

$$\left\{ \begin{array}{l} P(A|B) := \frac{P(A \cap B)}{P(B)} \\ P(B|A) = \frac{P(A \cap B)}{P(A)} \end{array} \right\}$$

Conditional probability

$$\left\{ \begin{array}{l} P(A \cap B) = P(B|A) \times P(A) \\ P(A \cap B) = P(A|B) \times P(B) \end{array} \right\} \rightarrow \textcircled{1}$$

$$P(B|A) \times P(A) = P(A|B) \times P(B)$$

$$P(B|A) = \frac{P(A|B) \times P(B)}{P(A)} \Rightarrow \text{Bayes Theorem}$$

$$\left\{ \begin{array}{l} \text{Sample Space} = \left[\{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \} \right] \\ A = \{ 1, 3, 8, 7, 2 \} \Rightarrow \text{length} = 5 \\ B = \{ 1, 3, 9, 10 \} \\ [A \cap B = \{ 1, 3 \} \Rightarrow 2] \\ \text{not } B \text{ or } B' = \{ \underbrace{2, 4, 5, 6, 7, 8} \} \Rightarrow 6 \\ A \cap (\text{not } B) \\ A \cap (B') = \{ 2, 7, 8 \} \Rightarrow 3 \\ N(A) = N(A \cap B) + N(A \cap B') \end{array} \right\}$$

$$\left\{ \begin{array}{l} P(A) = P(A \cap B) + P(\text{not } B) \\ P(B) = P(B \cap A) + P(\text{not } A) \end{array} \right.$$

$$\left\{ P(A | B) = \frac{P(A \cap B)}{P(B)} \right\} \text{ Conditional probability}$$

$$\left\{ \begin{array}{l} P(B|A) = \frac{P(A \cap B)}{P(A)} \\ \text{Conditional probability} \end{array} \right.$$

$$P(A \cap B) = \underbrace{\{[P(B|A) \times P(A)]\}}_{\rightarrow 1} \rightarrow 1$$

$$P(A \cap B) = \underbrace{P(A|B) \times P(B)}_{\rightarrow 2} \rightarrow 2$$

$$P(A) = \underbrace{P(A \cap B)} + \underbrace{P(A \cap B')} \quad \downarrow$$

$$P(A) = \underbrace{P(A|B) \times P(B)} + \underbrace{P(A|B') \times P(B')} \quad \downarrow$$

$$P(A \cap B) = \underbrace{P(A|B) \times P(B)}_1$$

$$P(A \cap B') = P(A|B') \times P(B')$$

$$P(A) = \underbrace{P(A|B) \times P(B)} + \underbrace{P(A|B') \times P(B')} \quad \downarrow$$

[Law of total probability]

$$P(B|A) = \frac{P(A|B) \times P(B)}{P(A)}$$

$$P(B|A) = \frac{P(A|B) \times P(B)}{\text{_____}}$$

$$\underbrace{P(A|B) \times P(B) + P(A|B') \times P(B')}_{\rightarrow P(A) \text{ by law of total probability}}$$

{ Extension of Bayes Theorem }

Example problem

10 October 2025 14:06

[A certain disease affects 1% of the population.]

There is a test for the disease with the following properties:

- If a person has the disease, the test is positive 99% of the time (true positive).
- If a person does not have the disease, the test is positive 5% of the time (false positive).

[If a person's test comes back positive, what is the probability that they actually have the disease?]

$$P(D) = \frac{1}{100} \quad \checkmark$$

$$P(ND) = \frac{99}{100} \quad \checkmark$$

$$\{ P(Pos|D) = \frac{99}{100} \quad \checkmark \}$$

$$P(Pos|ND) = \frac{5}{100} \quad \checkmark$$

$$\left[P(D | pos) \right]$$

$$\frac{P(B|A) = P(A|B) \times P(B)}{P(A|B) \times P(B) + P(A|B') \times P(B')}$$

$$\left[P(D|pos) = \frac{\frac{99}{100} \times \frac{1}{100}}{\frac{99}{100} \times \frac{1}{100} + \frac{5}{100} \times \frac{99}{100}} \right]$$

Classical vs Bayesian Probability

10 October 2025 14:19

(India Matches)

Example Dataset — "Cricket Match Outcome Predictor"

Imagine this simple dataset of past matches:

Match	Weather	Pitch	Sachin 50+	Result
1	Sunny	Flat	Yes	Win
2	Cloudy	Flat	No	Lose
3	Sunny	Grassy	No	Win
4	Rainy	Grassy	No	Lose
5	Sunny	Flat	Yes	Win
6	Cloudy	Flat	Yes	Win
7	Sunny	Grassy	No	Lose

$\Rightarrow P(W) = 4/7$
 $P(L) = 3/7 \Rightarrow 43\%$
 $\frac{1}{7} = 14.3\%$

Classical probability

Before 1800s

{Bayes} \Rightarrow Conditional probability $P(\text{Win} | \text{Sunny})$
 can
 be combined
 a few probability theories $P(\text{Win} | \text{Flat})$

We can find out probabilities based on
 conditions

{Bayesian probability} {Machine Learning} $\xrightarrow{\text{Predict}}$ {Predict} Probabilities

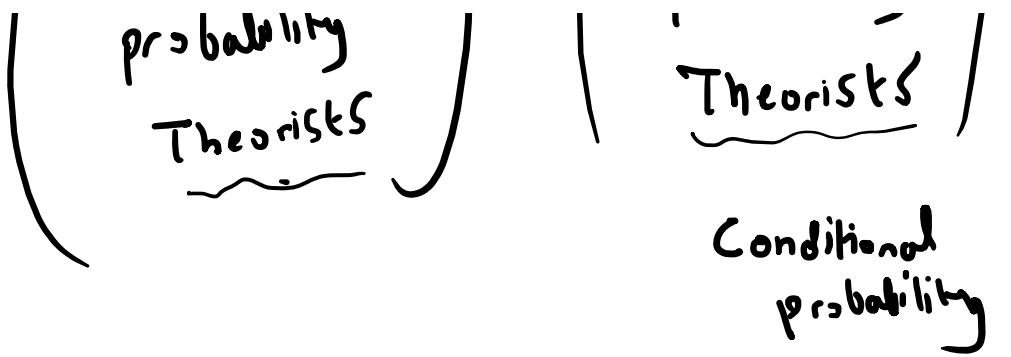
$P(\text{Win} | \text{Sunny, Flat})$

1850 - 1900

Classical probability

Bayesian probability
Theorists

ML Engineers
Data Science



-
- A hand-drawn diagram with two main points enclosed in brackets. The first point, labeled "i)", discusses "Assumptions (that are involved)". The second point, labeled "ii)", discusses the "Amount of data required is huge to be certain".

$(2^{1^{\text{st}}} \text{ (entropy)}) \Rightarrow$ Partially solved problem (ii)

Assumption \Rightarrow Small amount of uncertainty

[ML models all make certain assumptions]

Uncertainty \Rightarrow Noise + Randomness } \downarrow

98% 2% Almighty
on God.

{ Bayesian Probability
forerunner of
ML }

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Probability

$$\left\{ P(A \cap B) = P(B|A) \times P(A) \right\} \Rightarrow \text{Law of Multiplication}$$

Special case $\left\{ \begin{array}{l} A \text{ and } B \text{ are} \\ \text{independent} \end{array} \right\}$

$$\left[P(B|A) \right] = P(B)$$

$$P(A \cap B) = P(B|A) \times P(A)$$

Special case of
multiplication
law

$$P(A \cap B) = P(B) \times P(A)$$

$\left\{ \text{When } A \text{ and } B \text{ are independent} \right\}$

Law of addition

10 October 2025 14:24

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

[Addition law of probability]

If A and B are Mutually exclusive

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B)$$

- i) Addition Law
 - ii) Multiplication Law
 - iii) Law of Total probability
- }

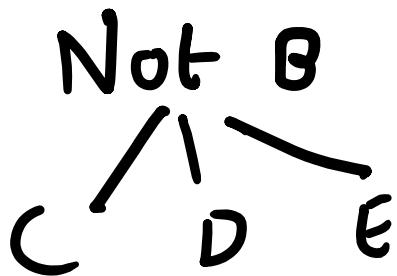
(Bayes Theorem)

Extension of Law of total probability

10 October 2025 23:05

$$P(A) :$$

$$P(A|B) \times P(B) + P(A|B') \times P(B')$$



$$P(A) = P(A|B) \times P(B) +$$

$$P(A|C) \times P(C) + P(A|D) \times P(D)$$

.....

$$P(A) = \sum_{x=1}^i P(A|E_x) \times P(E_x)$$

{ Correlation
Close to 0
No relationship age and Bill }

Correlation is \Rightarrow one increase
close to 1 with
other
 $age \uparrow$ Bill \uparrow

{ Correlation is } \Rightarrow one will decrease
close to -1 when other increases.
 $age \uparrow$ Bill \downarrow