Lesson 1 & module 1

H_0: coin fair

H_A: Coin isn't fair

Prior
0.66
0.34

Data: 5 Heads

likelihood:

$$P(HHHHH \mid H_0) = \left(\frac{1}{2}\right)^5 = \frac{1}{32} = 0.03125$$

$$P(HHHHH \mid H_A) = \int_0^1 p^5 \, dp = \left[\frac{p^6}{6}\right]_0^1 = \frac{1}{6}$$

Marginal:

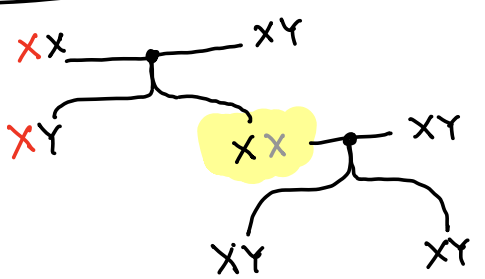
$$P(HHHHHH) = p(\text{data} \mid H_0)\, p(H_0) + p(\text{data} \mid H_A)\, p(H_A)$$

$$= (0.03125)(0.66) + (1/6)(0.34$$

$$\approx 0.078$$

$$p(H_0 \mid \text{data}) = \frac{p(\text{data} \mid H_0)\, p(H_0)}{p(\text{data})} = \frac{(0.03125)(0.66)}{(0.078)} \approx 0.268$$

Hemophilia Example. X-linked trait



← Evidence

Prior
50/50

Prior: 0.5

$\theta = 0$ (not a carrier)

Prior: 0.5

$\theta = 1$ (a carrier)

Data: $\vec{y} = (0, 0)$

likelihood:

$$p(\vec{y} \mid \theta=1) = 1/2 \cdot 1/2 = 1/4$$

$$p(\vec{y} \mid \theta=0) = 1 \cdot 1 = 1$$

marginal:

$$p(\vec{y}) = p(\vec{y} \mid \theta=1)\, p(\theta=1) + p(\vec{y} \mid \theta=0)\, p(\theta=0)$$

$$= 1/4 \cdot 1/2 + 1 \cdot 1/2 = 5/8$$

posterior:

$$p(\theta=1 \mid \vec{y}) = \frac{p(\vec{y} \mid \theta=1)\, p(\theta=1)}{p(y)} = \frac{1/8}{5/8} = 1/5 = 0.2$$

(1/4 · 1/2)

or 20%

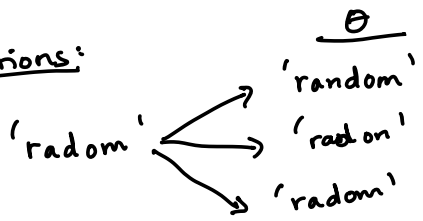
What if there is third child who is also XY and not afflicted.

$y = (0)$

$$p(\theta=1 \mid y) = \frac{p(y \mid \theta=1)\, p(\theta=1)}{p(y)} = \frac{(\frac{1}{2})(1/5)}{\frac{1}{2} \cdot \frac{1}{5} + 1 \cdot 4/5}$$

$$= \frac{1}{9} \approx 11\%$$

Spelling Corrections:



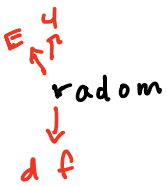
data: y = radom

$$p(\theta|y) \propto p(\theta)p(y|\theta)$$

Prior:

θ	relative freq.	prob.
random	760×10^{-7}	0.923
radon	60.5×10^{-7}	0.073
radom	3.12×10^{-7}	0.004

$$\frac{760}{760+60.5+3.12}$$



Likelihood:

θ	$p('radom'|\theta)$
random	0.00193
radon	0.000143
radom	0.975

Posterior:

θ	$p(\theta)p('radom'|\theta)$	$p(\theta|'radom')$
random	1470×10^{-10}	~ 0.325
radon	8.65×10^{-10}	~ 0.002
radom	3040×10^{-10}	~ 0.673