MLE Binomial: MSE

The variance, bias and MSE of the θ_{mle} estimator are repsectively

$$var(\hat{\theta}_{mle}) = var\left(\sum_{i=1}^{n} \frac{x_i}{n}\right) = \frac{1}{n^2} var(\sum_{i=1}^{n} x_i) = \frac{1}{n^2} \sum_{i=1}^{n} var(x_i)$$
$$= \frac{1}{n^2} n\theta(1-\theta) = \frac{\theta(1-\theta)}{n}$$

$$bias(\theta_{mle}) = E\left[\hat{\theta}_{mle} - \theta\right] = E\frac{1}{n}\left[\sum_{i=1}^{n} x_i\right] - \theta = \theta - \theta = 0$$

$$MSE(\theta_{mle}) = var(\hat{\theta}_{mle}) + bias(\theta_{mle})^2 = \frac{\theta(1-\theta)}{n} + 0^2 = \frac{\theta(1-\theta)}{n}$$

Bayes estimator Binomial: MSE

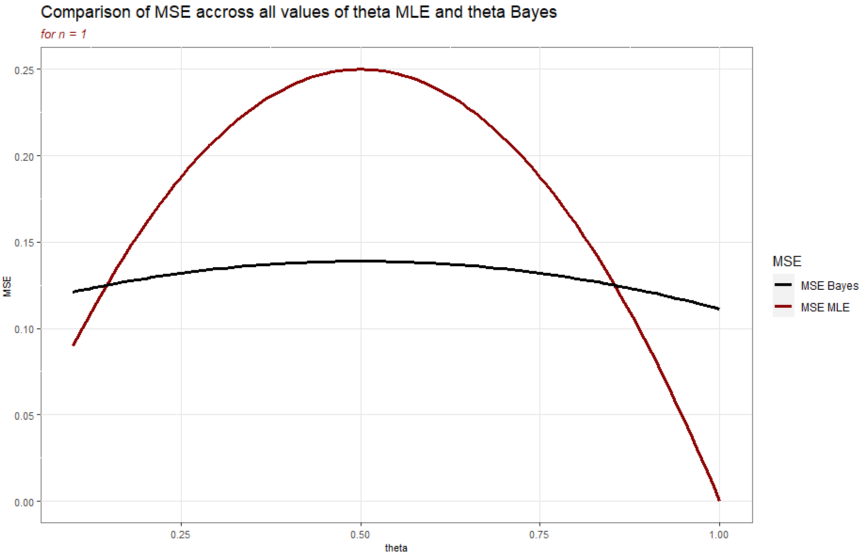
The variance, bias and MSE of the θ_{bayes} estimator assuming a $Beta(\alpha = 1, \beta = 1)$ prior on θ are repsectively

$$var(\hat{\theta}_{bayes}) = \frac{(n\theta(1-\theta))}{(n+2)^2}$$

$$bias(\theta_{bayes}) = E\left[\hat{\theta}_{bayes} - \theta\right] = \frac{1 + nE[x_i]}{(n+2)} - \theta = \frac{1 - 2\theta}{(n+2)}$$

$$MSE(\theta_{mle}) = var(\hat{\theta}_{mle}) + bias(\theta_{mle})^2 = \frac{1 + n\theta - n\theta^2 + n^2 + 2\theta}{(n+2)^2}$$

MSE comparison in R



MSE comparison in Python