Java code: tower of HaNoi

```
// 1. Implement a recursive solution to the mathematical game known as Tower of Hanoi.

public class ToH {
   // # (1.)
    public static void main(String[] args) {
              Hanoi (1,"A","B","C");
         }
         //move n disks from position "from" to "to" via "other"

         private static void Hanoi(int n, String left , String middle, String right)
           {
              if (n < 0)

            // define exception
               throw new IllegalArgumentException("n must be a nonnegative integer.");
            if (n == 0)
                return;

            // base case
            if (n > 0)
            Hanoi(n-1, left, right, middle);

            // moves n-1 disks from left(A) to middle (B)
            System.out.printf("Move one disk from rod %s to rod %s \n ", left, right);

            // display the progress
            Hanoi(n-1, middle, left, right);
            // moves n-1 disks from middle(B) to right(C)
          }
}
```

How does the total number of disk movements relate to n?

The total (minimal) number of movements is $(2^n) - 1$. That is $(2^1) - 1 = 1$ moves, if $n = 1$, $(2^2) - 1 = 3$ moves, if $n = 2$, $(2^3) - 1 = 7$ moves if $n = 3$, $(2^4) - 1 = 15$ moves, if $n = 4, (2^5) - 1 = 31$, moves if $n = 5$ etc. It can be proved by induction, i.e $P(1) = 1$ move (from A to C, trivial) For $P(n + 1)$: if our formula is true, we should obtain $(2^n + 1) - 1$. Demonstration: to move the top n disks from rod A to B requires $(2^n) - 1$ moves. Then we have to move the largest disk from A to C : 1 additional move. Then move the n disks from B to C: again $(2^n) - 1$ moves. So for $n + 1$ disks, we require $(2^n) - 1 + 1 + (2^n) - 1 = 2((2^n) - 1) + 1$ (recursion: $2 * f_n - 1 + 1) = (2^n + 1) - 2 + 1 = (2^n + 1) - 1$.

By induction principle the initial formula $P(n) = (2^n) - 1$ is true.

Example of Tower of Hanoi

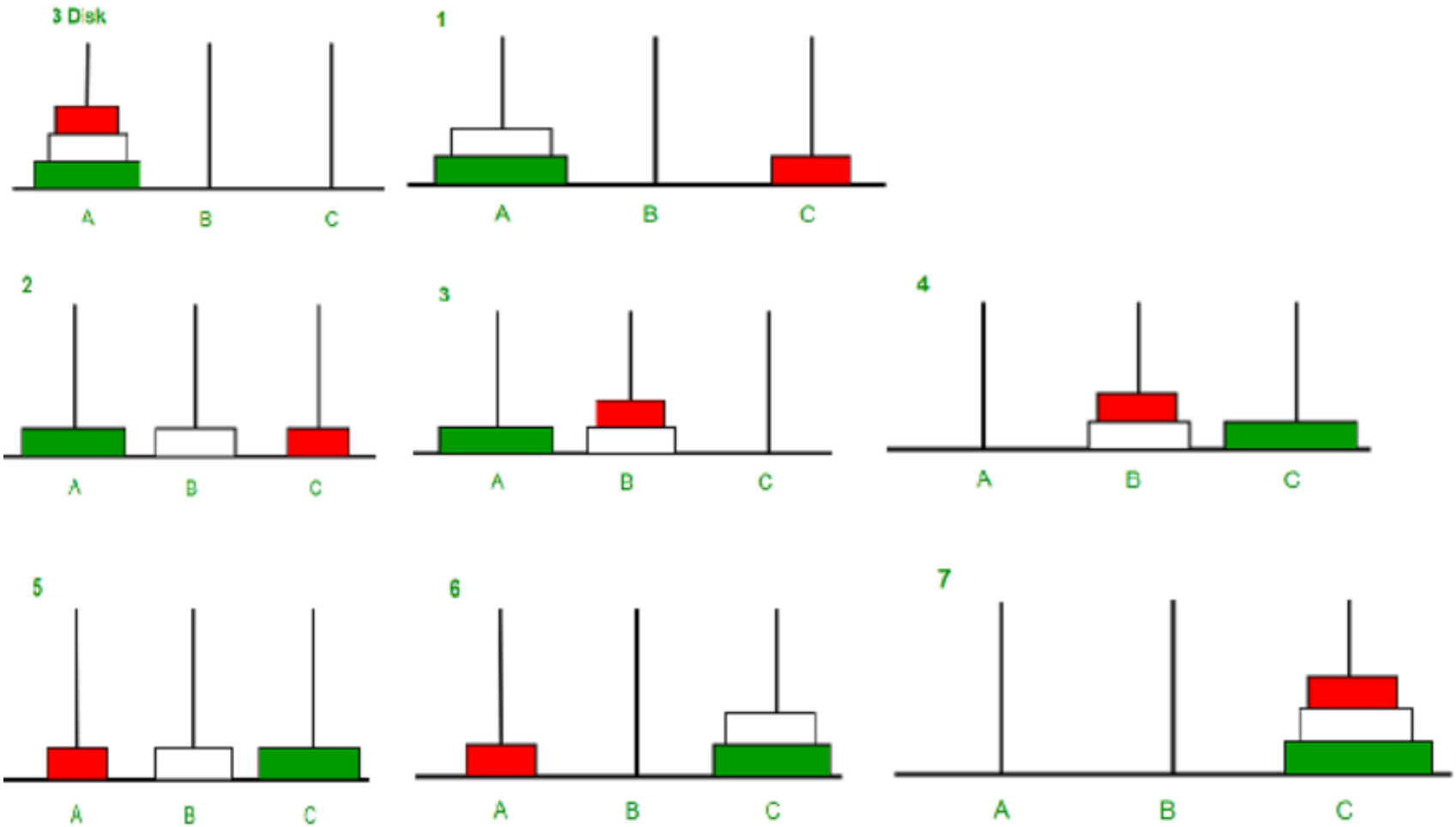

Image illustration for 3 disks

Sources

Java code: own implementation

Illustration:
https://www.geeksforgeeks.org/c-program-for-tower-of-hanoi/