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Information and admissible sets

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Abstract

«Abstract here»

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1 Introduction

I explore the effect of incorporating information on the identified set of values for the average causal effect of an endogenous variable on an outcome variable (the parameter of interest) that is delivered by a non-parametric model that embeds an exclusion restriction and an independence restriction that together characterise an instrumental variable. Endogenous variation enters the model since agents are permitted to non-randomly select a scalar observable characteristic. I consider the effect of combining many instrumental variables into a composite instrumental variable with many points of support on the identified set of values for counterfactual outcome distributions. Further, I consider the effect of enriching individual behaviour by allowing relevant exogenous variables to affect individual choice; specifically, I allow these variables to enter the structural equations that determine the endogenous variable and that determine the outcome variable. I establish the conditions under which the parameter of interest in the enriched model is equivalent to the parameter of interest in a model that does not explicitly account for the contribution of additional relevant exogenous variables.

The model that I consider partially identifies the parameter of interest. That is, the restrictions on the set of admissible structures (or data generating processes) that are implied by the model are insufficient to exclude observationally equivalent structures that deliver different values of the parameter of interest. The restrictions that are implied are nonetheless sufficient to restrict the set of values of the parameter of interest up to a non-trivial set. This concept is described graphically in Figure 4.

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The model that I consider is partially identifying. That is, the restrictions on the set of admissible structures (or data generating processes) that are implied by the model are insufficient to exclude observationally equivalent structures. Furthermore, the model is unable to identify the structural characteristic of interest

3 B

I explore the effect of incorporating information on the identified set of values for the average causal effect of an endogenous variable on an outcome variable (the parameter of interest) that is delivered by a non-parametric model. The model is weakly restrictive in that it does not embed a structural equation

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Points to make:

- Non-parametric IV model with endogenous variable
- Non-random selection
- Average causal effect of endogenous variable on outcome variable is parameter of interest
- Model partially identifies the parameter of interest
- Model is weakly restrictive in that it does not specify a distribution for unobserved heterogeneity (imposes a normalisation and allows all distributional assumption to enter the non-parametric threshold function) and does not specify a selection equation
- Credible identification
- Want to incorporate additional information for the purpose of efficiency and to potentially provide tighter bounds on parameter of interest; may also be that conditional parameter of interest is useful

Related literature:

- Chesher and Rosen (2013) The starting point for analysis
- Chesher et al. (2013) Characterise level sets and identification
- Balke and Pearl (1997) Binary treatment characterise the identified set under non-compliance; weaker restrictions than presented here
- Kitagawa (2009) Continuous outcome variable; nesting of identified set under different model assumptions
- Blundell and Powell (2003) Average structural function when continuous endogenous regressor
- Beresteanu et al. (2012) Random set theory

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I explore the effect of incorporating information for a non-parametric model that permits non-random selection. In permitting non-random selection the model introduces endogenous variation in a scalar random variable, and it is the average causal effect of this endogenous variable on the outcome variable that is of interest. The model embeds an exclusion restriction and an independence restriction that together characterise an instrumental variable with this instrumental variable providing exogenous variation that is used to measure the causal effect of the endogenous variable on the outcome variable. Asides from the existence of an instrumental variable the model is agnostic with respect to the relationship between the endogenous variable and the instrumental variable, and only specifies the relationship between the outcome variable and the endogenous variable up to a non-parametric index function. As such, the model is credible (Manski, 2013) in that it embeds only weak non-verifiable restrictions but does not identify the average causal effect of the endogenous variable on the outcome variable.[1] Rather the model partially identifies the average causal effect of the endogenous variable on the outcome variable.

Notation

There is a probability space $(\Omega, \Sigma, \mathbb{P})$ on which are defined random variables (Y, D, X, Z, U). Here, (Y, D, X, Z) are observable with supports $(\mathcal{R}_Y, \mathcal{R}_D, \mathcal{R}_X, \mathcal{R}_Z)$, and U is unobservable with as yet unspecified support. I allow (X, Z, U) to be vectors, in which case the support is given by the Cartesian product of the supports of each element in the vector. I refer to Y as the outcome variable, to D as the endogenous variable, to X as the exogenous variable, to Z as the instrumental variable, and to U as unobservable heterogeneity. The logic of this naming convention will be made clear by the restrictions that are imposed upon these random variables in the main text. Lower case letters are used to represent specific values of these random variables.

I denote by $Y(d)$ the counterfactual value of Y when D is externally fixed, and by $D(z)$ the counterfactual value of D when Z is externally fixed. I denote by $\mathbb{E}$ the expectation operator, and by $\mathbb{1}$ the indicator function. Related to these concepts are the average causal effects $ACE(D \to Y)$ and $ACE(Z \to D)$ that are defined as $\mathbb{E}[Y(d_1) - Y(d_0)]$ and $\mathbb{E}[D(z_1) - D(z_0)]$ that are well-defined when D and Z are binary, respectively. To distinguish between population and sample quantities, I subscript sample quantities by n.

Further terminology and notation is introduced in Figure 1 through Figure 4. This specifically relates to models and structures, and is consistent with the approach that is formally laid out in Hurwicz (1950) and in Koopmans and Reiersøl (1950). Following Hurwicz (1950) I also adopt the notation $S :. P$ that signifies that a structure S generates a probability distribution (of observable variables) P, and $P .: G$ that signifies that P is generated by S.

6 A threshold crossing model

References

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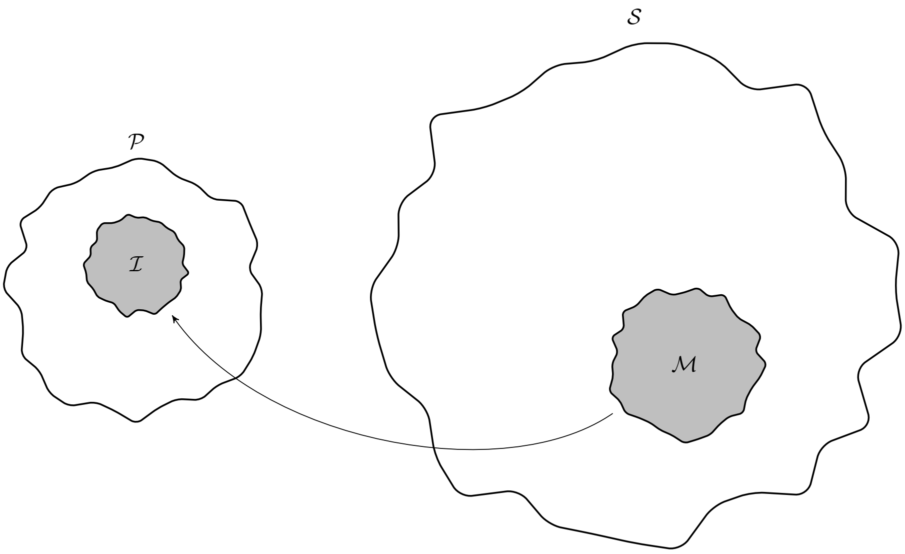
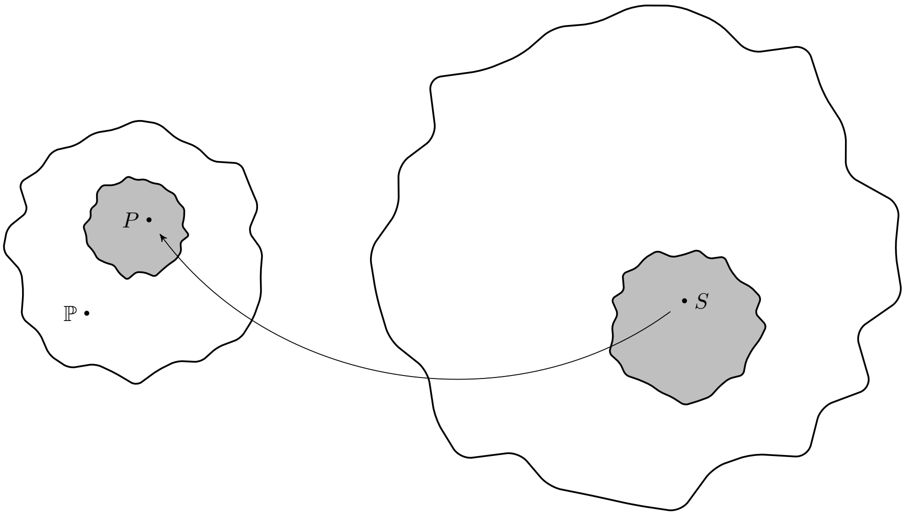
[1] Assumptions that cannot be tested using data; the model does embed some non-trivial non-verifiable restrictions that might be relaxed.

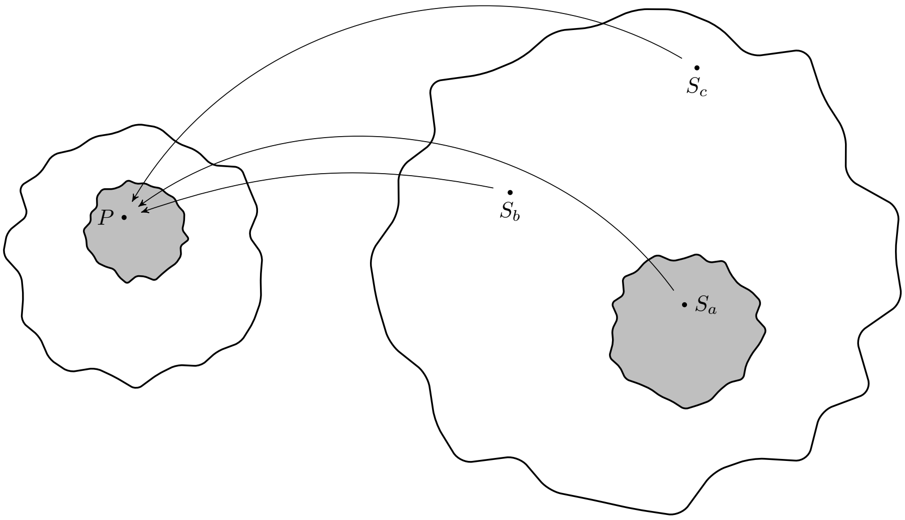


(a) A model $\mathcal{M}$ is a set of structures that forms a proper subset of the class of all structures $\mathcal{S}$. Each structure in $\mathcal{M}$ generates a probability distribution in the class of all probability distributions (of observable variables) $\mathcal{P}$. Then the image $\mathcal{I}$ is the set of all probability distributions that are generated by structures in $\mathcal{M}$.

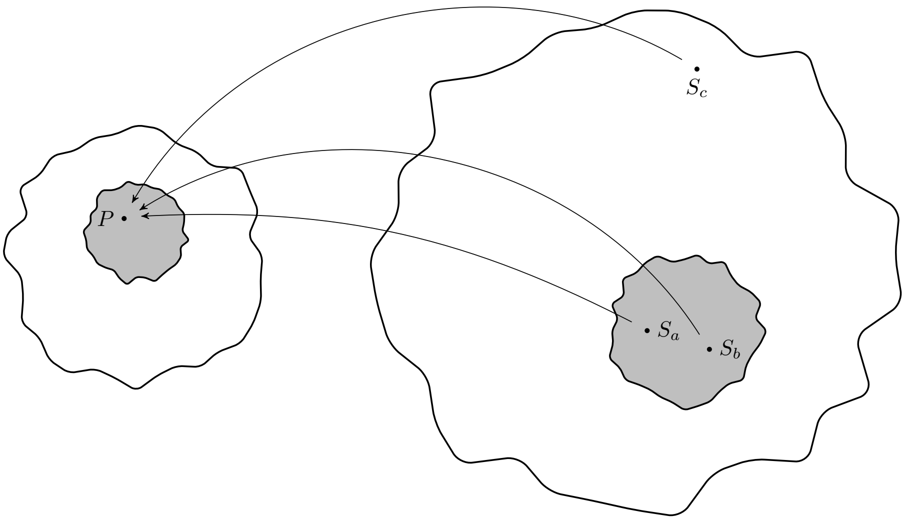


(b) A structure S is incompatible with data if it generates a probability distribution (of observable variables) P that is distinct from a realised probability distribution $\mathbb{P}$. If all structures in $\mathcal{M}$ are incompatible with data then $\mathcal{M}$ is said to be observationally restrictive, and is falsified. This condition is equivalent to $\mathbb{P} \in \mathcal{P} \setminus \mathcal{I}$.

Figure 1: Structures, models, probability distributions (of observable variables), and falsifiability.

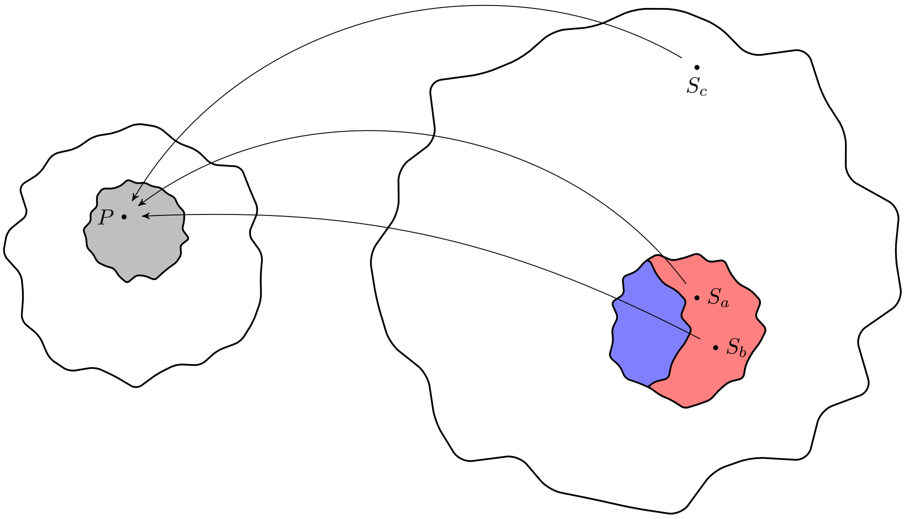


(a) A model $\mathcal{M}$ is said to identify a structure S if the probability distribution (of observable variables) P that is generated by S is distinct from those generated by other structures in $\mathcal{M}$. The structures S_a, S_b and S_c are said to be observationally equivalent as they all generate P but S_b and S_c are not admitted by $\mathcal{M}$. As S_a is the only structure that is admitted by $\mathcal{M}$ and that generates P, S_a is identified by $\mathcal{M}$. For completeness, $\mathcal{M}$ is said to be uniformly identifying if it identifies each structure that it admits.

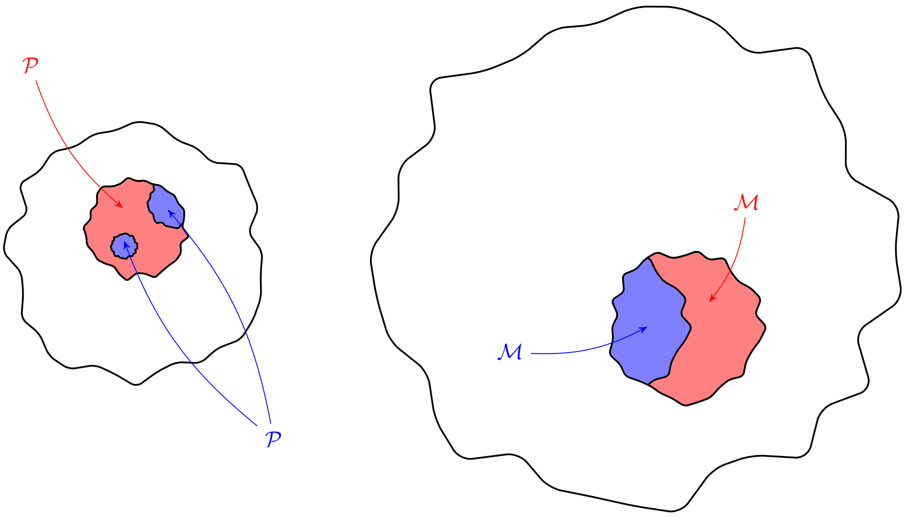


(b) As S_a and S_b are observationally equivalent and are both admitted by $\mathcal{M}$ then $\mathcal{M}$ does not identify either S_a or S_b. Nonetheless, as $\mathcal{M}$ restricts the set of observationally equivalent structures that generate P to S_a and S_b then $\mathcal{M}$ partially identifies S_a (and S_b to within $\{S_a, S_b\}$).

Figure 2: Identification and non-identification of a structure, and partial identification of a structure.



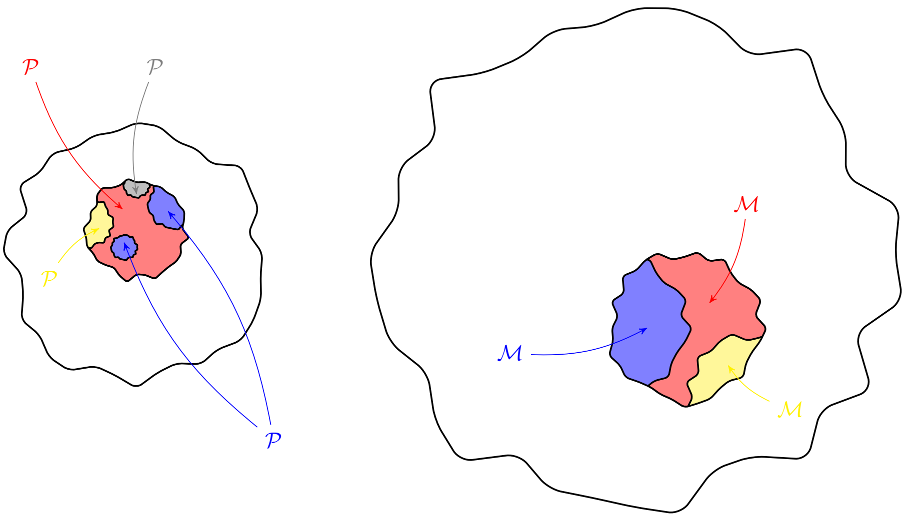
(a) A structural characteristic χ is a function of a structure S. A model $\mathcal{M}$ can be partitioned such that structures in a partition deliver the same value for χ. Structures in the red partition $\mathcal{M}$ deliver the value a for χ, and structures in the red partition $\mathcal{M}$ deliver the value b for χ. If χ is constant across all observationally equivalent structures that $\mathcal{M}$ admits then $\mathcal{M}$ is said to identify χ. As $\chi(S_a)$ is equal to $\chi(S_b)$ (is equal to a) $\mathcal{M}$ identifies χ.



(b) If $\mathcal{M}$ identifies χ for all structures in $\mathcal{M}$ then $\mathcal{M}$ is said to uniformly identify χ. The class of all probability distributions (of observable variables) is partitioned into the blue partition $\mathcal{P}$ and into the red partition $\mathcal{P}$. Probability distributions in $\mathcal{P}$ are generated by (potentially many) structures in $\mathcal{M}$, and probability distributions in $\mathcal{P}$ are generated by (potentially many) structures in $\mathcal{M}$. It is important that the number of partitions in $\mathcal{M}$ and in $\mathcal{P}$ are equal, although that number can be countably infinite. In the context of Figure 3b $\mathcal{M}$ uniformly identifies χ since observationally equivalent structures that $\mathcal{M}$ admits are in the same colour of $\mathcal{M}$. More conveniently, whether $\mathcal{M}$ uniformly identifies χ can be determined by the existence of an identifying correspondence G, a functional. P is a probability distribution in $\mathcal{P}$, and P is a probability distribution in $\mathcal{P}$. Then $\mathcal{M}$ uniformly identifies χ if the value of $G(P)$ is a and if the value of $G(P)$ is b, holding for any such P and P. Notice that if $\mathcal{M}$ uniformly identifies all χ then $\mathcal{M}$ also uniformly identifies structures.

Figure 3: The identification of structural characteristics, and identifying correspondences.

(a) A structural characteristic χ is a function of a structure S. A model $\mathcal{M}$ can be partitioned such that structures in a partition deliver the same value for χ. Structures in the red partition $\mathcal{M}$ deliver the value a for χ, structures in the blue partition $\mathcal{M}$ deliver the value b for χ, and structures in the yellow partition $\mathcal{M}$ deliver the value c for χ. The class of all probability distributions (of observable variables) $\mathcal{P}$ is partitioned into the red partition $\mathcal{P}$, into the blue partition $\mathcal{P}$, into the yellow partition $\mathcal{P}$ and into the grey partition $\mathcal{P}$. Probability distributions in a colour of $\mathcal{P}$ are generated by (potentially many) structures in the same colour of $\mathcal{M}$; the exception is probability distributions in $\mathcal{P}$ which are generated by (potentially many) structures in $\mathcal{M}$ and in $\mathcal{M}$. P is a probability distribution in $\mathcal{P}$ with probability distributions defined similarly for each colour in $\mathcal{P}$.



(b) That probability distributions in $\mathcal{P}$ are generated by structures in $\mathcal{M}$ and in $\mathcal{M}$ creates a complication; the value of χ is not constant across observationally equivalent structures that $\mathcal{M}$ admits and that generate a probability distribution in $\mathcal{P}$. So $\mathcal{M}$ does not uniformly identify χ. Consideration of the identifying correspondence G determines that this corresponds to there being structures in $\mathcal{M}$ for which G does not deliver the value of χ when applied to the probability distributions that these structures generate. Nonetheless, if $\mathcal{M}$ restricts the set of values of χ for any probability distribution in $\mathcal{P}$ then $\mathcal{M}$ does have some non-trivial identifying power for χ. Then $\mathcal{M}$ is said to uniformly partially identify χ if $\mathcal{M}$ and $\mathcal{P}$ can each be partitioned into countably many disjoint subsets and that a probability distribution in a partition of $\mathcal{P}$ is not generated by a structure in at least one partition of $\mathcal{M}$, holding for any such partition of $\mathcal{P}$. In the context of Figure 4 $\mathcal{M}$ identifies χ up to $\{a, c\}$, $\mathcal{M}$ identifies χ uniquely to b, and $\mathcal{M}$ identifies χ up to $\{a, c\}$. Each partition of $\mathcal{P}$ includes probability distributions that are generated by structures in at least one partition of $\mathcal{M}$. Equivalently, if G is permitted to be a multivalued functional (or one-to-many) then $\mathcal{M}$ uniformly partially identifies χ if G exists and if $G(P)$ contains the set of values of χ that are delivered by structures that generate P, holding for all such P. A caveat must be applied here; G cannot be trivial in the sense that it is constant across all such P. Clearly this definition of G does not exclude the possibility that there is multiplicity of identifying correspondences that satisfy this property. Sharpness is a desirable property in such circumstances; a functional G that can be shown to deliver smaller sets according to some well-defined distance measure across all possible P (and that satisfies the properties above) should be preferred to any alternative identifying correspondence.

Figure 4: Partial identification of a structural characteristic.