Smoothing Splines: introduction

Smooting splines is a type of nonparametric regression in which we fit a smooth curve to our set of data points. This techniques finds a balance between smoothness of the curve and a perfect fit in the extreme case for which we would have (almost) as many piecewise polynomials as observations. Suppose that we observe a set of n pairs $(x_i, y_i), i = 1, ..., n$. The relationship is of the form

$$y_i = f(x_i) + \epsilon_i$$

where $f(.)$ is some 'smooth' function that we can estimate by

$$\hat{f} = \frac{1}{n} \sum_{i=1}^{n} (y_i - f(x_i))^2 + \lambda \int [f''(x)]^2 dx$$

It is some kind of penalized least squares regression problem where λ is a 'smoothness' parameter and $\int [f''(x)]^2 dx$ is a 'roughness' penalty.

'mcycle ' dataset

mcycle: A data frame of 133 observations x 2 variables, giving a series of measurements of head acceleration in a simulated motorcycle accident, used to test crash helmets

times: in milliseconds after impact
accel: in g.

```
> head(mcycle)
  times accel
1   2.4   0.0
2   2.6  -1.3
3   3.2  -2.7
4   3.6   0.0
5   4.0  -2.7
6   6.2  -2.7
```

Source: Silverman, B. W. (1985) Some aspects of the spline smoothing approach to non-parametric curve fitting. Journal of the Royal Statistical Society series B 47, 1–52.

Reference: Venables, W. N. and Ripley, B. D. (2002) Modern Applied Statistics with S-PLUS. Fourth Edition. Springer

Summary

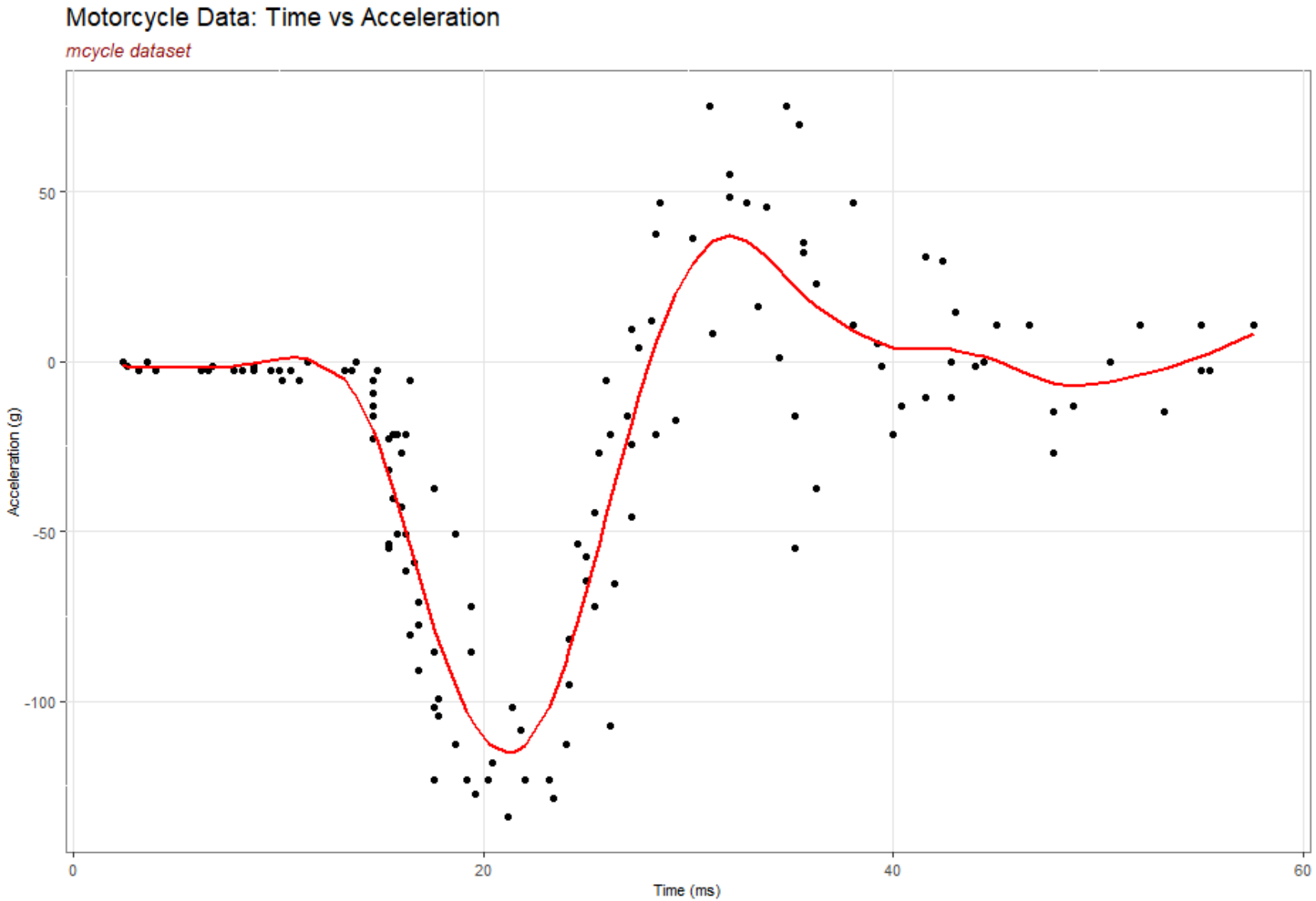
Here is the minimal code to run Smooting Splines regression in R.

```
> # fit smoothing splines model (ss) with default number of knots
> modss = with(mcycle, ss(times, accel))
> summary(modss)
..
Residuals:
     Min       1Q   Median       3Q      Max
-76.7951 -12.5654  -0.8346  12.5823  50.5576

Approx. Signif. of Parametric Effects:
            Estimate Std. Error t value  Pr(>|t|)
(Intercept)  -14.234      2.313  -6.154 1.018e-08 ***
x              9.549     21.603   0.442 6.593e-01
..
Approx. Signif. of Nonparametric Effects:
               Df Sum Sq Mean Sq F value Pr(>F)
s(x)        10.21 210130 20585.3   40.08      0 ***
Residuals 120.79  62035   513.6
..

> # using smooth.spline function, then plotting
> mod1ss = with(mcycle, smooth.spline(times, accel))
> fit = data.frame(times = mod1ss$x, accel = mod1ss$y)
> head(fit)
  times     accel
1   2.4 -1.373830
2   2.6 -1.435130
```

Plot of the fitted model



Main observations

- The smoothing spline reveals a sharp increase in acceleration after 20 milliseconds, indicating a rapid change in speed.
- Following the initial spike, the acceleration decreases and shows a fluctuating pattern over time, suggesting variable speed changes during the motorcycle ride.
- The fitted spline captures several local maxima and minima, highlighting the periods of acceleration and deceleration throughout the observed time span.
- Towards the latter part of the time series, the smoothing spline indicates a gradual stabilization of acceleration, suggesting that the motorcycle's speed changes become less extreme.

References

Faraway, J. J., Extending the Linear Model (2006), ISBN 0-203-62105-0 (e-book)

Venables, W. N. and Ripley, B. D. (2002) Modern Applied Statistics with S-PLUS. Fourth Edition. Springer

The R Project for Statistical Computing:
https://www.r-project.org/