

MLE Binomial: MSE

The variance, bias and MSE of the $\hat{\theta}_{mle}$ estimator are respectively

$$\begin{aligned} var(\hat{\theta}_{mle}) &= var\left(\sum_{i=1}^n \frac{x_i}{n}\right) = \frac{1}{n^2} var\left(\sum_{i=1}^n x_i\right) = \frac{1}{n^2} \sum_{i=1}^n var(x_i) \\ &= \frac{1}{n^2} n\theta(1-\theta) = \frac{\theta(1-\theta)}{n} \end{aligned}$$

$$bias(\hat{\theta}_{mle}) = E\left[\hat{\theta}_{mle} - \theta\right] = E\frac{1}{n}\left[\sum_{i=1}^n x_i\right] - \theta = \theta - \theta = 0$$

$$MSE(\hat{\theta}_{mle}) = var(\hat{\theta}_{mle}) + bias(\hat{\theta}_{mle})^2 = \frac{\theta(1-\theta)}{n} + 0^2 = \frac{\theta(1-\theta)}{n}$$

Bayes estimator Binomial: MSE

The variance, bias and MSE of the $\hat{\theta}_{bayes}$ estimator assuming a $Beta(\alpha = 1, \beta = 1)$ prior on θ are respectively

$$var(\hat{\theta}_{bayes}) = \frac{(n\theta(1 - \theta))}{(n + 2)^2}$$

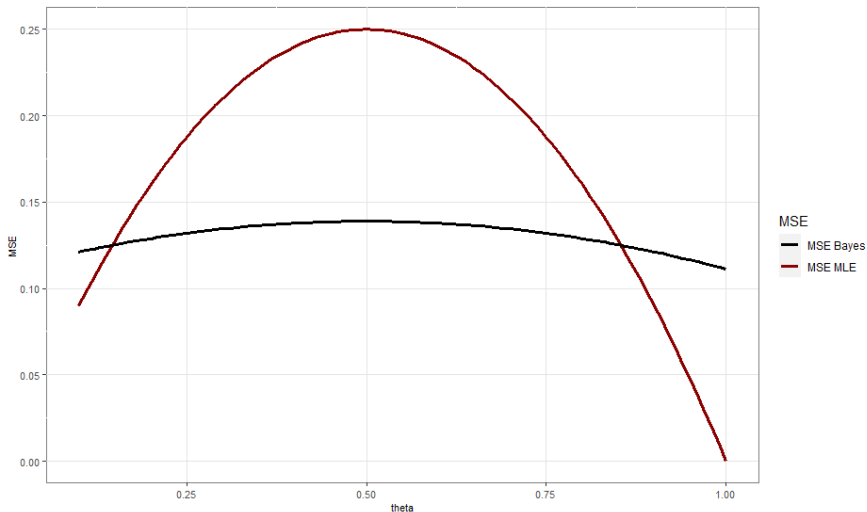
$$bias(\hat{\theta}_{bayes}) = E\left[\hat{\theta}_{bayes} - \theta\right] = \frac{1 + nE[x_i]}{(n + 2)} - \theta = \frac{1 - 2\theta}{(n + 2)}$$

$$MSE(\hat{\theta}_{bayes}) = var(\hat{\theta}_{bayes}) + bias(\hat{\theta}_{bayes})^2 = \frac{1 + n\theta - n\theta^2 + n^2 + 2\theta}{(n + 2)^2}$$

MSE comparison in R

Comparison of MSE accross all values of theta MLE and theta Bayes

for $n = 1$



MSE comparison in Python

