$$\vec{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \quad \vec{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad \Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

$$|\Sigma| = 1 - \rho^2$$

$$\vec{\theta} \,|\, y \sim N(\vec{y}, \Sigma)$$

$$\Sigma^{-1} = \frac{1}{1-\rho^2} \begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix}$$

Joint probability distribution

$$f(\vec{\theta} \,|\, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2} (\vec{\theta} - \vec{y})^T \Sigma^{-1} (\vec{\theta} - \vec{y}) \right)$$

$$= \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\frac{-1}{2(1-\rho^2)} (\theta_1 - y_1 \quad \theta_2 - y_2) \begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix} \begin{pmatrix} \theta_1 - y_1 \\ \theta_2 - y_2 \end{pmatrix} \right)$$

$$= \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\frac{-1}{2(1-\rho)^2} \left[(\theta_1 - y_1)^2 - 2\rho(\theta_1 - y_1)(\theta_2 - y_2) + (\theta_2 - y_2)^2 \right] \right)$$

$$f(\theta_1 \,|\, \theta_2) \propto \exp\left(-\frac{1}{2(1-\rho^2)} \left[(\theta_1 - y_1)^2 - 2\rho(\theta_2 - y_2)(\theta_1 - y_1) \right] \right)$$

$$\propto \exp\left(-\frac{1}{2(1-\rho^2)} \left[(\theta_1 - (y_1 + \rho(\theta_2 - y_2)))^2 \right] \right)$$

$$\theta_1 \,|\, \theta_2 \sim N\left(y_1 + \rho(\theta_2 - y_2),\ 1 - \rho^2 \right)$$

$$\theta_2 \,|\, \theta_1 \quad \text{is the same calculation.}$$

Suppose $\bar{y} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\rho = 0.8$

$$\theta_1 | \theta_2, y \sim N\left(.8\theta_2, .36\right)$$

$$\theta_2 | \theta_1, y \sim N\left(.8\theta_1, .36\right)$$

θ_1 is first and θ_2 is second. $\theta_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\underline{t=1}$ $\theta_1^1 \sim N(0, .36) \xrightarrow{\text{draw}} \theta_1^1 = 0.12$

$\theta_2^1 \sim N\left((.8)(.12), .36\right) \xrightarrow{\text{draw}} \theta_2^1 = 0.40$

$\underline{t=2}$ $\theta_1^2 \sim N\left((.8)(.4), .36\right) \xrightarrow{\text{draw}} \theta_1^2 = 0.001$

$\theta_2^2 \sim N\left((.8)(.001), .36\right) \xrightarrow{\text{draw}} \theta_2^2 = 0.148$

$\vdots$