Bayes Theorm Revision

10 October 2025 13:28

$$\left\{ P(A|B) = \frac{P(A \cap B)}{P(B)} \right.$$

$$\left\{ P(B|A) = \frac{P(A \cap B)}{P(A)} \right\}$$

Conditional probability

$$P(A \cap B) = P(B|A) \times P(A) \rightarrow 1$$

$$P(A \cap B) = P(A|B) \times P(B) \rightarrow 2$$

$$\underbrace{P(B|A) \times P(A) = P(A|B) \times P(B)}$$

$$P(B|A) = \frac{P(A|B) \times P(B)}{P(A)}$$

$\Rightarrow$ Bayes Theorm

Bayes Theorm Extension

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Sample Space = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 3, 8, 7, 2\} \Rightarrow$ Length = 5

$B = \{1, 3, 9, 10\}$

$A \cap B = \{1, 3\} \Rightarrow 2$

not B or $B' = \{2, 4, 5, 6, 7, 8\} \Rightarrow 6$

$A \cap (\text{not } B)$

$A \cap (B') = \{2, 7, 8\} \Rightarrow 3$

$$N(A) = N(A \cap B) + N(A \cap B')$$

$$P(A) = P(A \cap B) + P(A \cap B')$$

$$P(B) = P(B \cap A) + P(B \cap A')$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$ Conditional probability

$\rightarrow P(B)$) Conditional probability

$$\left\{ P(B|A) = \frac{P(A\cap B)}{P(A)} \right\}$$

$$P(A\cap B) = \{[P(B|A) \times P(A)]\} \rightarrow ①$$

$$P(A\cap B) = \underbrace{P(A|B) \times P(B)} \rightarrow ②$$

$$P(A) = \underbrace{P(A\cap B)} + P\underbrace{(A\cap B')}$$

$$P(A) = \underbrace{P(A|B) \times P(B)} + \underbrace{P(A|B') \times P(B')}$$

$$P(A\cap B) = \{P(A|B) \times P(B)\}$$

$$P(A\cap B') = P(A|B') \times P(B')$$

$$P(A) = \underbrace{P(A|B) \times P(B)} + \underbrace{P(A|B') \times P(B')}$$

[law of total probability]

$$P(B|A) = \frac{P(A|B) \times P(B)}{P(A)}$$

$$P(B|A) = \frac{P(A|B) \times P(B)}{\underbrace{P(A|B) \times P(B) + P(A|B') \times P(B')}_{\to P(A) \text{ by law of total probability}}}$$

{Extension of Bayes Theorm}

Example problem

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A certain disease affects **1% of the population.**

There is a test for the disease with the following properties:

- If a person has the disease, the test is **positive 99%** of the time (true positive).
- If a person does **not** have the disease, the test is **positive 5%** of the time (false positive).

If a person's test comes back **positive**, what is the probability that they **actually have the disease**?

$$P(D) = \frac{1}{100} \checkmark$$

$$P(ND) = \frac{99}{100} \checkmark$$

$$P(Pos \mid D) = \frac{99}{100} \checkmark$$

$$P(Pos \mid ND) = \frac{5}{100} \checkmark$$

$$P(D \mid pos)$$

$$P(B/A) = \frac{P(A/B) \times P(B)}{P(A/B) \times P(B) + P(A/B') \times P(B')}$$

$$P(D/pos) = \frac{\frac{99}{100} \times \frac{1}{100}}{\frac{99}{100} \times \frac{1}{100} + \frac{5}{100} \times \frac{99}{100}}$$

Classical vs Bayesian Probability

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(India Matches)

Classical probability

🏏 **Example Dataset — "Cricket Match Outcome Predictor"**

Imagine this simple dataset of past matches:

Match	Weather	Pitch	Sachin 50+	Result
1	Sunny	Flat	Yes	Win
2	Cloudy	Flat	No	Lose
3	Sunny	Grassy	No	Win
4	Rainy	Grassy	No	Lose
5	Sunny	Flat	Yes	Win
6	Cloudy	Flat	Yes	Win
7	Sunny	Grassy	No	Lose

Climate

$\Rightarrow P(W) = 4/7$

$P(L) = 3/7 \Rightarrow 43\%$

$\frac{4}{7} = 57\%$

Before 1800s

{Bayes} ⇒ Conditional probability can be combined a few probability theorem

$P(\text{Win} \mid \text{Sunny})$

$P(\text{Win} \mid \text{Flat})$

$P(\text{Win} \mid \text{Sunny, Flat})$

We can find out probabilities based on Conditions

{Bayesian Probability}

{Machine Learning} ⇐ {Predict} Probabilities

1850 - 1900

(Classical probability)

(Bayesian probability Theorists)

ML Engineers

Data Scien.

probability
Theorists

Theorists

Conditional
probability

Results ⇒ using Math + stats
Bayesian
ML

i) [Assumptions that are involved]
ii) Amount of data required is huge
to be certain

21^{st} Century ⇒ Partially solved
problem (ii)

Assumption ⇒ Small amount of
Uncertainity

ML models all make certain assumptions

$\underbrace{\text{Uncertainity} \Rightarrow \text{Noise}}_{98\%}$ + Randomness → Almighty or God.

Randomness ⇑ 2%

Bayesian Probability

forerunner of

ML

Law of multiplication

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$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Probability

$$\left\{ P(A \cap B) = P(B|A) \times P(A) \right\} \Rightarrow \text{Law of Multiplication}$$

Special case {A and B are independent}

$$\left[P(B|A) \right] = P(B)$$

$$P(A \cap B) = P(B|A) \times P(A)$$

(Special case of multiplication law)

$$P(A \cap B) = P(B) \times P(A)$$

{When A and B are independent}

Law of addition

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$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

[Addition law of probability]

If A and B are Mutually exclusive

$$P(A \cap B) = 0$$

$$\left[P(A \cup B) = P(A) + P(B) \right]$$

i) Addition law

ii) Multiplication law

iii) law of total probability

(Bayes Theorem)

Extension of Law of total probability

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$P(A) :$

$$P(A|B) \times P(B) + \underbrace{P(A|B') \times P(B')}_{\text{Not } B}$$

C D E

$$P(A) = P(A|B) \times P(B) + P(A|C) \times P(C) + P(A|D) \times P(D) \ldots$$

$$\boxed{P(A) = \sum_{x=1}^{i} P(A|E_x) \times P(E_x)}$$

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Correlation
Close to 0

No relationship age and Bill

Correlation is close to 1 ⇒ one increase with other

age ↑ Bill ↑

Correlation is close to -1 ⇒ one will decrease when other increases.

age ↑ bill ↓