Stirling's formula

Stirling's formula to approximate the value of a large factorial:

$$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^{n}$$

Note: The approximation by Stirling's formula tend to underestimate slightly the value of the factorial, as we will see on the graph (slide4).

Time complexity:

Stirling's approximation: $\mathcal{O}(1)$

$n! : \mathcal{O}(n)$

Stirling's formula in R

```r
 1 stirling = function(n) {
 2
 3   sqrt(2 * pi * n) * (n^n) * exp(-n)
 4
 5 }
 6
 7 # comparison
 8 approximation = stirling(15)
 9 exact = factorial(15)
10 approximation # 1.300431e+12
11 exact # 1.307674e+12
```

Stirling's formula in Python

```python
import math

def stirling(n):
    return math.sqrt(2 * math.pi * n) * (n**n) * math.exp(-n)

# Comparison
approximation = stirling(15)
exact = math.factorial(15)

print("Approximation:", approximation)
Approximation: 1300430722199.4658
print("Exact value:", exact)
Exact value: 1307674368000
```

Visualization

Exact factorial vs Stirling approximation
exact(orange) vs stirling (green)