New Decision Rules for Fuzzy Statistical Inferences

Julien Rosset(✉) and Laurent Donzé

Department of Informatics, University of Fribourg, ASAM Group, Fribourg, Switzerland
{julien.rosset,laurent.donze}@unifr.ch

Abstract. We profit from the concept of the fuzzy confidence interval to implement two decision rules based on two new measures called levels of acceptance and rejection, to perform fuzzy hypothesis testing. The procedure can make a decision when both the data and hypotheses are fuzzy. Through a numerical application, we illustrate our proposed method. We compare the levels of acceptance and rejection of different fuzzy null hypotheses at a 5% significant level for a confidence interval at a 95% confidence level.

Keywords: Fuzzy inference · Fuzzy bootstrapped sample · Fuzzy confidence interval · Fuzzy test on the mean

1 Introduction

Statisticians working with fuzzy data have tried for decades to handle fuzzy inferences properly and have already worked on extending the classical inference concepts to the fuzzy world with more or less success. Nowadays, one disposes of several methods specially implemented to perform fuzzy inference (see, for example, Berkachy and Donzé ([2–5]) and Berkachy [1]). But, one major difficulty of the fuzzy inferences remains. Indeed, one cannot adequately treat the fuzzy distribution the fuzzy test statistic should follow. In this context, a straight comparison with the classical approach is sometimes not pertinent and could lead us in false directions.

In the following, we offer two new rules of decision to test a parameter θ when both fuzzy data and hypotheses are fuzzy. Our method is general and allows fuzzy inference at a given significant level. The procedure consists essentially of calculating a fuzzy confidence interval at a given confidence level for the considered parameter and comparing it to the fuzzy null hypotheses. Evidently, the fuzzy interval depends on the fuzziness of the data, which in turn influences the distribution of the test statistic. To manage this problem, Rosset and Donzé [9] recently proposed using bootstrapped techniques to construct fuzzy confidence intervals. The advantages of our proposed method are that it is intuitive in the sense that the results are easy to interpret and are not heavy computationally.

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The following illustrates our fuzzy testing approach with a numerical application. We thoroughly discuss and compare the results for several sets of hypotheses, which also gives us the opportunity to show how to interpret the obtained results.

In Sect. 2, we comment on recent developments and summarise the notation used throughout the study. We develop, proof and explain our new fuzzy test in Sect. 3. Then, an empirical application is given in Sect. 4. Finally, Sect. 5 concludes our work.

2 Recent Developments and Notation

In the past decades, many researchers explored the fuzzy extension of a classical confidence interval with the aim of using the confidence interval to make statistical inferences. Several fuzzy statistical testing procedures are based on this principle. Regarding the generalisation of a confidence interval to the fuzzy world, we can mention the work of Chachi and Taheri [6], Kahraman, Oztaysi and Onar [7], Berkachy [1] and Berkachy and Donzé [5]. Chachi and Taheri [6] studied the construction of fuzzy confidence intervals for fuzzy parameters based on normal fuzzy random variables and then performed tests for the mean. The restriction to normal fuzzy random variables made it unsuitable for most applications since this assumption is rarely verified. Kahraman, Oztaysi and Onar [7] developed interval-valued intuitionistic fuzzy confidence intervals (IVIF) to test population means. Despite its interesting method, the procedure remains limited since it is built upon using a crisp standard deviation and student t-distribution. The method proposed by Berkachy and Donzé [5] is overcoming this problem by considering bootstrapping techniques to construct fuzzy confidence intervals and performing a fuzzy testing procedure via the likelihood ratio. Since the concept of fuzzy likelihood ratio is difficult to generalise, we were interested in developing a testing procedure that maintains the general construction of fuzzy confidence intervals through Bootstrap while overcoming the limitation of the fuzzy likelihood ratio method to be applicable to any parameter.

In the following, we will define a fuzzy number by $\tilde{x}$. We write by $\mu_{\tilde{x}}(\cdot)$, the membership function. We consider also the α-cuts of $\tilde{x}$ denoted by $\tilde{x}^{\alpha}$ or by its equivalent in interval form by $[x^{L,\alpha}, x^{R,\alpha}]$. In practice, triangular fuzzy numbers are often used. We denote them by a triplet $\tilde{x} = (x^L, x, x^R)$ with $x^L \leq x \leq x^R \in \mathbb{R}$. We denote by X a random variable and by $\tilde{X}$ a fuzzy random variable in the epistemic sense. At last, let us designate by $\complement$ the complement of a set and by $\neg$ the negation of a preposition.

3 Fuzzy Inference for a Parameter θ

Let θ be an unknown distribution parameter, and let us draw a random sample $X_1, \ldots, X_n$ from this distribution. Assuming fuzzy modelling of the data, we can write as $\tilde{X}_1, \ldots, \tilde{X}_n$, the fuzzy equivalent of the random sample, and as $\mathbf{X} = (\tilde{X}_1, \ldots, \tilde{X}_n)$ the fuzzy vector with its membership function $\mu_{\tilde{X}}$, such that

$\mu_{\tilde{X}} : \mathbb{R}^n \to [0,1]^n$. On the other hand, let $\phi : \mathbb{R}^n \to \mathbb{R}$ a real-valued function, and assume that ϕ is the appropriate test function for the parameter θ. In the case of fuzzy data, this function is applied to a fuzzy vector, $\phi(\underset{\sim}{\mathbf{X}}) = \phi(\tilde{X}_1, \ldots, \tilde{X}_n)$. The latter gives a fuzzy number, which we note simply by $\tilde{\phi}$ and designate its α-cuts by $\tilde{\phi}^\alpha$. Viertl [11] gives a complete definition of a fuzzy test statistic. Let us define a fuzzy test in the sense of Kruse and Meyer [8]. See also Berkachy [1, p. 122].

Definition 1. (Fuzzy Test) Consider a fuzzy random vector $\tilde{X}_1, ..., \tilde{X}_n$ related to the original $X_1, ..., X_n$ in the space Ω. Let δ be a particular value of the interval $[0, 1]$. A function $\tilde{\phi}$ is called a fuzzy test for the null H_0 and the alternative H_1 hypotheses at the significance level δ if

$$\sup_{\alpha\in[0,1]} P_{H_0} \left\{\omega_i \in \Omega, i = 1, \ldots, n : \tilde{\phi}^\alpha \left(\tilde{X}_1(\omega_1), ..., \tilde{X}_n(\omega_n)\right) \in \{1\}\right\} \leq \delta \tag{1}$$

where $\tilde{\phi}^\alpha$ is the α-cut of $\tilde{\phi}$ and P_{H_0} means that the probability is evaluated under the condition of the null hypothesis, i.e. $\tilde{\phi}$ is constructed to test H_0.

Adopting the procedure described, for example, in Berkachy [1], we can propose new decision rules for a test at significance level δ with fuzzy data and fuzzy hypotheses. Recall that a fuzzy hypothesis is the transcription of a crisp hypothesis into a fuzzy number, generally a triangular one. The fuzzy null and alternative hypotheses are respectively noted by $\tilde{H}_0$ and $\tilde{H}_1$.

Let us first assume that $\tilde{\sqcap}$ is a fuzzy confidence interval for the parameter θ at the confidence level $1 - \delta$. We explain below how to build such a confidence interval.

Let $\tilde{\phi}^{\text{rej}}$ and $\tilde{\phi}^{\text{acc}}$, two fuzzy statistical tests on $\mathbb{F}_c^*(\mathbb{R})$, the class of the non-empty compact, convex and normal fuzzy sets on $\mathbb{R}$, such that

$$\tilde{\phi}^{\text{rej}} : (\mathbb{F}_c^*(\mathbb{R}))^n \to [0,1], \qquad \tilde{\phi}^{\text{acc}} : (\mathbb{F}_c^*(\mathbb{R}))^n \to [0,1].$$

For a given α-cut, we assume the following functions:

$$\tilde{\phi}^{\text{rej},\alpha}(\tilde{X}_1, ..., \tilde{X}_n) = \begin{cases} 0 & \text{if} \quad \tilde{H}_0^\alpha \subseteq \tilde{\sqcap}^\alpha \quad \text{or if} \quad \alpha = 1, \\ 1 & \text{if} \quad \tilde{H}_0^\alpha \subseteq (\tilde{\sqcap}^\alpha)^{\complement}, \\ b_\alpha & \text{if} \quad \left(\tilde{H}_0^\alpha \cap (\tilde{\sqcap}^\alpha)^{\complement} \neq \emptyset \quad \text{and} \ \neg(\tilde{H}_0^\alpha \supset \tilde{\sqcap}^\alpha) \ \text{and} \ \alpha < 1\right), \\ c_\alpha & \text{if} \ \left(\tilde{H}_0^\alpha \cap (\tilde{\sqcap}^\alpha)^{\complement} \neq \emptyset \quad \text{and} \ \tilde{H}_0^\alpha \supset \tilde{\sqcap}^\alpha \quad \text{and} \quad \alpha < 1\right), \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

$$\tilde{\phi}^{\text{acc},\alpha}(\tilde{X}_1, ..., \tilde{X}_n) = \begin{cases} 0 & \text{if} \quad \left(\tilde{H}_0^\alpha \subseteq (\tilde{\sqcap}^\alpha)^{\complement} \quad \text{or} \quad \alpha = 1\right), \\ 1 & \text{if} \quad \tilde{H}_0^\alpha \subseteq \tilde{\sqcap}^\alpha, \\ d_\alpha & \text{if} \quad \left(\tilde{H}_0^\alpha \cap \tilde{\sqcap}^\alpha \neq \emptyset \quad \text{and} \quad \alpha < 1\right), \\ 0 & \text{otherwise}, \end{cases} \tag{3}$$

where

$$b_\alpha = \left(\frac{r_R - r_L}{H_0^{R,\alpha} - H_0^{L,\alpha}}\right), c_\alpha = \left(\frac{r_R - r_L - \left(\sqcap^{R,\alpha} - \sqcap^{L,\alpha}\right)}{H_0^{R,\alpha} - H_0^{L,\alpha}}\right) \text{ and } d_\alpha = \left(\frac{a_R - a_L}{H_0^{R,\alpha} - H_0^{L,\alpha}}\right).$$

When $\alpha = 1$, $H_0^{R,\alpha} - H_0^{L,\alpha} = 0$, we cannot evaluate b_α, c_α, d_α, hence we have to define a specific rule. The quantities a_L, a_R, r_L and r_R are given by:

$$\begin{aligned} a_L &= \inf(\tilde{H}_0^\alpha \cap \tilde{\sqcap}^\alpha), & a_R &= \sup(\tilde{H}_0^\alpha \cap \tilde{\sqcap}^\alpha), \\ r_L &= \inf(\tilde{H}_0^\alpha \cap (\tilde{\sqcap}^\alpha)^\complement), & r_R &= \sup(\tilde{H}_0^\alpha \cap (\tilde{\sqcap}^\alpha)^\complement). \end{aligned}$$

Proposition 1. *$\tilde{\phi}^{rej}$ and $\tilde{\phi}^{acc}$ are fuzzy tests.*

Similar as in Berkachy [1], a proof of Prop. (1) is given as follows:

Proof. Let us examine the fuzzy test $\tilde{\phi}^{\text{rej}}$. We must verify that the test conforms to Def. 1. Consider the two-sided confidence interval $\tilde{\sqcap}$ of a parameter θ at the significance level δ. By definition, for every value of θ at the null hypothesis H_0,

$$P_{H_0}(\sqcap^{L,\alpha} \leq \theta \leq \sqcap^{R,\alpha}) \geq 1 - \delta \quad \forall \alpha \in [0, 1].$$

On the other hand, we can also see that

$$P_{H_0}\left\{\omega_i \in \Omega, i = 1, \ldots, n : \tilde{\phi}^{\text{rej},\alpha}(\tilde{X}_1(\omega_1), \ldots, \tilde{X}_n(\omega_n)) \in \{1\}\right\}$$

is equivalent to

$$1 - P_{H_0}\left\{\omega_i \in \Omega, i = 1, \ldots, n : \tilde{\phi}^{\text{rej},\alpha}(\tilde{X}_1(\omega_1), ..., \tilde{X}_n(\omega_n)) \in \left\{\{0\}, \{b_\alpha\}, \{c_\alpha\}\right\}\right\}.$$

Moreover, by construction, we have that

$$P_{H_0}\left\{\omega_i \in \Omega, i = 1, \ldots, n : \tilde{\phi}^{\text{rej},\alpha}(\tilde{X}_1(\omega_1), ..., \tilde{X}_n(\omega_n)) \in \left\{\{0\}, \{b_\alpha\}, \{c_\alpha\}\right\}\right\}$$

is equal to $P_{H_0}\left\{\omega_i \in \Omega, i = 1, \ldots, n : \theta \in \tilde{\sqcap}^\alpha(\omega_1, \ldots, \omega_n)\right\}$. Thus, we can write that

$$\begin{aligned} &P_{H_0}\left\{\omega_i \in \Omega, i = 1, \ldots, n : \theta \in \tilde{\sqcap}^\alpha(\omega_1, \ldots, \omega_n)\right\} \\ &= P_{H_0}\left\{\omega_i \in \Omega, i = 1, \ldots, n : \sqcap^{L,\alpha}(\omega_1, \ldots, \omega_n) \leq \theta \leq \sqcap^{R,\alpha}(\omega_1, \ldots, \omega_n)\right\} \\ &\geq 1 - \delta. \end{aligned}$$

Hence,

$$P_{H_0}\left\{\omega_i \in \Omega, i = 1, \ldots, n : \tilde{\phi}^{\text{rej},\alpha}(\tilde{X}_1(\omega_1), \ldots, \tilde{X}_n(\omega_n)) \in \{1\}\right\} \leq \delta,$$

which is true for all α-cuts. Therefore,

$$\sup_{\alpha \in [0,1]} P_{H_0}\left\{\omega_i \in \Omega, i = 1, \ldots, n : \tilde{\phi}^{\text{rej},\alpha}(\tilde{X}_1(\omega_1), \ldots, \tilde{X}_n(\omega_n)) \in \{1\}\right\} \leq \delta,$$

and $\tilde{\phi}^{rej}$ is a fuzzy test. The proof is exactly the same for $\tilde{\phi}^{\text{acc}}$. □

The above constructed fuzzy tests $\tilde{\phi}^{rej}$ and $\tilde{\phi}^{\text{acc}}$ permit us to propose a set of two new decision rules given in the following definition:

Definition 2. (*Decision rules*) Let be

$$L^{\text{rej}} = \int_{[0,1]} \tilde{\phi}^{\text{rej},\alpha} \mathrm{d}\alpha \qquad \text{and} \qquad L^{\text{acc}} = \int_{[0,1]} \tilde{\phi}^{\text{acc},\alpha} \mathrm{d}\alpha. \tag{4}$$

L^{rej} and L^{acc} are called, respectively, the levels of rejection and acceptance (not rejection) of the (fuzzy) null hypothesis $\tilde{H}_0$.

1. If $L^{\text{rej}} = 1$, $\tilde{H}_0$ is rejected. In this case $L^{\text{acc}} = 0$;
2. If $L^{\text{rej}} = 0$, $\tilde{H}_0$ is not rejected. In this case $L^{\text{acc}} = 1$;
3. If $0 < L^{\text{rej}} < 1$, the test shows how strong $\tilde{H}_0$ is rejected. The higher L^{rej} is, the stronger the test rejects $\tilde{H}_0$;
4. If $0 < L^{\text{acc}} < 1$, the test shows how strong $\tilde{H}_0$ is not rejected. The higher L^{acc} is, the stronger the test doesn't reject $\tilde{H}_0$;
5. If $L^{\text{rej}} > L^{\text{acc}}$, $\tilde{H}_0$ tends to be rejected;
6. If $L^{\text{rej}} < L^{\text{acc}}$, $\tilde{H}_0$ tends to be not rejected;
7. If $L^{\text{rej}} = L^{\text{acc}}$, no decision can be taken.

The idea for this fuzzy inference is to construct a fuzzy decision rule based on the two ratios of the area of the considered fuzzy hypothesis inside the rejection domain, noted by $\neg\tilde{\sqcap}$ or/and inside the acceptance domain, given by $\tilde{\sqcap}$, divided by the area of $\tilde{H}_0$ and to compare them.

The fuzzy tests $\tilde{\phi}^{rej}$ and $\tilde{\phi}^{\text{acc}}$ depend on a fuzzy confidence interval $\tilde{\sqcap}$ at a given confidence level $1-\delta$. The question of the construction of such a fuzzy confidence interval becomes a central point of the inference procedure. In Rosset and Donzé [10], we propose constructing a fuzzy confidence interval using bootstrap techniques. Our method is attractive because no specific (fuzzy) distribution should be assumed. Let us recall our method.

Let be $\tilde{t}$ a fuzzy estimator for the fuzzy parameter $\tilde{\theta}$. By using bootstrap techniques, we can estimate k new fuzzy parameters $\tilde{t^*}_1, \ldots, \tilde{t^*}_k$ and build an estimated empirical fuzzy distribution $\hat{\tilde{F}}$ with the $k+1$ estimates ("augmented distribution"). Left and right parts of a fuzzy confidence interval at a given significance level γ for each α-cut are given by:

$$\hat{\theta}^{L,\alpha}_{\gamma/2} = t^{L,\alpha} - ({t^*}^{L,\alpha}_{(k+1)(1-\gamma/2)} - t^{L,\alpha}) = 2t^{L,\alpha} - {t^*}^{L,\alpha}_{(k+1)(1-\gamma/2)} \tag{5}$$

$$\hat{\theta}^{R,\alpha}_{1-\gamma/2} = t^{R,\alpha} - ({t^*}^{R,\alpha}_{(k+1)(\gamma/2)} - t^{R,\alpha}) = 2t^{R,\alpha} - {t^*}^{R,\alpha}_{(k+1)(\gamma/2)}. \tag{6}$$

where, for the left α-cut, ${t^*}^{L,\alpha}_{(k+1)(1-\gamma/2)}$ is the $(1-\gamma/2)$-quantile, and for the right α-cut, ${t^*}^{R,\alpha}_{(k+1)(\gamma/2)}$ is the $\gamma/2$-quantile of the augmented distribution. The bootstrapped fuzzy confidence interval at confidence level $1-\gamma$ is given by $\tilde{\sqcap}^{\alpha} = [\sqcap^{L,\alpha}, \sqcap^{R,\alpha}] = [\hat{\theta}^{L,\alpha}_{\gamma/2}, \hat{\theta}^{R,\alpha}_{1-\gamma/2}]$.

4 Application

We intend to test the mean of a distribution at a 5% significant level. We consider the observations from Berkachy and Donzé [5] given in Table 1. The data is supposed to be fuzzy. Table 1 shows how the data has been fuzzified. The crisp mean is 2.8, and the fuzzy one is $\tilde{\bar{x}} = (1.8, 2.8, 3.8)$. In the first step, we generate a fuzzy confidence interval at a 95% confidence level with the method described above. We obtain $(0.9333, 1.9333, 3.6, 4.6)$. The two fuzzy tests can be computed after postulating a fuzzy null hypothesis. We offer in Table 2 the levels of acceptance L^{acc} and rejection L^{rej} for 13 different null fuzzy hypotheses.

Table 1. Fuzzy observations from Berkachy and Donzé [5]

x_i	$\tilde{x}_i$	x_i	$\tilde{x}_i$	x_i	$\tilde{x}_i$	x_i	$\tilde{x}_i$	x_i	$\tilde{x}_i$
4	$(3, 4, 5)$	3	$(2, 3, 4)$	3	$(2, 3, 4)$	5	$(4, 5, 6)$	3	$(2, 3, 4)$
1	$(0, 1, 2)$	2	$(1, 2, 3)$	2	$(1, 2, 3)$	2	$(1, 2, 3)$	3	$(2, 3, 4)$

Note $\bar{x} = 2.8$, $\tilde{\bar{x}} = (1.8, 2.8, 3.8)$

4.1 Discussion: Acceptance and Rejection Levels

Looking at Table 2, which shows the levels of acceptance and rejection of given fuzzy null hypotheses, we first notice that when the fuzzy null hypothesis is outside the fuzzy confidence interval, its level of rejection is 1 and its level of acceptance is 0 and conversely, when the fuzzy null hypothesis is inside the fuzzy confidence interval, its level of rejection is 0, and its level of acceptance is 1. This result is not surprising and is in accordance with crisp classical tests. Let us now discuss the cases where the null hypothesis overlaps the fuzzy confidence interval. We see that when we consider a fuzzy null hypothesis larger than the fuzzy confidence interval, given by $\tilde{H}_0 = (0, 3.8, 6)$ in Table 2 (Test 11), we find a level of acceptance of 0.68 and a level of rejection of 0.43. These levels describe how much $\tilde{H}_0$ is inside, respectively, outside the fuzzy confidence interval. In this case, we would tend not to reject the fuzzy null hypothesis. However, if we take another larger $\tilde{H}_0 = (0.5, 4.5, 8)$ (Test 12), this time, a more significant portion is out of the fuzzy confidence interval and is inside its complementary, thus telling us we should tend to reject the null hypothesis more often than accepting it. It is pretty natural to find such results. The levels of acceptance and rejection reflect the portion of the null hypothesis contained in the confidence interval, respectively, included in its complementary. Lastly, we can also see that when we use crispier but still fuzzy null hypotheses (Tests 2,3,4,6,8,9,10), the results tend to be more and more binary. This comes from the fact that the thinner $\tilde{H}_0$ is, the easier it is for it to be outside or inside the confidence interval. Notice that when a thin $\tilde{H}_0$ (Tests 5,7) is outside the fuzzy confidence interval's core but still inside its fuzzy left and right bounds, we again get non-binary rejection and acceptance levels. This is because $\tilde{H}_0$ is simultaneously inside the fuzzy confidence interval and its complementary.

Table 2. Fuzzy Hypothesis testing at a 5% significant level

Tests	$\tilde{H}_0$	acceptance	rejection
1	(−0.3, −0.2, −0.1)	0	1
2	(0, 0.5, 0.8)	0	1
3	(0.8, 0.85, 0.9)	0	1
4	(0.95, 1, 1.05)	0.12	0.99
5	(1, 1.5, 2)	0.75	0.58
6	(1.8, 1.9,2)	0.99	0.08
7	(1.7, 1.75, 1.78)	0.96	0.34
8	(2, 2.5, 3.5)	1	0
9	(2.4, 2.45, 2.5)	1	0
10	(2.6, 2.8, 3.2)	1	0
11	(0, 3.8, 6)	0.68	0.43
12	(0.5, 4.5, 8)	0.44	0.62
13	(7, 8, 9)	0	1

This Table shows the levels of acceptance and rejection of different fuzzy null hypotheses. The fuzzy trapezoidal number $(0.933, 1.933, 3.6, 4.6)$ is a fuzzy confidence interval at a 95% confidence level for the distribution's mean

5 Conclusion

Our work introduced a fuzzy statistical testing procedure based on precedent works. Our new method revolves around constructing a fuzzy confidence interval for the parameter of interest and comparing it to a fuzzy null hypothesis. More precisely, we compare the ratios of how much of the null hypothesis is contained respectively in the confidence interval or its complementary. We then illustrated our testing procedure through a numerical application and discussed the results. The crisp nature of the results of our testing method, namely the levels of acceptance and rejection, allows one to interpret them easily. Moreover, the underlying simplicity of the method is of great advantage. Indeed, it is relatively easy to implement and is not intensive computation-wise. Furthermore, its general aspect should also be mentioned. Indeed, to perform statistical testing, one just calculates a fuzzy confidence interval for one's parameter of interest to perform statistical testing. Lastly, let us again mention that it generalises the classical testing approach. Indeed, the crispier the null hypothesis is, the more binary the results are. In the future, for further research, we will be interested in investigating the behaviour of our testing procedure for different parameters.

References

1. Berkachy, R.: The Signed Distance Measure in Fuzzy Statistical Analysis. Some theoretical, empirical and programming advances. PhD thesis University of Fribourg (2020). https://folia.unifr.ch/unifr/documents/308986
2. Berkachy, R., Donzé, L.: Testing fuzzy hypotheses with fuzzy data and defuzzification of the fuzzy p-value by the signed distance method. In: Proceedings of the 9th International Joint Conference on Computational Intelligence, pp. 255–264 (2017). https://doi.org/10.5220/0006500602550264
3. Berkachy, R., Donzé, L.: Fuzzy confidence interval estimation by likelihood ratio. In: Proceedings of the 11th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT 2019). Atlantis Studies in Uncertainty Modelling Vol. 1., pp. 150–157. Atlantis Press (2019a). https://doi.org/10.2991/eusflat-19.2019.23
4. Berkachy, R., Donzé, L.: Testing hypotheses by fuzzy methods: a comparison with the classical approach. In: Meier, A., Portmann, E., Terán, L. (Eds.) Applying Fuzzy Logic for the Digital Economy and Society, pp. 1–22. Fuzzy Management Methods. Springer, Cham (2019b). https://doi.org/10.1007/978-3-030-03368-2_1
5. Berkachy, R., Donzé, L.: Fuzzy Confidence Intervals by the Likelihood Ratio with Bootstrapped Distribution. In: Proceedings of the 12th International Joint Conference on Computational Intelligence (IJCCI 2020) - Volume 1, , pp. 231–242. SciTePress (2020). https://doi.org/10.5220/0010023602310242
6. Chachi, J., Taheri, S.: Fuzzy confidence intervals for mean of Gaussian fuzzy random variables. Expert Syst. Appl. 38(5), 5240–5244 (2011). https://doi.org/10.1016/j.eswa.2010.10.034
7. Kahraman, C., Oztaysi, B., Onar, S.C. Interval-Valued Intuitionistic Fuzzy Confidence Intervals. J. Intell. Syst. **28**(2), 307–319 (2019). https://doi.org/10.1515/jisys-2017-0139
8. Kruse, R., Meyer, K.D.: Statistics with Vague Data. Theory and Decision Library, Series B: Mathematical and Statistical Methods. D. Reidel Publishing Company (1987). https://doi.org/10.1007/978-94-009-3943-1
9. Rosset, J., Donzé, L.: Fuzzy least squares and fuzzy orthogonal least squares linear regressions. In: Proceedings of the 15th International Joint Conference on Computational Intelligence - FCTA, pp. 359–368. SciTePress (2023). https://doi.org/10.5220/0012182700003595
10. Rosset, J., Donzé, L.: Fuzzy confidence intervals by bootstrapped fuzzy distributions. In: Proceedings of the 20th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU 2024, Lisboa, July 22–26, 2024) (2024)
11. Viertl, R.: Statistical Methods for Fuzzy Data. John Wiley & Sons (2011). https://doi.org/10.1002/9780470974414