<u>Example</u>: Ice Cream

$y = (450, 300, 250)$

noninformative prior: $\alpha = (1, 1, 1)$

posterior: Dirichlet $\alpha = (451, 301, 251)$

Answer: simulation : draws and compute $\theta_{vanilla} - \theta_{chocolate}$ $\sim 100\%$

Sampling distribution for multivariate normal

$y_1, \dots, y_n \sim N(\mu, \Sigma)$

$$p(y_1, y_2, \dots, y_n \mid \mu, \Sigma) \propto \det(\Sigma)^{-n/2} \exp\left(-\frac{1}{2}(y-\mu)\Sigma^{-1}(y-\mu)^T\right)$$

$$= \det(\Sigma)^{-n/2} \exp\left(-\frac{1}{2} tr\left(\Sigma^{-1} s_0\right)\right)$$

where $s_0 = (y-\mu)(y-\mu)^T$

Pen Example:

$$\mu_0 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \quad \Lambda_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \kappa_0 = 1, \quad \nu_0 = 0$$

$$y = \left\{ \begin{pmatrix} 1.8 \\ 4.8 \end{pmatrix}, \begin{pmatrix} 2.1 \\ 5.1 \end{pmatrix}, \begin{pmatrix} 2.0 \\ 5.3 \end{pmatrix}, \begin{pmatrix} 1.9 \\ 4.9 \end{pmatrix} \right\} \qquad \bar{y} = \begin{pmatrix} 1.95 \\ 5.025 \end{pmatrix}$$

$$S = \sum_{i=1}^{n} (y_i - \bar{y})(y_i - \bar{y})^T = \begin{pmatrix} -0.15 \\ -0.225 \end{pmatrix} \begin{pmatrix} -0.15 & -0.225 \end{pmatrix} + \begin{pmatrix} 0.15 \\ 0.075 \end{pmatrix} \begin{pmatrix} 0.15 & 0.075 \end{pmatrix} + \cdots$$

$$= \begin{pmatrix} 0.0225 & 0.03375 \\ 0.03375 & 0.50625 \end{pmatrix} + \cdots$$

$$\approx \begin{pmatrix} 0.0167 & 0.2167 \\ 0.02167 & 0.049167 \end{pmatrix}$$

$$\mu_n = \frac{\kappa_0}{\kappa_0 + n} \mu_0 + \frac{n}{\kappa_0 + n} \bar{y} = \frac{1}{1+4} \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \left(\frac{4}{1+4} \right) \begin{pmatrix} 1.95 \\ 5.025 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 1.96 \\ 5.02 \end{pmatrix}}$$

$$\kappa_n = 5 \qquad \nu_n = 4$$

$$\Lambda_n = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + S + \frac{4}{5+4} \left(\begin{pmatrix} 1.95 \\ 5.025 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix} \right) \left(\begin{pmatrix} 1.95 \\ 5.025 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix} \right)^T$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0.017 & 0.217 \\ 0.022 & 0.049 \end{pmatrix} + \frac{4}{9} \begin{pmatrix} 0.003 & -0.001 \\ -0.001 & 0.001 \end{pmatrix} \approx \boxed{\begin{pmatrix} 1.02 & 0.217 \\ 0.022 & 1.05 \end{pmatrix}}$$