The partial auto-correlation function

Let $\{y_t\}$ be a zero-mean stationary process. Let

$$\hat{y_t}^{h-1} = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \ldots + \beta_{h-1} y_{t-(h-1)}$$

be the best linear predictor of y_t based on the previous $h-1$ values $\{y_{t-1}, \ldots, y_{t-h+1}\}$. The best linear predictor of y_t based on the previous $h-1$ values of the process is the linear predictor that minimizes

$$E[(y_t - \hat{y}_t^{h-1})^2].$$

The partial autocorrelation of this process at lag h, denoted by $\phi(h,h)$ is defined as

$$\phi(h,h) = Corr(y_{t+h} - \hat{y}_{t+h}^{h-1}, y_t - \hat{y}_t^{h-1}),$$

for $h \geq 2$ and $\phi(1,1) = Corr(y_{t+1}, y_t) = \rho(1)$.

The partial autocorrelation function can also be computed via the Durbin-Levinson recursion for stationary processes as $\phi(0,0) = 0$,

$$\phi(n,n) = \frac{\rho(n) - \sum_{h=1}^{n-1} \phi(n-1,h)\rho(n-h)}{1 - \sum_{h=1}^{n-1} \phi(n-1,h)\rho(h)},$$

for $n \geq 1$, and

$$\phi(n,h) = \phi(n-1,h) - \phi(n,n)\phi(n-1,n-h),$$

for $n \geq 2$, and $h = 1, \ldots, (n-1)$.

Note that the sample PACF can be obtained by substituting the sample autocorrelations and the sample auto-covariances in the Durbin-Levinson recursion.