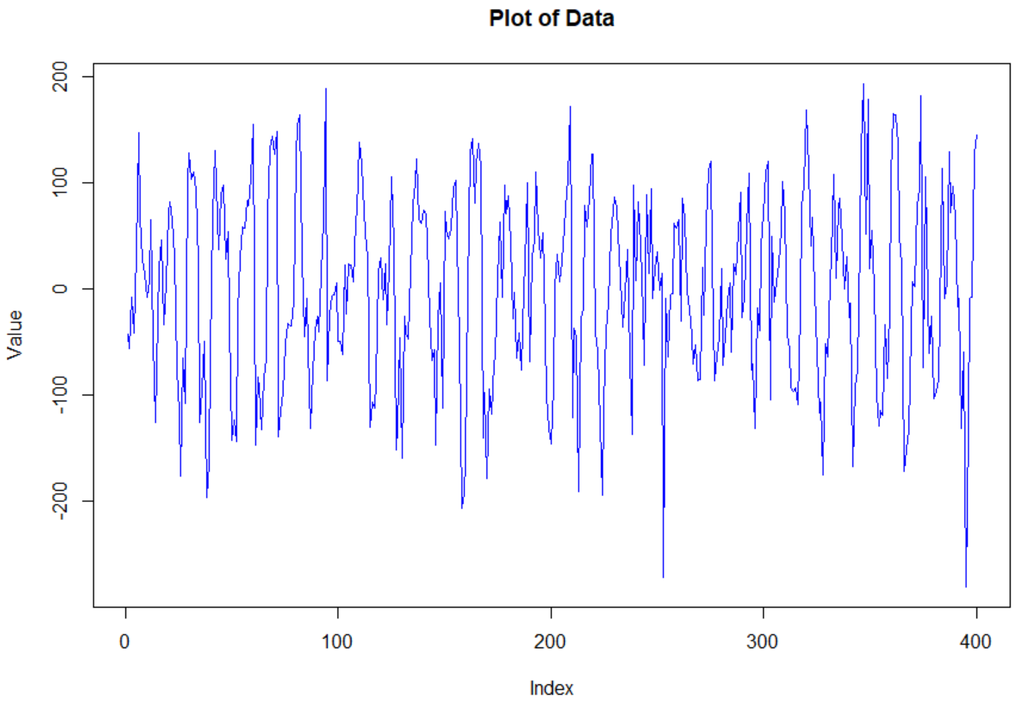
Question 1

Q: Download the dataset, and plot it in R. Upload a picture of your graph displaying the data and comment on the features of the data. Does it present any trends or quasi-periodic behavior?



A: The data, from the plot, does not exhibit any particular linear trend or periodic behavior and appear to be stationary: constant variance and mean over time.

Code:

```
# import data
yt = read.delim("C:/Users/julia/OneDrive/Desktop/Statistics notes/w50y2024/data.txt",
header = FALSE)
yt = as.numeric(data$V1)
plot(yt, type = 'l', col = "blue", main = 'Plot of data',
      xlab = "Index", ylab = "Value",
      main = "Plot of Data")
```

Question 2

Q: Modify the code below to obtain the maximum likelihood estimators (MLEs) for the AR coefficients under the conditional likelihood. For this you will assume an autoregressive model of order $p = 8$. The parameters of the model are $\boldsymbol{\phi} = (\phi_1, ..., \phi_8)^T$ and v. You will compute the MLE of $\boldsymbol{\phi}$, denoted as $\hat{\boldsymbol{\phi}}$.

A: The maximum likelihood estimates are $\boldsymbol{\phi} = (1.60987, -0.8934844, -0.0004550485, 0.008435584,$ $-0.02283475, 0.001826861, -0.01358144, 0.00909051)^T$.

Code:

```
## Case 1: Conditional likelihood
set.seed(2024)
p=8
y=rev(yt[(p+1):T]) # response
X=t(matrix(yt[rev(rep((1:p),T-p)+rep((0:(T-p-1)),rep(p,T-p)))],p,T-p));
XtX=t(X)%*%X
XtX_inv=solve(XtX)
phi_MLE=XtX_inv%*%t(X)%*%y # MLE for phi
s2=sum((y - X%*%phi_MLE)^2)/(length(y) - p) #unbiased estimate for v

cat("\n MLE of conditional likelihood for phi: ", phi_MLE, "\n",
    "Estimate for v: ", s2, "\n")
 MLE of conditional likelihood for phi:  1.60987 -0.8934844 -0.0004550485 0.008435584 -0.02283475 0.001826861
-0.01358144 0.00909051
 Estimate for v:  0.9568277
```

Question 3

Q: Obtain an unbiased estimator for the observational variance of the $AR(8)$.. You will compute the unbiased estimator for v denoted as s^2.

A: The estimator for v is and $s^2 = 0.9568277$.

Question 4

Q: Modify the code below to obtain 500 samples from the posterior distribution of the parameters $\boldsymbol{\phi} = (\phi_1, ..., \phi_8)^T$ and v. Once you obtain samples from the posterior distribution you will compute the posterior means of $\boldsymbol{\phi}$ and v, denoted as $\hat{\boldsymbol{\phi}}$ and $\hat{v}$ respectively.

A: The posterior means are $\hat{\boldsymbol{\phi}} = (1.605355159, -0.885135355, -0.004239663, 0.007291180, -0.026931834, 0.011587524, -0.021160654, 0.011207555)^T$. The rounded results are $\hat{\boldsymbol{\phi}} = (1.605, -0.885, -0.004, 0.007, -0.027, 0.012, -0.021, 0.011)^T$ and $\hat{v} = 0.967$.

Code:

```
n_sample=500 # posterior sample size
library(MASS)

## step 1: sample v from inverse gamma distribution
v_sample=1/rgamma(n_sample, (T-2*p)/2, sum((y-X%*%phi_MLE)^2)/2)

## step 2: sample phi conditional on v from normal distribution
phi_sample=matrix(0, nrow = n_sample, ncol = p)
for(i in 1:n_sample){
  phi_sample[i, ]=mvrnorm(1,phi_MLE,Sigma=v_sample[i]*XtX_inv)
}

#posterior means
apply(phi_sample, 2, mean)
# [1]  1.605355159 -0.885135355 -0.004239663  0.007291180
# -0.026931834  0.011587524 -0.021160654  0.011207555

round(apply(phi_sample, 2, mean),3)

#rounded variance estimate
round(mean(v_sample), 3)
```

Question 5

Q: Modify the code below to use the function polyroot and obtain the moduli and periods of the reciprocal roots of the AR polynomial evaluated at the posterior mean $\hat{\phi}$.

A: The moduli and periods of the reciprocal roots would then be:

Moduli: 0.963, 0.472, 0.963, 0.506, 0.506, 0.495, 0.472, 0.428

Periods: -1.179700e+01, -3.019000e+00, 1.179700e+01, 5.800000e+00, -5.800000e+00, 2.000000e+00, 3.019000e+00, -1.580598e+16

Code:

```
# Assume the folloing AR coefficients for an AR(8)
phi= apply(phi_sample, 2, mean)
roots=1/polyroot(c(1, -phi)) # compute reciprocal characteristic roots
r=Mod(roots) # compute moduli of reciprocal roots
lambda=2*pi/Arg(roots) # compute periods of reciprocal roots
# print results modulus and frequency by decreasing order
print(cbind(r, abs(lambda))[order(r, decreasing=TRUE), ][c(2,4,6,8),])

#                    r
# [1,] 0.9632800 1.179690e+01
# [2,] 0.5057569 5.799588e+00
# [3,] 0.4720255 3.019354e+00
# [4,] 0.4280434 1.580598e+16
```