

Model Documentation of the:

PVTOL with 2 Forces

1 Nomenclature

1.1 Nomenclature for Model Equations

x	horizontal displacement
y	vertical displacement
θ	roll angle
F_1, F_2	Forces on the left and right site of the PVTOL
g	acceleration due to gravity
l	distance between mass center of PVTOL and target point of the forces
m	mass of the PVTOL
J	moment of inertia of the PVTOL

1.2 Nomenclature for Derivation

2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6)^T = (x \ \dot{x} \ y \ \dot{y} \ \theta \ \dot{\theta})^T$$
$$\underline{u} = (u_1 \ u_2)^T = (F_1 \ F_2)^T$$

System Equations:

$$\dot{x}_1 = x_2 \tag{1a}$$

$$\dot{x}_2 = -\frac{\sin(x_5)}{m}(u_1 + u_2) \tag{1b}$$

$$\dot{x}_3 = x_4 \tag{1c}$$

$$\dot{x}_4 = \frac{\cos(x_5)}{m}(u_1 + u_2) - g \tag{1d}$$

$$\dot{x}_5 = x_6 \tag{1e}$$

$$\dot{x}_6 = \frac{l}{J}(u_2 - u_1) \tag{1f}$$

Inputs: F_1, F_2

Parameters: m, J, l

Outputs: x, y, θ

2.1 Assumptions

1. forces target the body of the PVTOL in a 90° angle

2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
acceleration due to gravity	g	9.81	$\frac{m}{s^2}$
distance of forces to mass center	l	0.1	m
mass	m	0.25	kg
moment of inertia	J	0.00076	$kg \cdot m^2$

3 Derivation and Explanation

References

- [1] Knoll, C: *Regelungstheoretische Analyse- und Entwurfsansätze für unteraktivierte mechanische Systeme*, p. 169, TU Dresden, 2016