Bayesian Computational Statistics

reference book : Bayesian Data Analysis;
Gelman et al.

Bayes' rule and its consequences

conditional probabilities

$P(A|B)$ probability of event A given that event B has occured.

$$P(A|B) = P(A \cap B) / P(B)$$

example : We have a standard 6 sided die

$$A = \{5\}, \quad B = \{1, 3, 5\}$$

$$P(A|B) = P(A \cap B) / P(B) = 1/3$$

$$P(A) = 1/6$$

$C = \{2, 4, 6\}$ so $P(A \cap C)/P(C) = 0/3 = 0$

Note: $P(A|B) \neq P(B/A)$

Bayes' rule relates $P(A|B)$ to $P(B/A)$:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

It gives us a framework for making and updating estimates of $P(A|B)$ based on evidence.
Updating our beliefs in the face of new information.

<u>example</u>: medical testing

event A: having the disease
event B: testing positive

prior $P(A)$: 0.01 (1%)

likelihood $P(B/A)$: 0.99

false positive rate to compute the marginal 0.05

marginal: $P(B) = P(B|A) \cdot P(A) + P(B/A^c) \cdot P(A^c)$

$$= 0.99 \cdot 0.01 + 0.05 \cdot 0.99$$

$$\approx 0.0594$$

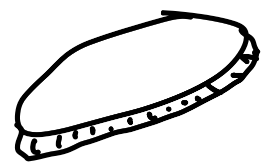
posterior $P(A|B) = \dfrac{P(B/A) \cdot P(A)}{P(B)} = \dfrac{0.99 \cdot 0.01}{0.0594} \approx \underset{2}{\underline{0.1\overline{6}}}$

a bit surprising.

Bayesian inference

1. Start with a prior distribution
2. Collect data
3. Compute the likelihood
4. Compute the marginal probabilit
5. Compute the posterior via Bayes' rule (updated beliefs)

example: Is it a fair coin?



H_0 : It is fair

H_1 : It is biased prior: 0.5

Data: T, T, T

$$P(H_0 \mid TTT) = \frac{P(TTT \mid H_0)\, P(H_0)}{P(TTT)}$$

$$P(TTT) = P(TTT \mid H_0) \cdot P(H_0) + P(TTT \mid H_1) \cdot P(H_1)$$

$$P(TTT \mid H_1) = \int_0^1 p^3 dp = \frac{p^4}{4}\Big|_0^1 = 1/4, \quad \text{so}$$

$$P(H_0 \mid TTT) = \underline{1/3}$$