Master en Técnicas de conservación de la biodiversidad y ecología

Modelización:
Modelos biofísicos en ecología

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¿Si aumenta **1ºC la temperatura ambiente**, cuánto cambia la **temperatura del cuerpo**?

Modelos microclimáticos

Modelo biofísico

Modelo comportamental

Aumento esperado (nivel macroclimático)

+1ºC

➡

Aumento real (microclima)

+1.2 ºC (sol)
+0.8 ºC (sombra)

➡

Aumento de temperatura corporal

➡

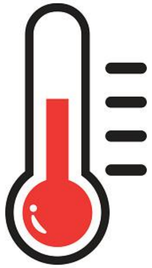
Respuesta comportamental

e.g., pasa más tiempo a la sombra

Modelo ectotermo

Modelo biofísico

$$C \frac{d\,Tb}{dt} = \text{Entrada de calor} - \text{Salida de calor}$$

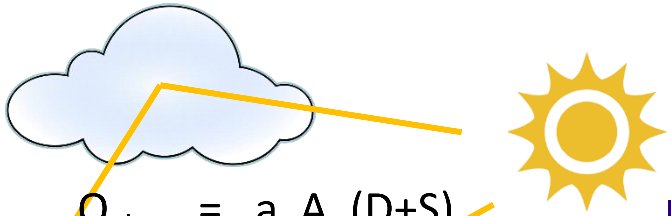


Modelo biofísico

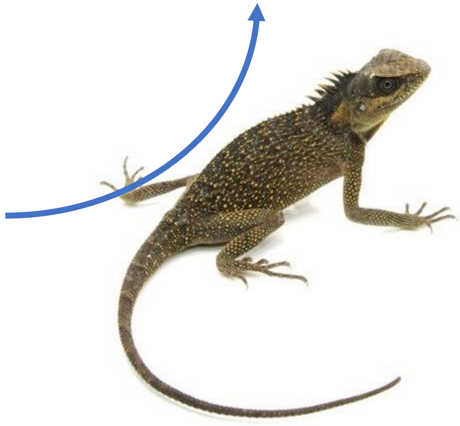
$$Q_{abs} = a \; A \; (D+S)$$

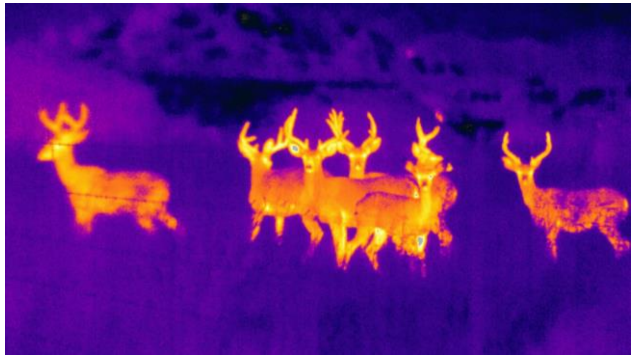
[W] [m^2][W m^{-2}]

Modelo biofísico



$$Q_{abs} = a\ A\ (D+S)$$





Radiación térmica

$$Q_{IR} = -A\ \varepsilon\ \sigma\ [(T_b + 273)^4 - (T_a + 273)^4]$$

Ley de Stefan-Boltzmann

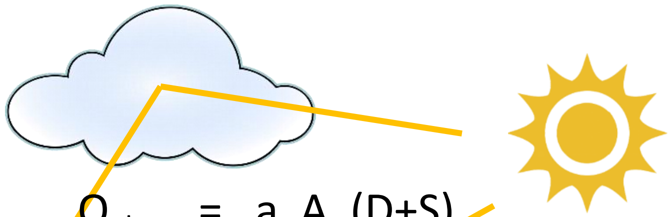
Modelo biofísico

$$Q_{conv} = - A h_c (T_b - T_a)$$

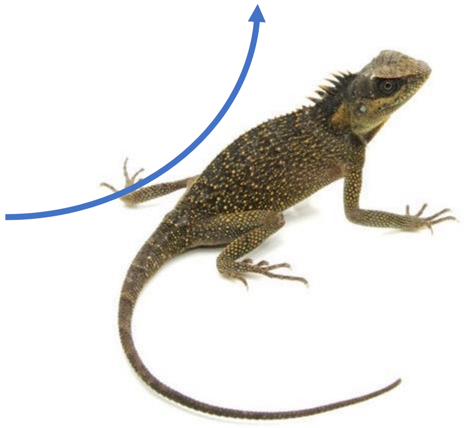
[W] [m²] [Wm⁻² ºC⁻¹] [ºC]

h_c: coeficiente de convección

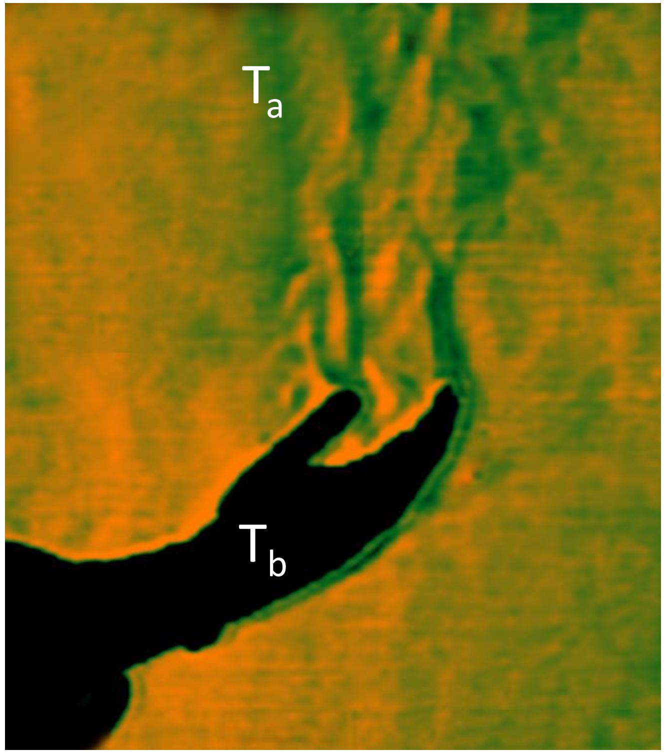
$$\mathbf{h_c = a} \times \text{velocidad viento}^{\mathbf{b}} \times \text{longitud cuerpo}^{\mathbf{-c}}$$



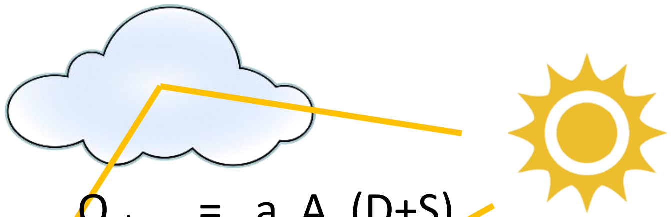
$$Q_{abs} = a \; A \; (D+S)$$



$$Q_{IR} = - A \, \varepsilon \, \sigma \, [(T_b + 273)^4 - (T_a + 273)^4]$$



Modelo biofísico



$$Q_{abs} = a\ A\ (D+S)$$

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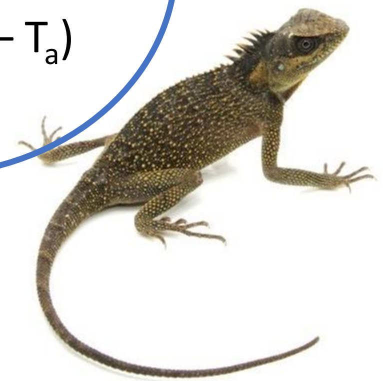

$$Q_{IR} = -\ A\ \varepsilon\ \sigma\ [(T_b + 273)^4 - (T_a + 273)^4]$$

$$Q_{net} = Q_{abs} + Q_{IR} + Q_{conv}$$

Modelo biofísico

Transferencia de calor

$Q_{net} = Q_{abs} + Q_{IR} + Q_{conv}$ $[W] = [J\ s^{-1}]$ Tasa de transferencia de calor

Temperatura corporal, T_b

$$\frac{dT_b}{dt} = \frac{1}{MC} Q_{net}$$ $$\frac{[ºC]}{[s]} = \frac{1}{[g][J\ g^{-1}\ ºC^{-1}]} [J\ s^{-1}]$$ Tasa de cambio de temperatura

M = masa (g)

C = capacidad calorífica ($J\ g^{-1}\ ºC^{-1}$)

Temperatura corporal, T_b

$$Q_{net} = Q_{abs} + Q_{IR} + Q_{conv}$$

$$Q_{abs} = A\ a\ (D + S)$$

$$Q_{IR} = -A\ \varepsilon\ \sigma\ (T_b^4 - T_a^4)$$

$$Q_{conv} = -A\ h_c\ (T_b - T_a)$$

$$\Delta T_b = \frac{1}{MC} Q_{net}$$

Parámetros del lagarto (tamaño, forma y color)

M = 5g

A = 0.0029 m^2

C = 3.7 J g^{-1} ºC^{-1}

a = 0.9

Parámetros ambientales

D = 400 Wm^{-2}

S = 20% de D

Ta = 20 ºC

v = 0.8 m s^{-1}

Tiempo	Tb	Q_{abs}	Q_{IR}	Q_{conv}
1	20.000	1.263	0.000	0.000
2	20.792	1.263	-0.012	-0.044
3	21.620	1.263	-0.024	-0.089
4	22.490	1.263	-0.038	-0.137
5	23.408	1.263	-0.052	-0.188
6	24.385	1.263	-0.067	-0.242
7	25.433	1.263	-0.083	-0.300
8	26.569	1.263	-0.101	-0.362
9	27.820	1.263	-0.121	-0.431
10	29.227	1.263	-0.144	-0.509

Temperatura corporal, T_b

$$Q_{net} = Q_{abs} + Q_{IR} + Q_{conv}$$

$$Q_{abs} = A\ a\ (D + S)$$

$$Q_{IR} = -A\ \varepsilon\ \sigma\ (T_b^4 - T_a^4)$$

$$Q_{conv} = -A\ h_c\ (T_b - T_a)$$

$$\Delta T_b = \frac{1}{MC} Q_{net}$$

Parámetros del lagarto (tamaño, forma y color)

M = 5g

$A = 0.0029\ m^2$

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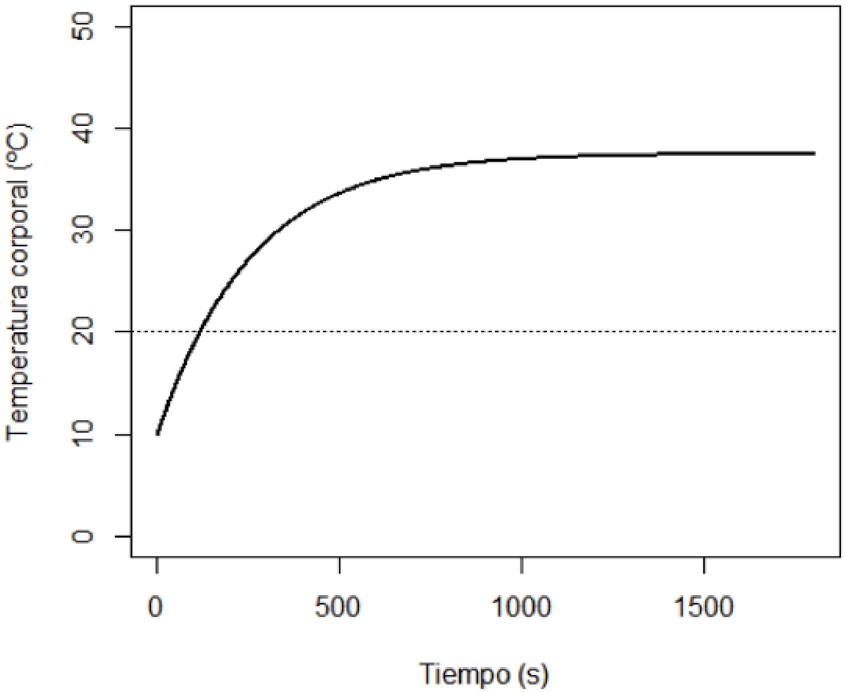

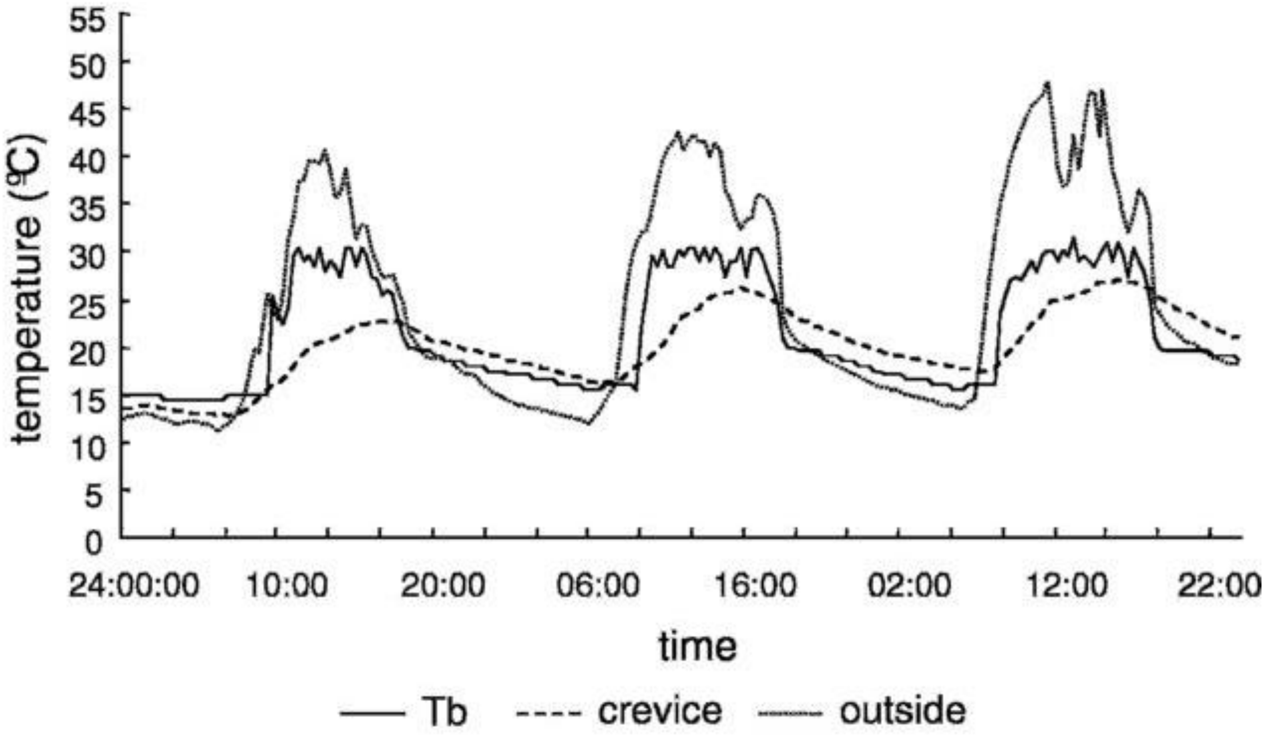

Pseudocordylus melanotus

McConnachie et al. 2009 *Herp monogr*

Técnicas de conservación de la biodiversidad y ecología

Modelos biofísicos: Práctica

Modelo de ectotermo

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McGill Department of Biology

MARIE CURIE ACTIONS

Solución analítica:

modelo transitorio de transferencia de calor

$$\frac{dT_b}{dt} = \frac{1}{cM}(Q_{abs} + Q_{IR} + Q_{conv})$$

$$T_b(t) = T_b(t-1) + \Delta t \frac{1}{cM}(Q_{abs} + Q_{IR} + Q_{conv})$$

Solución numérica por iteración (método de Euler)

Solución analítica:
modelo transitorio de transferencia de calor

$$\frac{dT_b}{dt} = \frac{1}{cM}(Q_{abs} + Q_{IR} + Q_{conv})$$

$$\frac{dT_b}{Q_{abs} + Q_{IR} + Q_{conv}} = \frac{1}{cM}dt$$

$$\int \frac{dT_b}{Q_{abs} + Q_{IR} + Q_{conv}} = \int \frac{1}{cM}dt$$

…

$$\boxed{T_b(t) = \cdots \exp(\ldots tiempo)}$$

Solución analítica:
modelo transitorio de transferencia de calor

$$cM\frac{dT_b}{dt} = Q_{abs} + Q_{IR} + Q_{conv}$$

$$cM\frac{dT_b}{dt} = Aa(D+S) - A\varepsilon\sigma\left(T_b^4 - T_a^4\right) - Ah_c(T_b - T_a)$$

Solución analítica:
modelo transitorio de transferencia de calor

$$cM\frac{dT_b}{dt} = Q_{abs} + Q_{IR} + Q_{conv}$$

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$$A4\varepsilon\sigma T_a^3(T_b - T_a)$$

Bakken 1976 *J Theor Biol*

Solución analítica:
modelo transitorio de transferencia de calor

$$cM\frac{dT_b}{dt} = Q_{abs} + Q_{IR} + Q_{conv}$$

$$cM\frac{dT_b}{dt} = Aa(D+S) - A\varepsilon\sigma\left(T_b^4 - T_a^4\right) - Ah_c(T_b - T_a)$$

$$A4\varepsilon\sigma T_a^3(T_b - T_a)$$

$$AR(T_b - T_a)$$

Solución analítica:
modelo transitorio de transferencia de calor

$$cM\frac{dT_b}{dt} = Q_{abs} + Q_{IR} + Q_{conv}$$

$$cM\frac{dT_b}{dt} = Aa(D+S) - AR(T_b - T_a) - Ah_c(T_b - T_a)$$

Solución analítica:
modelo transitorio de transferencia de calor

$$cM\frac{dT_b}{dt} = Q_{abs} + Q_{IR} + Q_{conv}$$

$$cM\frac{dT_b}{dt} = Aa(D+S) - AR(T_b - T_a) - Ah_c(T_b - T_a)$$

$$\frac{dT_b}{dt} = \frac{Aa}{cM}(D+S) - \frac{AR}{cM}(T_b - T_a) + \frac{Ah_c}{cM}(T_b - T_a)$$

$$\frac{dT_b}{dt} = \frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right)(T_b - T_a)$$

Solución analítica:
modelo transitorio de transferencia de calor

$$cM\frac{dT_b}{dt} = Q_{abs} + Q_{IR} + Q_{conv}$$

$$cM\frac{dT_b}{dt} = Aa(D+S) - AR(T_b - T_a) - Ah_c(T_b - T_a)$$

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$$\frac{dT_b}{dt} = \frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right)T_b - \left(\frac{A(R+h_c)}{cM}\right)T_a$$

Solución analítica:
modelo transitorio de transferencia de calor

$$\frac{dT_b}{dt} + \left(\frac{A(R + h_c)}{cM}\right)T_b = \frac{Aa}{cM}(D + S) - \left(\frac{A(R + h_c)}{cM}\right)T_a$$

$$\frac{dy}{dt} + \theta y = j$$

Solución analítica:
modelo transitorio de transferencia de calor

$$\underbrace{\boxed{\frac{dT_b}{dt}}}_{\frac{dy}{dt}} + \underbrace{\boxed{\left(\frac{A(R+h_c)}{cM}\right)T_b}}_{\theta y} = \underbrace{\boxed{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right)T_a}}_{j}$$

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$$\frac{dy}{dt} + \theta y = j$$

$$y = y_{equilibrio} + y_{transición}$$

$$y_{transición}$$

$$\frac{dy}{dt} + \theta y = 0$$

$$\frac{dy}{dt} = -\theta y$$

Solución analítica:
modelo transitorio de transferencia de calor

$$\underbrace{\boxed{\frac{dT_b}{dt}}}_{\frac{dy}{dt}} + \underbrace{\boxed{\left(\frac{A(R+h_c)}{cM}\right)T_b}}_{\theta y} = \underbrace{\boxed{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right)T_a}}_{j}$$

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$$y_{transición}$$

$$\frac{dy}{dt} + \theta y = 0$$

$$\frac{dy}{dt} = -\theta y$$

$$y_{transición}(t) = C_1 e^{-\theta t}$$

Solución analítica:

modelo transitorio de transferencia de calor

$$\underset{\frac{dy}{dt}}{\boxed{\frac{dT_b}{dt}}} + \underset{\theta y}{\boxed{\left(\frac{A(R+h_c)}{cM}\right)}T_b} = \underset{j}{\boxed{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right)T_a}}$$

$$\frac{dy}{dt} + \theta y = j$$

$$y = y_{equilibrio} + y_{transición}$$

$$y_{transición}(t) = C_1 e^{-\theta t}$$

$$y_{equilibrio}$$

Solución analítica:
modelo transitorio de transferencia de calor

$$\underbrace{\boxed{\frac{dT_b}{dt}}}_{\frac{dy}{dt}} + \underbrace{\boxed{\left(\frac{A(R+h_c)}{cM}\right)T_b}}_{\theta y} = \underbrace{\boxed{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right)T_a}}_{j}$$

$$\frac{dy}{dt} + \theta y = j$$

$$y = y_{equilibrio} + y_{transición}$$

$$y_{transición}(t) = C_1 e^{-\theta t}$$

$$y_{equilibrio}$$

$$0 + \theta y = j$$

$$\theta y = j$$

$$y_{equilibrio} = \frac{j}{\theta}$$

Solución analítica:
modelo transitorio de transferencia de calor

$$\underbrace{\boxed{\frac{dT_b}{dt}}}_{\frac{dy}{dt}} + \underbrace{\boxed{\left(\frac{A(R+h_c)}{cM}\right) T_b}}_{\theta y} = \underbrace{\boxed{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right) T_a}}_{j}$$

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$$\underset{\frac{dy}{dt}}{\boxed{\frac{dT_b}{dt}}} + \underset{\theta y}{\boxed{\left(\frac{A(R+h_c)}{cM}\right) T_b}} = \underset{j}{\boxed{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right) T_a}}$$

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Solución analítica:

modelo transitorio de transferencia de calor

$$\underbrace{\boxed{\frac{dT_b}{dt}}}_{\frac{dy}{dt}} + \underbrace{\boxed{\left(\frac{A(R+h_c)}{cM}\right)}T_b}_{\theta y} = \underbrace{\boxed{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right)T_a}}_{j}$$

Condiciones iniciales

$$y_{transición}(t) = C_1 e^{-\theta t}$$

$$y_{equilibrio} = \frac{j}{\theta}$$

$$y = \frac{j}{\theta} + C_1 e^{-\theta t}$$

Solución analítica:
modelo transitorio de transferencia de calor

$$\underbrace{\boxed{\frac{dT_b}{dt}}}_{\frac{dy}{dt}} + \underbrace{\boxed{\left(\frac{A(R+h_c)}{cM}\right)}T_b}_{\theta y} = \underbrace{\boxed{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right)T_a}}_{j}$$

$$y_{transición}(t) = C_1 e^{-\theta t}$$

$$y_{equilibrio} = \frac{j}{\theta}$$

$$y = \frac{j}{\theta} + C_1 e^{-\theta t}$$

Condiciones iniciales

$$y(t=0) = y_0 = \frac{j}{\theta} + C_1 \cancel{e^{-\theta t}}$$

$$C_1 = y_0 - \frac{j}{\theta}$$

Solución analítica:

modelo transitorio de transferencia de calor

$$\underbrace{\boxed{\frac{dT_b}{dt}}}_{\frac{dy}{dt}} + \underbrace{\boxed{\left(\frac{A(R+h_c)}{cM}\right)T_b}}_{\theta y} = \underbrace{\boxed{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right)T_a}}_{j}$$

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Condiciones iniciales

$$y(t=0) = y_0 = \frac{j}{\theta} + C_1 \cancel{e^{-\theta t}}$$

$$C_1 = y_0 - \frac{j}{\theta}$$

$$y(t) = \frac{j}{\theta} + \left(y_0 - \frac{j}{\theta}\right) e^{-\theta t}$$

Solución analítica:
modelo transitorio de transferencia de calor

$$\underbrace{\boxed{\frac{dT_b}{dt}}}_{\frac{dy}{dt}} + \underbrace{\boxed{\left(\frac{A(R+h_c)}{cM}\right) T_b}}_{\theta y} = \underbrace{\boxed{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right) T_a}}_{j}$$

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$$y(t) = \frac{j}{\theta} + \left(y_0 - \frac{j}{\theta}\right)e^{-\theta t}$$

$$\frac{j}{\theta} = \frac{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right)T_a}{\frac{A(R+h_c)}{cM}}$$

$$\frac{j}{\theta} = T_a + \frac{a(D+S)}{R+h_c}$$

Solución analítica:

modelo transitorio de transferencia de calor

$$\underset{\frac{dy}{dt}}{\boxed{\frac{dT_b}{dt}}} + \underset{\theta y}{\boxed{\left(\frac{A(R+h_c)}{cM}\right)}T_b} = \underset{j}{\boxed{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right)T_a}}$$

$$y(t) = \frac{j}{\theta} + \left(y_0 - \frac{j}{\theta}\right)e^{-\theta t}$$

$$\frac{j}{\theta} = \frac{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right)T_a}{\frac{A(R+h_c)}{cM}}$$

$$\frac{j}{\theta} = T_a + \frac{a(D+S)}{R+h_c} = T_e$$

Solución analítica:
modelo transitorio de transferencia de calor

$$\underbrace{\boxed{\frac{dT_b}{dt}}}_{\frac{dy}{dt}} + \underbrace{\boxed{\left(\frac{A(R+h_c)}{cM}\right)}T_b}_{\theta y} = \underbrace{\boxed{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right)T_a}}_{j}$$

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$$\theta = \frac{A(R+h_c)}{cM}$$

Solución analítica:
modelo transitorio de transferencia de calor

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Solución analítica:

modelo transitorio de transferencia de calor

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$$y(t) = \frac{j}{\theta} + \left(y_0 - \frac{j}{\theta}\right)e^{-\theta t}$$

$$\frac{j}{\theta} = T_a + \frac{a(D+S)}{R+h_c} = T_e$$

$$\theta = \frac{A(R+h_c)}{cM}$$

$$y(t) = T_e + (T_0 - T_e)e^{-\frac{A(R+h_c)}{cM}t}$$

Solución analítica:

modelo transitorio de transferencia de calor

$$\underbrace{\boxed{\frac{dT_b}{dt}}}_{\frac{dy}{dt}} + \underbrace{\boxed{\left(\frac{A(R+h_c)}{cM}\right) T_b}}_{\theta y} = \underbrace{\boxed{\frac{Aa}{cM}(D+S) - \left(\frac{A(R+h_c)}{cM}\right) T_a}}_{j}$$

$$y(t) = \frac{j}{\theta} + \left(y_0 - \frac{j}{\theta}\right) e^{-\theta t}$$

$$\frac{j}{\theta} = T_a + \frac{a(D+S)}{R+h_c} = T_e$$

$$\theta = \frac{A(R+h_c)}{cM}$$

$$y(t) = T_e + (T_0 - T_e)e^{-\frac{A(R+h_c)}{cM}t}$$

$$T_e = T_a + \frac{a(D+S)}{R+h_c}$$