# Stationarity time series: rationale

**(weakly) stationary process**: Inuitively, we have a stationary process if its mean is finite and constant and the autocovariance function depends only on the lag, and not on the time $t$.

For a discrete or continuous **stationary** time series $\{X_t\}$, the mean and variance functions are respectively defined as:

$$\mu_t = E[X_t] = \mu$$

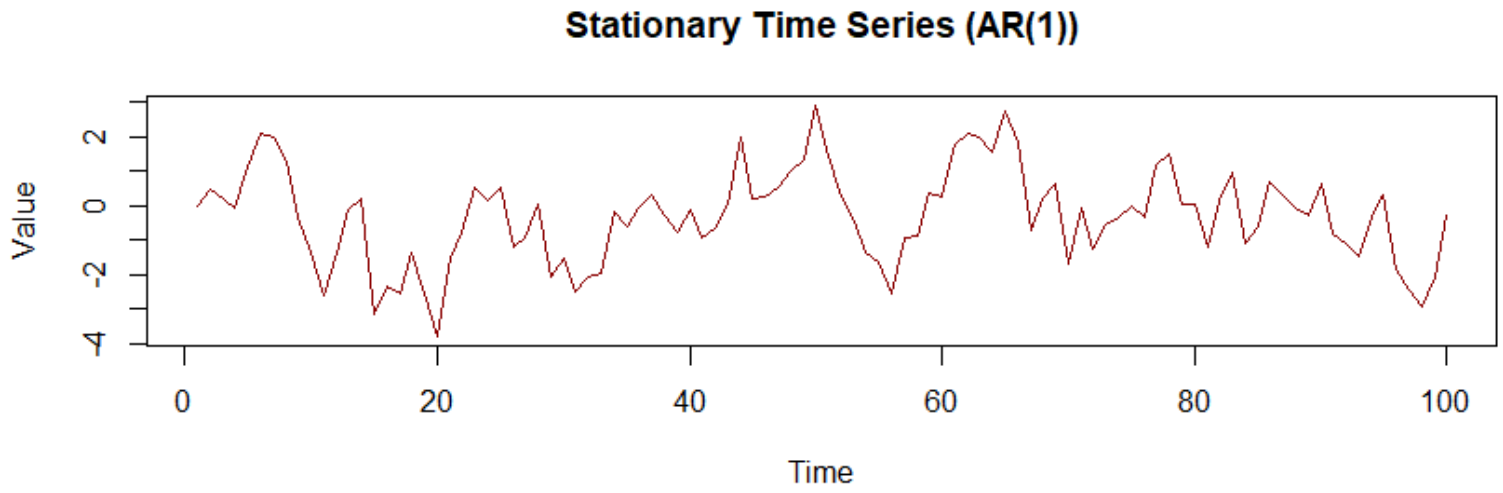
$$\sigma_t^2 = var(X_t) = E[(X_t - \mu)^2] = \sigma^2$$

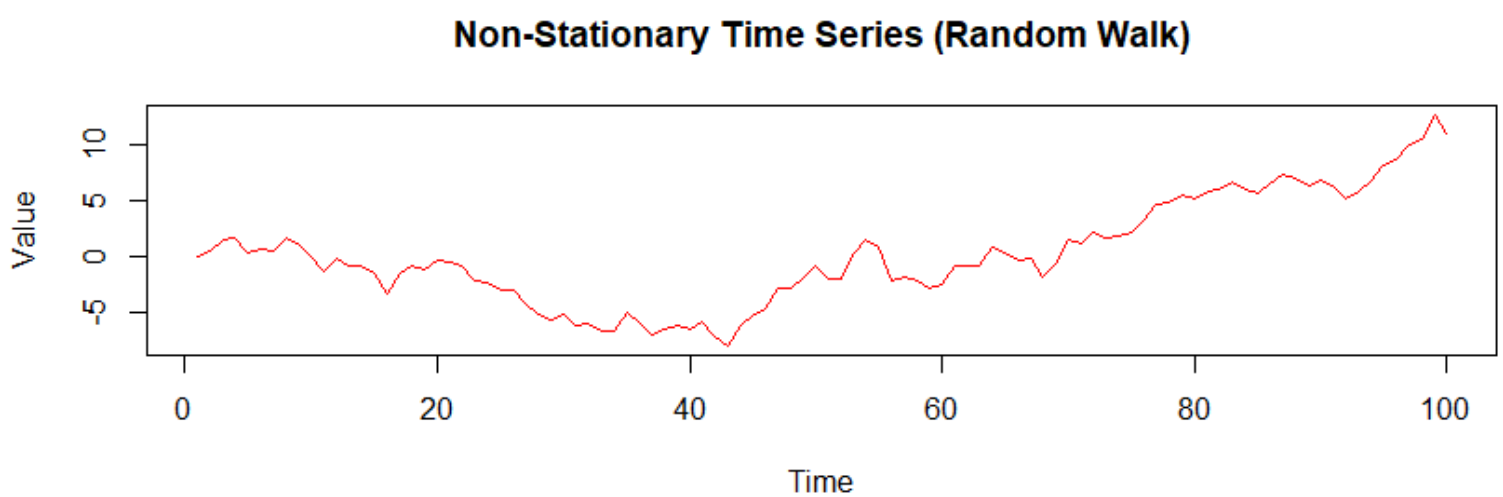
An example of a stationary series is an AR(1) process. An example of a non-stationary series is a Random Walk (RW). We consider those two to illustrate two test for stationary series, namely the augmented Dicky-Fuller test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test.

# R code to generate simple time series

```r
# simulate two times series (first = stationary, second = non stationary)
set.seed(2024)

# parameters
n = 100; phi = 0.7; sigma = 1 # number of observation, AR(1) parameter value and
      s.d. of WN

# stationary time series: AR(1)
t1 = numeric(n)
epsilon = rnorm(n, mean = 0, sd = sigma)  # White noise

for (t in 2:n) {
  t1[t] = phi * t1[t - 1] + epsilon[t]
}

# non-stationary time series: A random walk
t2 = numeric(n)
epsilon = rnorm(n, mean = 0, sd = sigma)  # White noise

for (t in 2:n) {
  t2[t] = t2[t - 1] + epsilon[t]
}

# plots
par(mfrow = c(2, 1))
plot(t1, type = "l", col = "darkred", main = "Stationary Time Series (AR(1))",
     ylab = "Value", xlab = "Time")
plot(t2, type = "l", col = "red", main = "Non-Stationary Time Series (Random
     Walk)",
     ylab = "Value", xlab = "Time")
```

# Plot of the two time series



# Augmented Dickey-Fuller tests in R

```
> # augmented Dickey-Fuller test
> library(tseries)
> adf.test(t1) # p-value: 0.0321; Ho: the series is non-stationary, is rejected

 Augmented Dickey-Fuller Test

data:  t1
Dickey-Fuller = -3.6518, Lag order = 4, p-value = 0.0321
alternative hypothesis: stationary

> adf.test(t2) # p-value: 0.8317; Ho: the series is non-stationary, is not
      rejected

 Augmented Dickey-Fuller Test

data:  t2
Dickey-Fuller = -1.3844, Lag order = 4, p-value = 0.8317
alternative hypothesis: stationary
```

# KPSS tests in R

```
> # KPSS test
> kpss.test(t1, null="Trend") # p-value: 0.0764; Ho: the series is trend
      stationary, is not rejected

 KPSS Test for Trend Stationarity

data:  t1
KPSS Trend = 0.13174, Truncation lag parameter = 4, p-value = 0.0764

> kpss.test(t2, null="Trend") # p-value: 0.01; Ho: the series is trend
      stationary, is rejected

 KPSS Test for Trend Stationarity

data:  t2
KPSS Trend = 0.45554, Truncation lag parameter = 4, p-value = 0.01

Warning message:
In kpss.test(t2, null = "Trend") : p-value smaller than printed p-value
```

# References

The R Project for Statistical Computing:
https://www.r-project.org/