mu}} e^{\frac{1}{2}\|\underline{t}\Sigma^{1/2}\|^2} \\
&= e^{\underline{t}^T \underline{\mu} + \frac{1}{2}\underline{t}^T \Sigma^{1/2 T} \Sigma^{1/2} \underline{t}} \\
&= e^{\underline{t}^T \underline{\mu} + \frac{1}{2}\underline{t}\Sigma\underline{t}}
\end{aligned}
$$

■

So we have that:

$$M_{\underline{X}}(t) = \exp\left(\underline{t}^T \underline{\mu} + \frac{1}{2}\underline{t}^T \Sigma \underline{t}\right)$$

Reference: See for ex. Probability and Statistics; M. Evans and J. Rosenthal; 2009.