Conditional Probability $P(A|B)$

$A = \{5\}$

$B = \{1, 3, 5\}$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \boxed{\dfrac{1}{3}}$

$P(A) = \dfrac{1}{6}$ $\{5\}$ $\{1, 2, 3, 4, 5, 6\}$

$C = \{2, 4, 6\}$

$P(A|C) = \dfrac{P(A \cap C)}{P(C)} = \boxed{\dfrac{0}{3}}$

Example: Medical Testing

Event A = Having the disease

Event B = Testing positive

Prior: $P(A) = 0.01$

Likelihood: $P(B|A) = 0.99$

False positive rate 0.05

Marginal: $P(B) = P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c)$

$= (0.99)(0.01) + (0.05) \cdot (.99)$

≈ 0.0594

Posterior: $P(A|B) = \dfrac{P(B|A)\, P(A)}{P(B)} = \dfrac{(.99)(.01)}{(0.0594)} \approx 0.167$!

Example: A fair coin?

	Prior
H_0: Coin is fair	0.5
H_A: Coin is biased	0.5

Data: TTT

$$P(H_0 \mid TTT) = \frac{p(TTT \mid H_0)\, p(H_0)}{p(TTT)} = \frac{(1/2)^3 (1/2)}{3/16} = \frac{1/16}{3/16} = \boxed{\frac{1}{3}}$$

$$p(TTT) = p(TTT \mid H_0)\, p(H_0) + p(TTT \mid H_A)\, p(H_A)$$

$$p(TTT \mid H_A) = \int_0^1 p^3\, dp = \frac{p^4}{4}\Big|_0^1 = \frac{1}{4}$$

$$= \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right) + \left(\frac{1}{4}\right) \cdot \frac{1}{2} = \frac{1}{16} + \frac{1}{8} = \frac{3}{16}$$