

Stirling's formula

Stirling's formula to approximate the value of a large factorial:

$$n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

Note: The approximation by Stirling's formula tend to underestimate slightly the value of the factorial, as we will see on the graph (slide4).

Time complexity:

Stirling's approximation: $O(1)$

$n! : O(n)$

Stirling's formula in R

```
1 stirling = function(n) {  
2  
3   sqrt(2 * pi * n) * (n^n) * exp(-n)  
4  
5 }  
6  
7 # comparison  
8 approximation = stirling(15)  
9 exact = factorial(15)  
10 approximation # 1.300431e+12  
11 exact # 1.307674e+12
```

Stirling's formula in Python

```
1 import math
2
3 def stirling(n):
4     return math.sqrt(2 * math.pi * n) * (n**n) * math.exp(-n)
5
6 # Comparison
7 approximation = stirling(15)
8 exact = math.factorial(15)
9
10 print("Approximation:", approximation)
11 Approximation: 1300430722199.4658
12 print("Exact value:", exact)
13 Exact value: 1307674368000
```

Visualization

Exact factorial vs Stirling approximation

exact(orange) vs stirring (green)

