Faulty Caliper Problem:

Prior

0.5

$\theta = 1 : \quad y \sim N(1, 1)$

0.5

$\theta = -1 : \quad y \sim N(-1, 1)$

likelihood

$$p(y=0.5 \mid \theta=1) = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{(y-1)^2}{2}\right)} \approx 0.1405$$

marginal

$$p(y=0.5) = p(y=0.5 \mid \theta=1)\, p(\theta=1) + p(y=0.5 \mid \theta=-1)\, p(\theta=-1)$$

$$= 0.09605$$

Posterior

$$p(\theta=1 \mid y=0.5) = \frac{0.07022}{0.09605} \approx 0.73$$