$$\vec{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \qquad \vec{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \qquad \Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

$$|\Sigma| = 1-\rho^2$$

$$\vec{\theta} \,|\, y \sim N(\vec{y}, \Sigma)$$

$$\Sigma^{-1} = \frac{1}{1-\rho^2}\begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix}$$

Joint probability distribution

$$f(\vec{\theta}\,|\,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2}(\vec{\theta}-\vec{y})^T \Sigma^{-1} (\vec{\theta}-\vec{y})\right)$$

$$= \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\frac{-1}{2(1-\rho^2)} (\theta_1-y_1 \quad \theta_2-y_2) \begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix} \begin{pmatrix} \theta_1-y_1 \\ \theta_2-y_2 \end{pmatrix}\right)$$

$$= \frac{1}{2\pi\sqrt{1-\rho^2}2} \exp\left(\frac{-1}{2(1-\rho)^2}\left[(\theta_1-y_1)^2 - 2\rho(\theta_1-y_1)(\theta_2-y_2) + (\theta_2-y_2)^2\right]\right)$$

$$f(\theta_1\,|\,\theta_2) \propto \exp\left(-\frac{1}{2(1-\rho^2)}\left[(\theta_1-y_1)^2 - 2\rho(\theta_2-y_2)(\theta_1-y_1)\right]\right)$$

$$\propto \exp\left(-\frac{1}{2(1-\rho^2)}\left[\left(\theta_1 - \left(y_1+\rho(\theta_2-y_2)\right)\right)\right]^2\right)$$

$$\theta_1 \,|\, \theta_2 \sim N\left(y_1+\rho(\theta_2-y_2),\ 1-\rho^2\right)$$

$\theta_2 \,|\, \theta_1$ is the same calculation.

Suppose $\vec{y} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\rho = 0.8$

$$\theta_1 | \theta_2, y \sim N(.8\theta_2, .36)$$

$$\theta_2 | \theta_1, y \sim N(.8\theta_1, .36)$$

θ_1 is first and θ_2 is second. $\theta_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

<u>$t=1$</u>

$$\theta_1^1 \sim N(0, .36) \xrightarrow{\text{draw}} \theta_1^1 = 0.12$$

$$\theta_2^1 \sim N((.8)(.12), .36) \xrightarrow{\text{draw}} \theta_2^1 = 0.40$$

<u>$t=2$</u>

$$\theta_1^2 \sim N((.8)(.4), .36) \xrightarrow{\text{draw}} \theta_1^2 = 0.001$$

$$\theta_2^2 \sim N((.8)(.001), .36) \xrightarrow{\text{draw}} \theta_2^2 = 0.148$$

$$\vdots$$