Survival Analysis Of Amoebae In a Nutrient-Poor Environment

Julian Sampedro

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Overview

(*i*) We will study from a survival analysis perspective the survival time of two particular amoebae in a nutrient-poor environment.

(*ii*) The methods used are Kaplan-Meier analysis, Cox regression as well as bayesian acceptance-rejection methods. We will perform our analyses in R programming.

(*iii*) Conclusions based on methods as well as domain specific conclusions are exposed in the last section of this presentation.

Survival times

Suppose that X are the survival time of two amoebae in a nutrient-poor environment and that $X_1 \sim Exp(\theta)$, with PDF and CDF given respectively by:

$$f_\theta(x) = \theta e^{-\theta x} \qquad F_{-\theta}(x) = 1 - e^{-\theta x}$$

In addition, suppose that we observed the following survival time:

Weeks	1	2	3	4	5	6	7	8	9
Amoeba alive	15	13	10	8	6	4	2	1	0
Amoeba dead	0	2	3	2	2	2	2	1	1

Bayesian Modeling

The probability of surviving 'one more week' is $e^{-\theta}$ and the probability of dying is $1 - e^{-\theta}$. So, the likelihood is given by:

$$p(x \mid \theta) \propto exp(-\theta)^{\sum_{i=1}^{alive}} \left(1 - exp(-\theta)\right)^{\sum_{i=1}^{dead}}$$

$$p(x \mid \theta) \propto exp(-44\theta) \left(1 - exp(-\theta)\right)^{15}$$

Since the Jeffrey's prior is $p(\theta) = 1/\theta$, we concludde that the posterior is given by:

$$p(\theta \mid x) \propto p(x \mid \theta)p(\theta) \propto \frac{1}{\theta} exp(-44\theta) \left(1 - exp(-\theta)\right)^{15}$$

Acceptance-Rejection Method

Since we are interested in the expected survival time (EST) $E[X] = 1/\theta$, we will use Monte Carlo integration to compute:

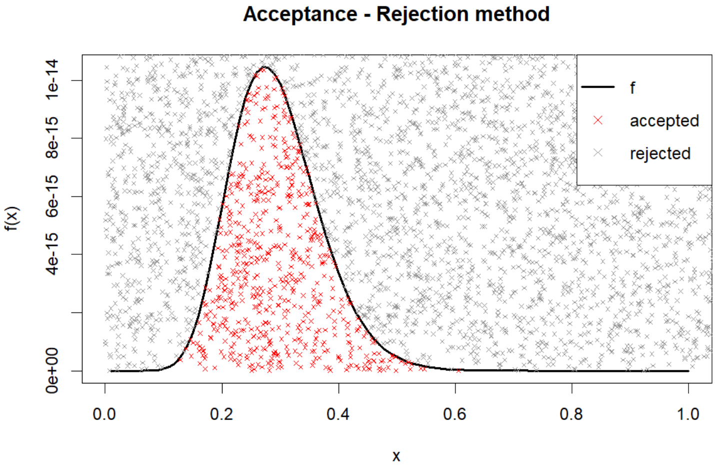
$$EST = \int_{\mathbb{R}+} \frac{1}{\theta} p(\theta \mid x) d\theta$$

To sample from the posterior $p(\theta \mid x)$, we proceed as follows:

- **Generate a proposal value:** Draw x (the abscissa) from an exponential distribution $x \sim \mathrm{Exp}(\theta = 1)$.
- **Generate a uniform ordinate:** Draw y uniformly on the interval $y \sim \mathrm{Unif}\big(0,\, c\, g(x)\big)$, where $c = 5 \times 10^{-13}$ and the proposal density is $g(x) = e^{-x}$.
- **Acceptance step:** Accept the proposed value x if $y \leq f(x)$, otherwise reject it and repeat the procedure. The target (posterior) is given by $f(x) = \frac{1}{x}\, e^{-44x}\big(1 - e^{-x}\big)^{15}$.

Using $n = 150,000$ simulations, the Acceptance-Rejection approximation yields $EST \approx 3.632$

Acceptance-Rejection area



Notes: *In red, the points in the plane that fall under the likelihood of the function that we try to approximate.*

Kaplan-Meier Estimator

The Kaplan-Meier estimator is given by the following formula:

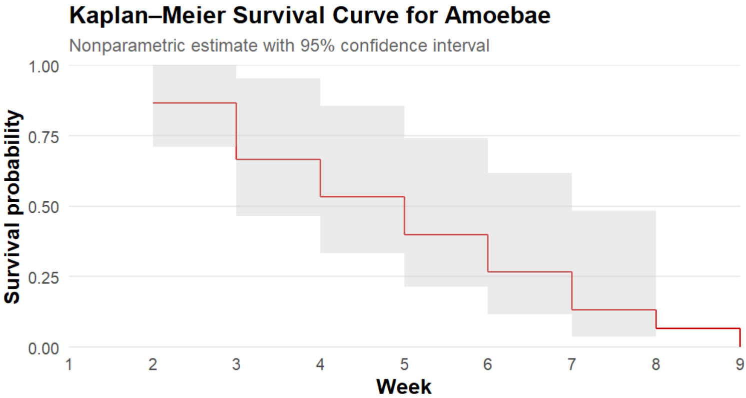
$$\hat{S}(t) = \prod_{t_i \leq t} (1 - \hat{q}_i)$$

where $\hat{S}(t)$ denotes the survival function at time t and $\hat{q}_i = \frac{d_i}{n_i}$ is the estimated probability of death at time t_i. The expected survival time can be approximated by the area under the Kaplan-Meier curve:

$$EST = \hat{E}[T] \approx \sum_{i=1}^{k} \left(t_i - t_{i-1}\right) \hat{S}(t_{i-1})$$

where $t_0 = 0$, $t_1, \ldots, t_k$ are the distinct observed event times, $\hat{S}(t_{i-1})$ is the Kaplan–Meier survival probability just before time t_i and $(t_i - t_{i-1})$ is the width of each step in the survival curve. In our example, $EST = 3.8$.

Kaplan-Meier Survival Curve



Discussion of the results

(*i*) The estimated expected survival time provides a biologically interpretable summary of amoeba viability and can inform experimental design in microbial survival studies.

(*ii*) The observed survival pattern suggests a steadily increasing mortality rate over time, consistent with biological stress accumulation in a nutrient-poor environment.

(*iii*) Bayesian inference with a Jeffreys prior combined with an acceptance–rejection Monte Carlo scheme allows estimation of the expected survival time when closed-form solutions are unavailable. The Expected Survival Time (EST) is about 3.62.

(*iv*) Nonparametric (Kaplan–Meier) and semi-parametric (Cox) survival models provide consistent estimates of amoeba survival in small samples, serving as robust benchmarks for parametric and Bayesian approaches. The Expected Survival Time (EST) is about 3.8.

References

The R Project for Statistical Computing