Stationarity time series: rationale

(weakly) stationary process: Inuitively, we have a stationary process if its mean is finite and constant and the autocovariance function depends only on the lag, and not on the time t.

For a discrete or continuous **stationary** time series $\{X_t\}$, the mean and variance functions are respectively defined as:

$$\mu_t = E[X_t] = \mu$$

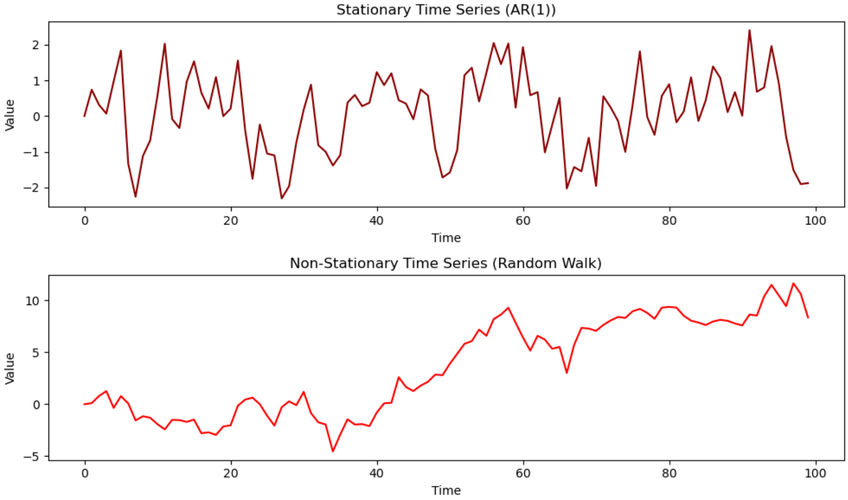
$$\sigma_t^2 = var(X_t) = E[(X_t - \mu)^2] = \sigma^2$$

An example of a stationary series is an AR(1) process. An example of a non-stationary series is a Random Walk (RW). We consider those two to illustrate two test for stationary series, namely the augmented Dicky-Fuller test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test.

Python code to generate simple time series

```
# simulate two times series (first = stationary, second = non stationary)
import numpy as np
import matplotlib.pyplot as plt

np.random.seed(2024)

# parameters
n = 100                 # Number of time points
phi = 0.7               # AR(1) coefficient for stationary series
sigma = 1               # Standard deviation of noise

# stationary time series: AR(1)
t1 = np.zeros(n)
epsilon = np.random.normal(0, sigma, n)  # White noise
for t in range(1, n):
    t1[t] = phi * t1[t - 1] + epsilon[t]

# Non-stationary time series: Random walk
t2 = np.zeros(n)
epsilon = np.random.normal(0, sigma, n)  # White noise
for t in range(1, n):
    t2[t] = t2[t - 1] + epsilon[t]
```

Plot of the two time series



Augmented Dickey-Fuller tests in Python

```
import numpy as np
import statsmodels.api as sm
from statsmodels.tsa.stattools import adfuller, kpss

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# Augmented Dickey-Fuller (ADF) Test
adf_result = adfuller(t1)
print("Augmented Dickey-Fuller Test:")
print(f"ADF Statistic: {adf_result[0]}")
print(f"p-value: {adf_result[1]}")

adf_result = adfuller(t2)
print("Augmented Dickey-Fuller Test:")
print(f"ADF Statistic: {adf_result[0]}")
print(f"p-value: {adf_result[1]}")

Augmented Dickey-Fuller Test:
ADF Statistic: -5.416705280883456
p-value: 3.1240289436797504e-06

Augmented Dickey-Fuller Test:
ADF Statistic: -1.0242832455279713
p-value: 0.7443048416671401
```

KPSS tests in Python

```
# KPSS Test
kpss_result = kpss(t1, regression='c')  # 'c' for constant, 'ct' for trend
print("\nKPSS Test:")
print(f"KPSS Statistic: {kpss_result[0]}")
print(f"p-value: {kpss_result[1]}")

kpss_result = kpss(t2, regression='c')  # 'c' for constant, 'ct' for trend
print("\nKPSS Test:")
print(f"KPSS Statistic: {kpss_result[0]}")
print(f"p-value: {kpss_result[1]}")

KPSS Test:
KPSS Statistic: 0.06590290927945074
p-value: 0.1

KPSS Test:
KPSS Statistic: 1.553286466403289 5
p-value: 0.01
```

Interpretation and References

For the Dickey-Fuller test:

AR(1) process: the p-value: 3.1240289436797504e-06; $H0$: the series is non-stationary, is rejected
RW process: the p-value: 0.7443048416671401; $H0$: the series is non-stationary, is not rejected

For the KPSS test:

AR(1) process: the p-value: 0.1; $H0$: the series is non-stationary, is not rejected
RW process: the p-value: 0.01; $H0$: the series is non-stationary, is rejected

Python:
https://www.python.org/