

# ANALYSIS AND MODELING OF LITHIUM FLOWS IN POROUS MATERIALS

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# Outline

- **Introduction**
- **Viscous Linear Stability Analysis**
- **OpenFOAM VOF-MHD Modelling**

# Introduction: Flowing Liquid Metal as PFC

▪ *flowing liquid metal (LM): attractive plasma facing component (PFC) in fusion engineering*

## ➤ *benefits of flowing LM PFC:*

- absorption & removal of *impurities* with LM flow
- absorption of *high heat fluxes* preventing *PFC melting & erosion*

## ➤ *some problems with flowing LM PFC:*

- need to achieve *smooth and controllable* LM flow in magnetic field
- prevent LM *thickness variation & formation of dry spots*
- mitigate LM *splashing & plasma contamination* by LM droplets formed during ELMs or plasma disruptions

# Introduction: Motivation for Research

- **goals of research:** *quantitative theoretical & computational analysis of LM free surface flows* in both porous & non-porous media
- **Flow of LM on substrate *without impact of gas or plasma*:**
  - dependence of flow regime on *speed & thickness* of LM layer
  - development/suppression of *waves* on the surface & their *wavelength*
  - influence of *magnetic field* on LM flow stability & surface waves
  - effects of *medium porosity & permeability* on LM flow
- **Flow of LM on substrate *under impact of gas or plasma*:**
  - development of *waves & their wavelengths* affected by gas or plasma flowing over LM layer with increasing speed
  - conditions for *LM splashing & droplet development*

# Viscous Linear Stability Analysis (VLSA)

## ■ viscous instability in porous media

*index  $m$*  - LM

*index  $p$*  - plasma (gas)

$V_p$  &  $V_m$  - velocities

$\rho_p$  &  $\rho_m$  - mass densities

$\mu_p$  &  $\mu_m$  - viscosities

$h_p$  &  $h_m$  - thicknesses

$\varepsilon_p$  &  $\varepsilon_m$  - porosities

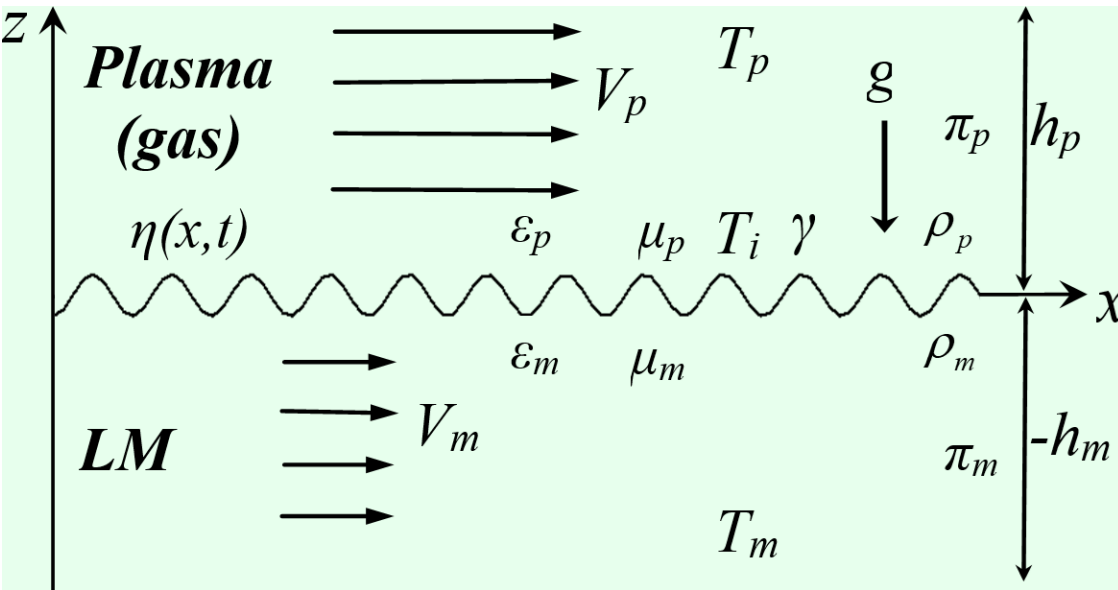
$\pi_p$  &  $\pi_m$  - permeabilities

$T_p$  &  $T_m$  - temperatures

$\gamma$  - surface tension       $g$  - gravity constant       $T_i$  - interfacial temperature

➤ **stability analysis:** includes *viscosity, thermal effects, and porosity*

- linearize *Navier-Stokes equations* & apply *harmonic normal modes*



*Sketch of the plasma-LM system*

# Viscous Linear Stability Analysis (VLSA)

- final expression for *the relative velocity*  $\Delta V = |V_p - V_m|$  of viscous instability in porous media

$$\begin{aligned}
 & k^2 \left[ \alpha^2 (\varepsilon_p \rho'_p \coth^2(kh_m) + \varepsilon_m \rho'_m \coth^2(kh_p)) + 4k^4 (\varepsilon_p \rho'_p \mu'^2_m + \varepsilon_m \rho'_m \mu'^2_p) + \right. \\
 & 4k^2 \varepsilon_m \varepsilon_p \left( \frac{\rho'_p}{\pi_m} \mu'^2_m + \frac{\rho'_m}{\pi_p} \mu'^2_p \right) + 2\alpha \varepsilon_m \varepsilon_p \left( \frac{\rho'_p}{\pi_m} \mu'_m \coth(kh_m) + \frac{\rho'_m}{\pi_p} \mu'_p \coth(kh_p) \right) + \\
 & \left. 4\alpha k^2 (\varepsilon_p \rho'_p \mu'_m \coth(kh_m) + \varepsilon_m \rho'_m \mu'_p \coth(kh_p)) + \varepsilon_m \varepsilon_p \left( \frac{\varepsilon_p \rho'_m \mu'^2_p}{\pi_p^2} + \frac{\varepsilon_m \rho'_p \mu'^2_m}{\pi_m^2} \right) \right] \Delta V^2 = \\
 & \left[ ((\rho_m - \rho_p)kg + \gamma k^3) + 2k^2 \alpha \left( \frac{\mu'_m}{\varepsilon_m \rho_m} + \frac{\mu'_p}{\varepsilon_p \rho_p} \right) + \alpha \left( \frac{\mu'_m}{\pi_m \rho_m} + \frac{\mu'_p}{\pi_p \rho_p} \right) \right] \times \\
 & \left[ \alpha (\varepsilon_p \coth(kh_m) + \varepsilon_m \coth(kh_p)) + 2k^2 (\varepsilon_p \mu'_m + \varepsilon_m \mu'_p) + \varepsilon_m \varepsilon_p \left( \frac{\mu'_m}{\pi_m} + \frac{\mu'_p}{\pi_p} \right) \right]^2
 \end{aligned}$$

$$\rho'_p = \rho_p \coth(kh_p) \quad \rho'_m = \rho_m \coth(kh_m) \quad \mu'_p = \mu_p \coth(kh_p) \quad \mu'_m = \mu_m \coth(kh_m)$$

$k = 2\pi/\lambda$  - wave number    $\lambda$  - wavelength    $\alpha$  - coefficient of heat transfer

# Viscous Linear Stability Analysis (VLSA)

- case of *viscous Li instability* in porous medium with parameters:

$$\rho_m \approx 0.516 \text{ g / cm}^3 \quad \text{at} \quad T_m = T_p = 454 \text{ K}$$

$$\rho_p \approx 1.06 \cdot 10^{-7} \text{ g / cm}^3 \Rightarrow N_p \approx 1.6 \cdot 10^{15} \text{ cm}^{-3} \text{ for Ar gas}$$

$$\mu_m \sim 6.4 \cdot 10^{-3} \text{ g / (cm} \cdot \text{s)} \quad \text{and} \quad \mu_p \sim 3.2 \cdot 10^{-5} \text{ g / (cm} \cdot \text{s)}$$

$$h_m = 1 \text{ cm} \quad h_p \rightarrow \infty \quad \alpha = 0 \quad \gamma = 331.3 \text{ g / s}^2 \quad g = 981 \text{ cm / s}^2$$

$$\varepsilon_m \rightarrow 0 \div 1 \quad < 0.01 \text{ for solids and } \sim 1 \text{ for non-porous medium}$$

$$\varepsilon_p = 1 \quad - \text{no porosity for Ar gas}$$

$$\pi_p \rightarrow \infty \quad - \text{no resistance for Ar gas to permeate in porous medium}$$

$$\pi_m \rightarrow \infty \div 0 \quad - \text{range of permeability of Li in porous medium}$$



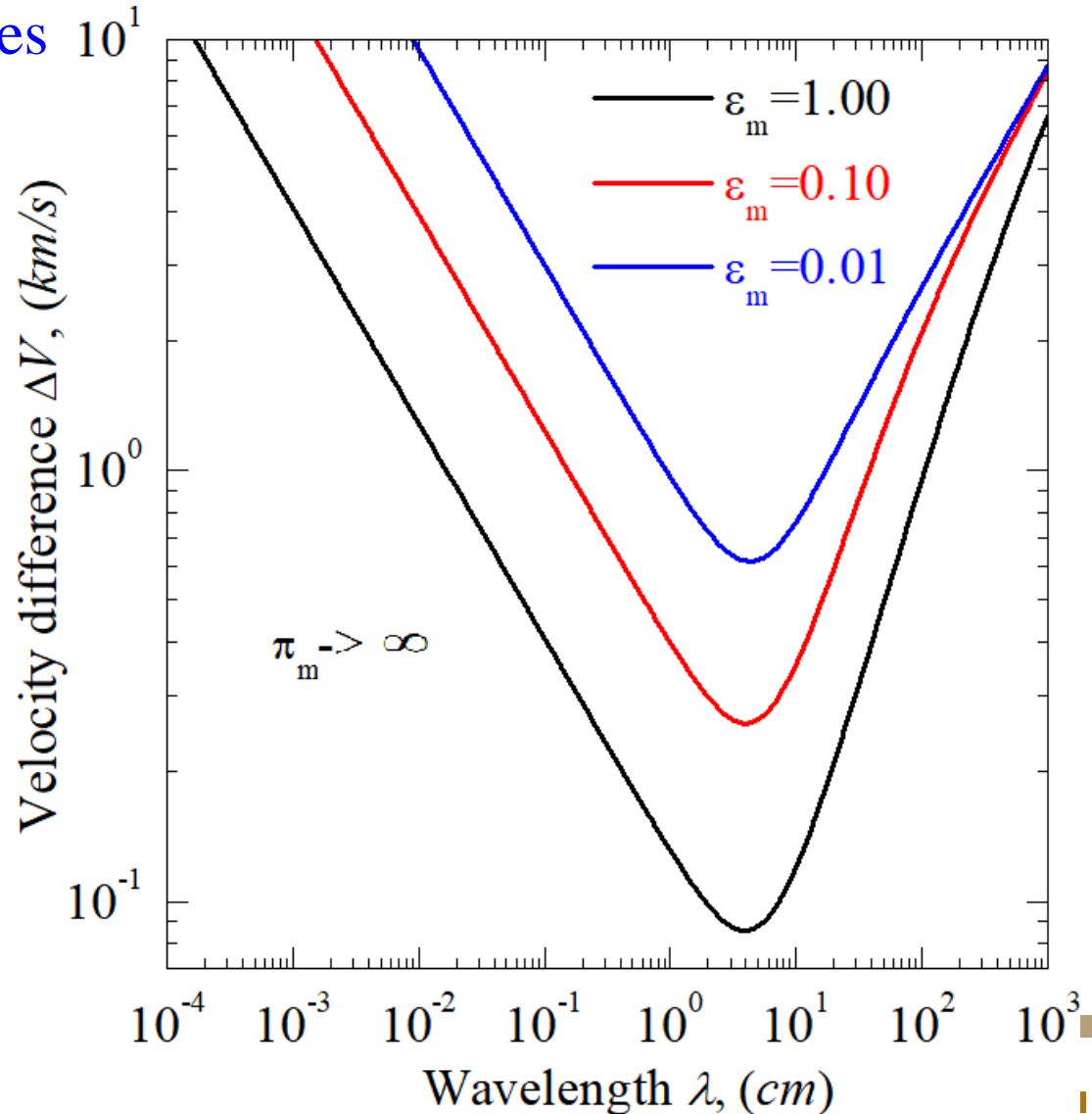
# Viscous Linear Stability Analysis (VLSA)

- effects of *porosity of medium* with very large permeability for Li on stabilization of surface waves

➤ case with  $\varepsilon_m = 1$  (non-porous medium): Ar gas flow with velocity of  $V_p > 100$  m/s induces *instability* with critical wavelength  $\sim 4$  cm

➤ with decrease in porosity: *whole range of wavelengths becomes more stable*

➤ porous medium: *has stabilizing effect* on Li flow

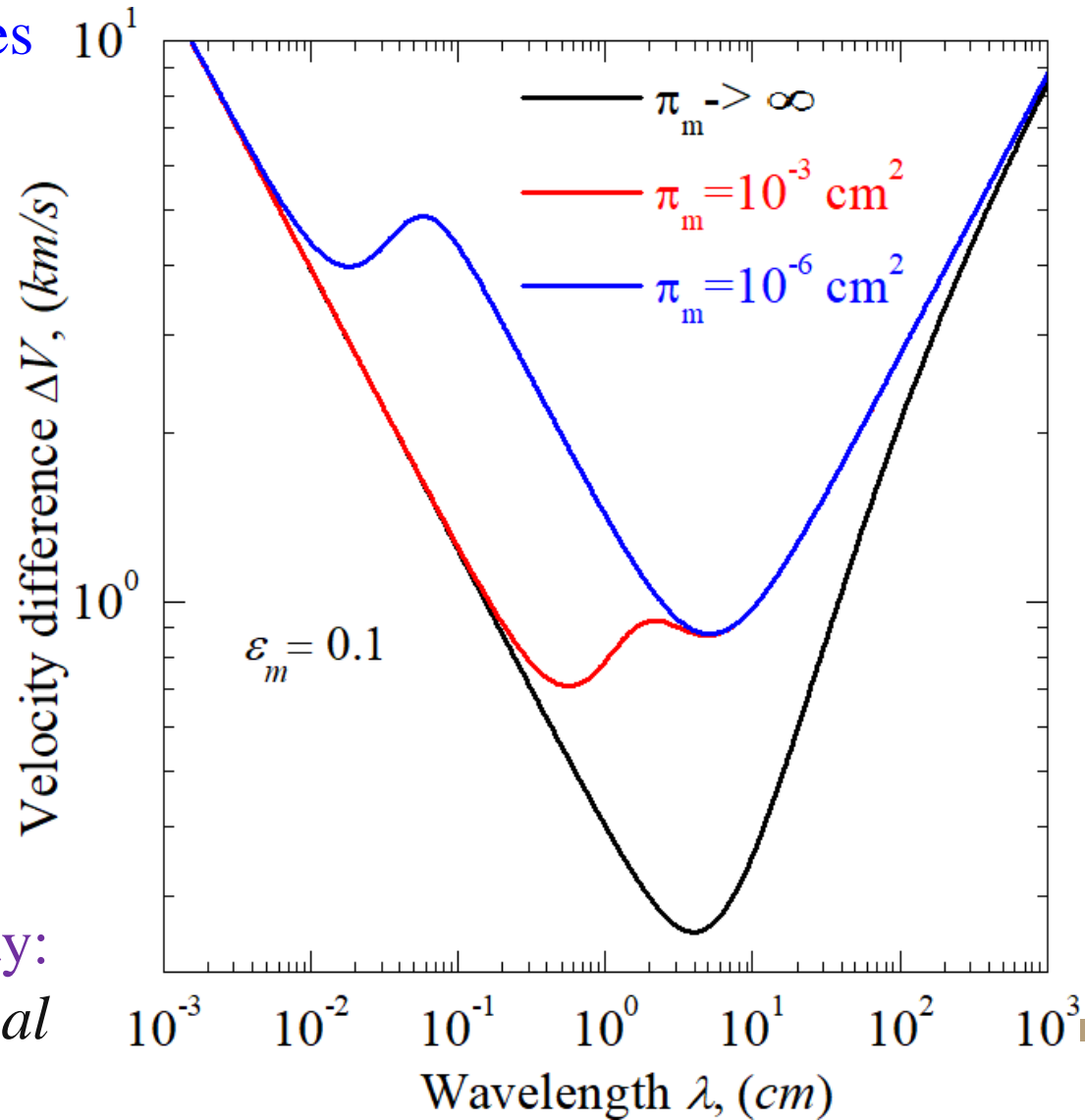




# Viscous Linear Stability Analysis (VLSA)

- effects of *permeability* of Li in the medium with porosity  $\varepsilon_m = 0.1$  on stabilization of surface waves

- case with  $\pi_m \rightarrow \infty$  (full permeability): Ar gas flow with velocity of  $V_p > 300$  m/s induces *instability* with *critical wavelength*  $\sim 4$  cm
- with decrease in Li permeability  $\pi_m$ :
  - short wavelengths  $< 4$  cm: become *more stable*
  - $\pi_m$  independent instability: at  $V_p > 1$  km/s with *critical wavelength*  $\sim 4$  cm



# Viscous Linear Stability Analysis (VLSA)

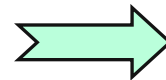
## ■ Summary

- **VLSA:** predicts only *the onset conditions* for development & growth of surface waves
  - values of *relative velocity*  $\Delta V$  as a function of *wavelength*  $\lambda$
  - **unstable & stable regions:** located *above & below* the  $\Delta V$  curve
  - *fastest growing wave* with “dangerous” wavelength  $\lambda_{cr}$
- **VLSA:** can not predict *the nonlinear stage* of wave growth, wave breakage & droplet ejection
- **VLSA:** can be used to investigate *Li flow behavior* as a function of
  - *velocity, mass density, and temperature* of gas (plasma)
  - *thickness, velocity, and temperature* of Li
  - *porosity & permeability* of porous medium

# OpenFOAM VOF-MHD Modelling

- **OpenFOAM:** Open Source CFD Toolbox with *extensive multi-physics capabilities*
- **C++ library of finite volume solvers & utilities:** under active development with capabilities of commercial CFD software
- **2D or 3D structured/unstructured mesh & parallel running**
- **possibility to extend and implement new physics models:** existing solvers can be used as *templates* for further development
- **representation of partial differential equations:** through natural language of *equation mimicking*:

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot \phi \mathbf{U} - \nabla \cdot \mu \nabla \mathbf{U} = -\nabla p$$



```
solve
(
    fvm::ddt(rho, U)
  + fvm::div(phi, U)
  - fvm::laplacian(mu, U)
  ==
  - fvc::grad(p)
);
```

<http://www.openfoam.com/>

# OpenFOAM VOF-MHD Modelling

Equations governing two-fluid flow:

$$\frac{\partial}{\partial t}(\varepsilon \rho u) + \nabla \cdot (\rho u u) = -\nabla p + \nabla \cdot (\mu (\nabla u + (\nabla u)^T)) + \gamma \kappa \nabla \alpha_m + \rho g + J \times B + \Pi$$

$$\frac{\partial \alpha_m}{\partial t} + \nabla \cdot (\alpha_m u) + \nabla \cdot (\alpha_m (1 - \alpha_m) u_c) = 0$$

$$\nabla \cdot \vec{u} = 0$$

$$\frac{\partial \rho C T}{\partial t} + \nabla \cdot (\rho C \vec{u} T) = \nabla \cdot (K \nabla T) + q$$

$$\vec{J} \times \vec{B} = -\nabla \left( \frac{\vec{B}^2}{2\mu_m} \right) + \frac{1}{\mu_m} \nabla \cdot (\vec{B} \vec{B})$$

$$q = \frac{J \cdot J}{\sigma}$$

$$\vec{J} = \frac{1}{\mu_m} \nabla \times \vec{B}$$

Magnetic Induction & Divergence equations:

$$\frac{\partial \vec{B}}{\partial t} + \nabla \cdot (\vec{u} \vec{B} - \vec{B} \vec{u}) = \nabla \cdot \frac{1}{\sigma \mu_m} \nabla \vec{B} + \nabla q$$

$$\nabla \cdot \vec{B} = 0$$

<http://www.openfoam.com/>

$$\alpha_p = 1 - \alpha_m$$

$$\rho = \alpha_m \rho_m + \alpha_p \rho_p$$

$$\mu = \alpha_m \mu_m + \alpha_p \mu_p$$

$$C = \alpha_m C_m + \alpha_p C_p$$

$$K = \alpha_m K_m + \alpha_p K_p$$

$$\sigma = \alpha_m \sigma_m + \alpha_p \sigma_p$$

# OpenFOAM VOF-MHD Modelling

## Equations governing two-fluid flow: (cont.)

$$\frac{\partial}{\partial t}(\varepsilon \rho u) + \nabla \cdot (\rho u u) = -\nabla p + \nabla \cdot (\mu (\nabla u + (\nabla u)^T)) + \gamma \kappa \nabla \alpha_m + \rho g + J \times B + \Pi$$

$\varepsilon \rightarrow 0 \div 1$  - porosity defining *the amount of pore space*

$$\Pi = -\left( \mu D + \frac{1}{2} \rho |u| F \right) u$$
 - Darcy-Forchheimer equation describing resistance (permeability) to LM flow

$D$  [1/m<sup>2</sup>] - viscous *Darcy* coefficient

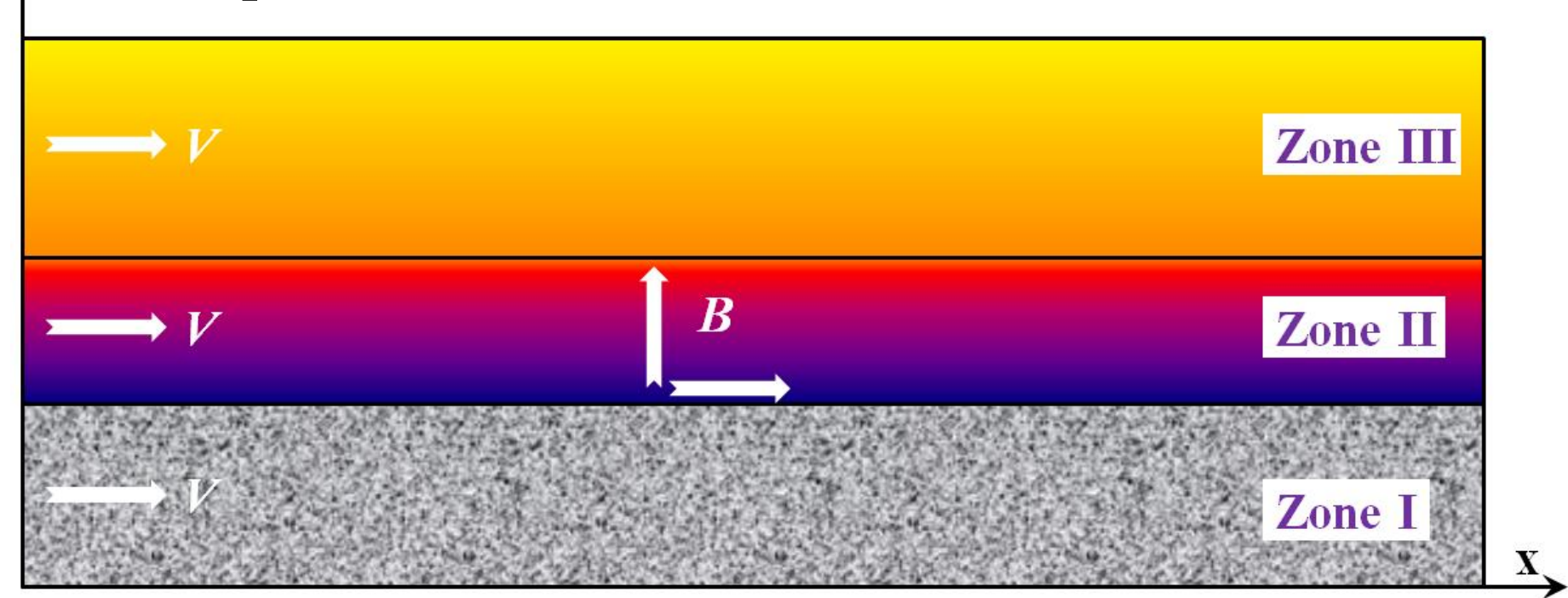
$F$  [1/m] - inertial *Forchheimer* coefficient

➤ **porous media:** introduced *in a certain zone* within the computational domain

➤ **porosity of media:** introduced as *holes/vanes* at some angle to *change the main flow direction*

# OpenFOAM VOF-MHD Modelling

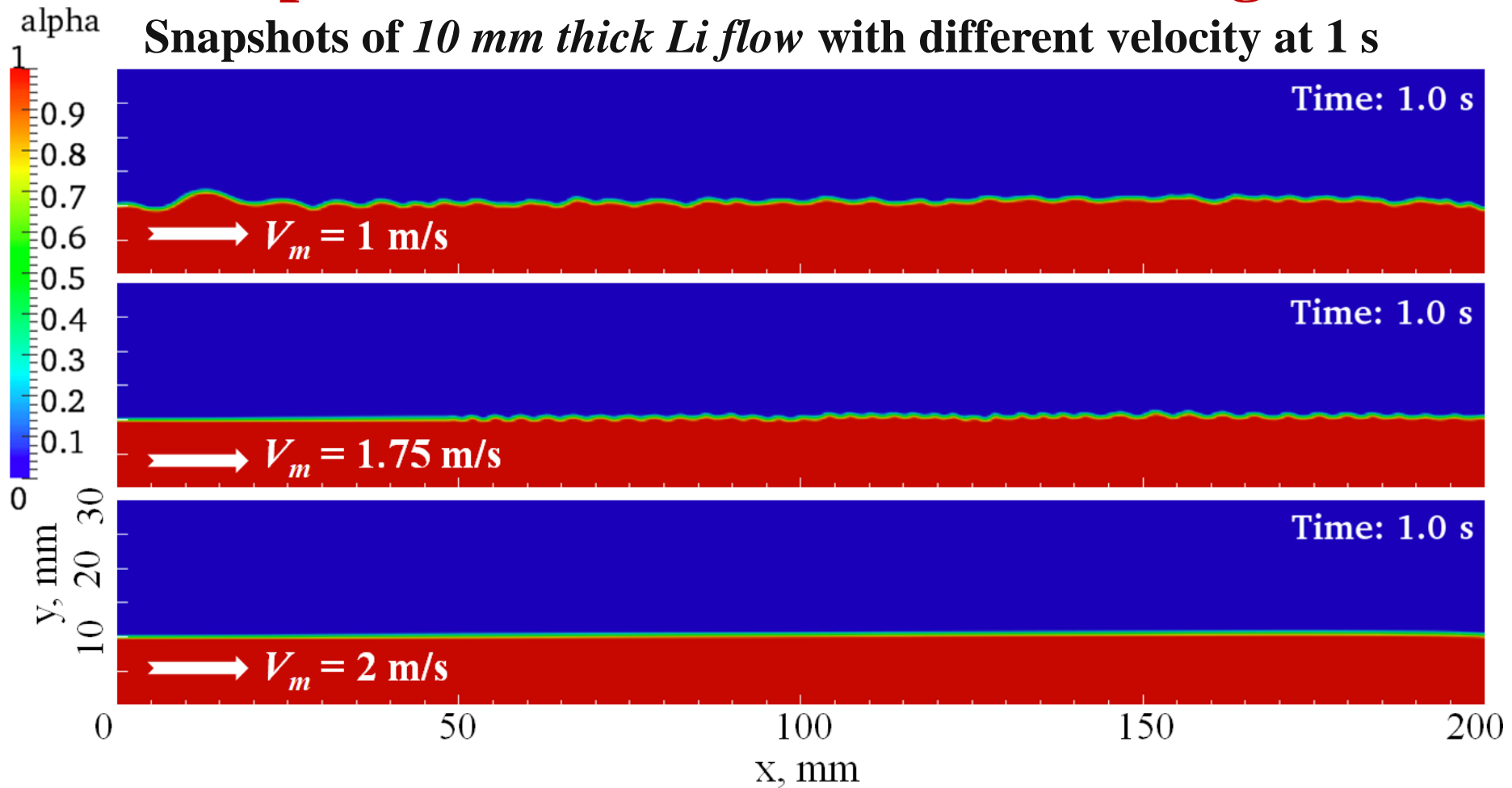
Set up of CFD 2D domain:



- Zone I: porous or nonporous, Li flow
- Zone II: nonporous, gas or Li flow
- Zone III: nonporous, gas flow

# OpenFOAM VOF-MHD Modelling

Snapshots of 10 mm thick Li flow with different velocity at 1 s

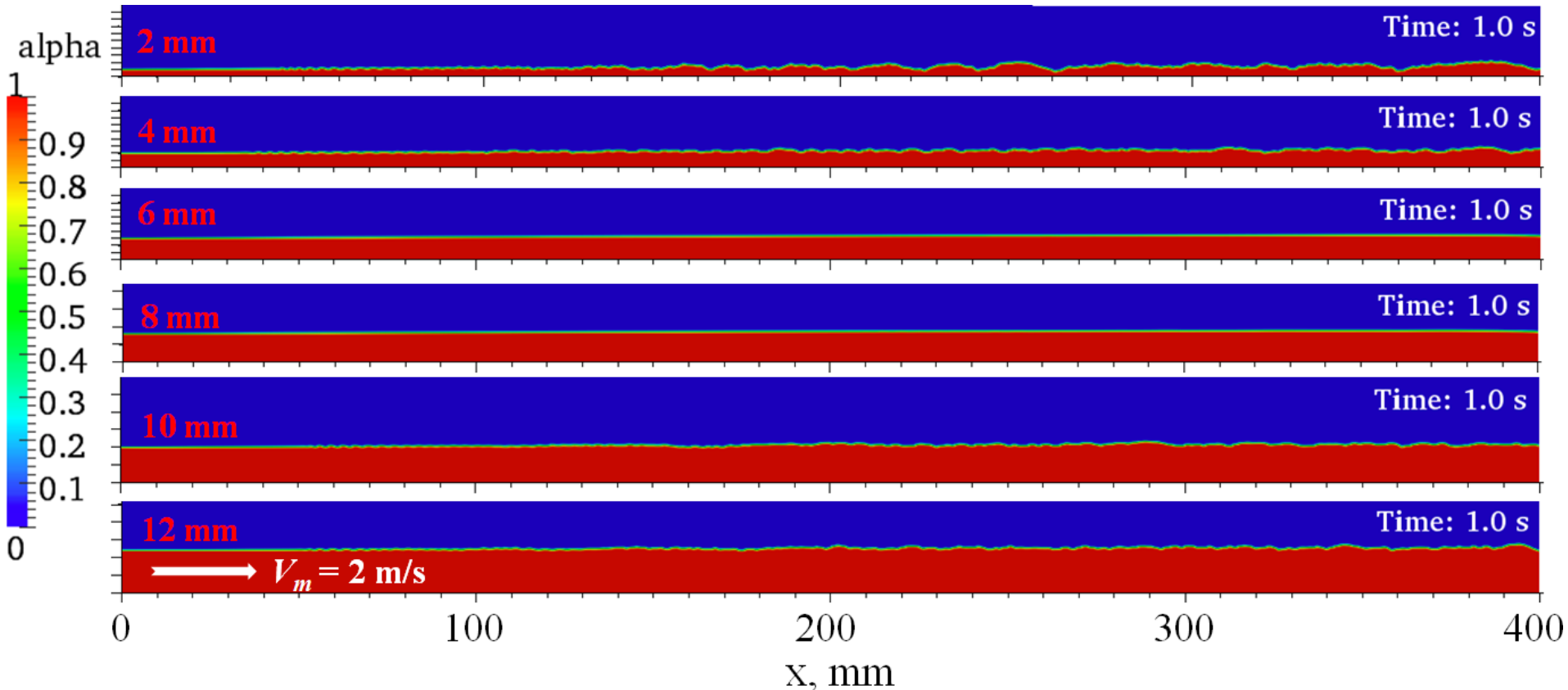


- for 1 m/s: *wavy surface* over the entire x-range of 200 mm
- for 1.75 m/s: *portion of flat surface* at  $x < 50 \text{ mm}$
- for 2 m/s: *flat surface* over the entire x-range of 200 mm



# OpenFOAM VOF-MHD Modelling

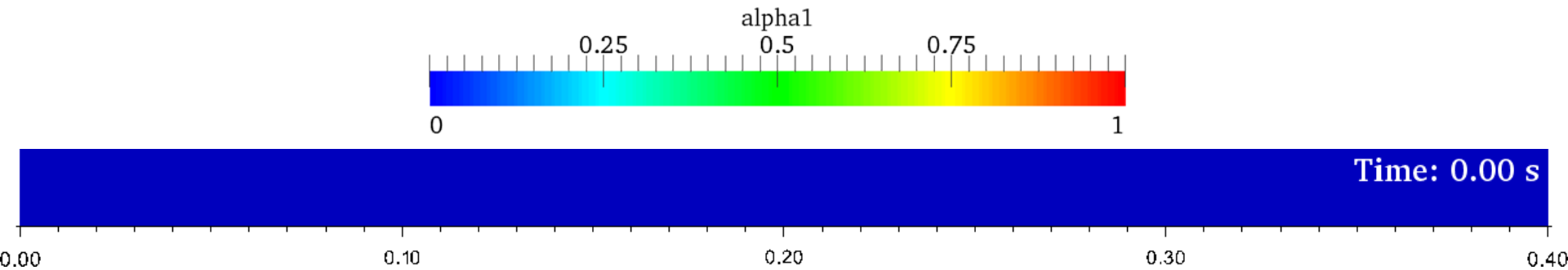
### Snapshots of *Li* flow of different thickness with velocity 2 m/s at 1 s



- $\leq 4$  mm thick LM: *long waves* with wavelength  $\sim 10$ - $20$  mm
- $\sim 6 - 8$  mm thick LM: *no waves, flat Li surface*
- $\sim 10 - 12$  mm thick LM: *wavy Li surface* with *short-length waves*
- $10$  mm thick LM: *flat & wavy* Li surface for  $200$  &  $400$  mm x-range

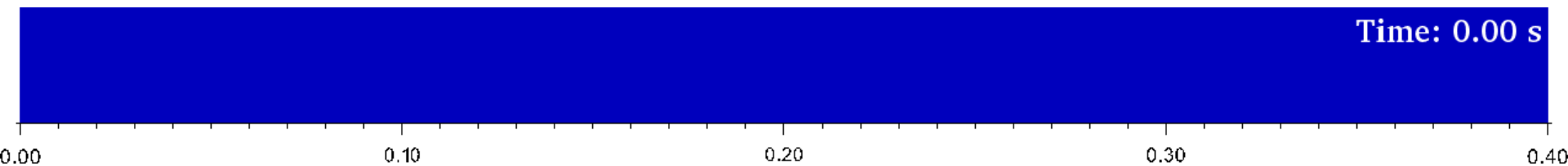
# OpenFOAM VOF-MHD Modelling

## Movies of Li flow on the surface



Li velocity 2 m/s & Li thickness 2 mm

- *large Li blob* in the front of Li flow
- *surface waves* with *long wavelengths* ~10-20 mm

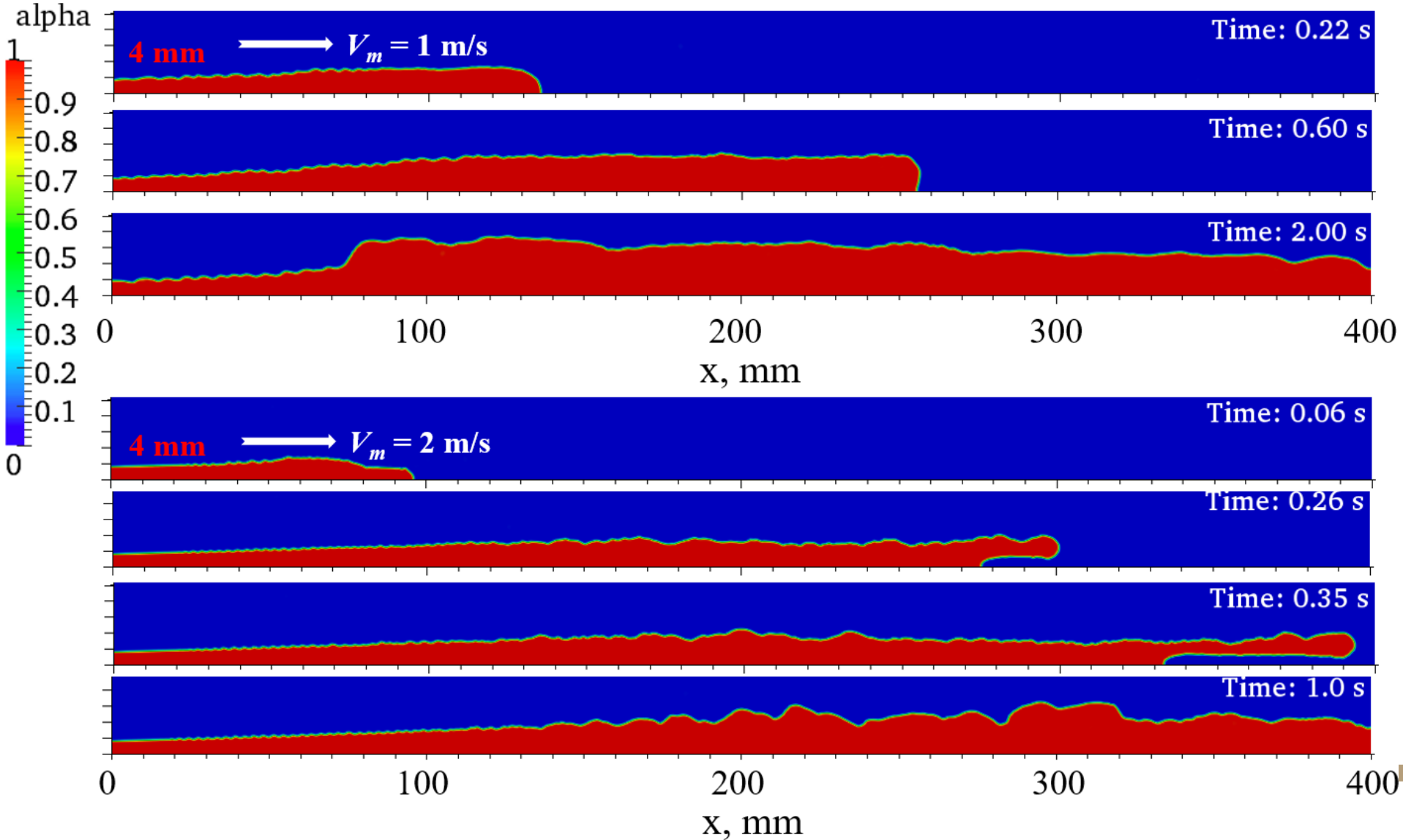


Li velocity 2 m/s & Li thickness 10 mm

- *convex Li shape* in the front of Li flow
- *surface waves* with *very short wavelengths* ~1-5 mm

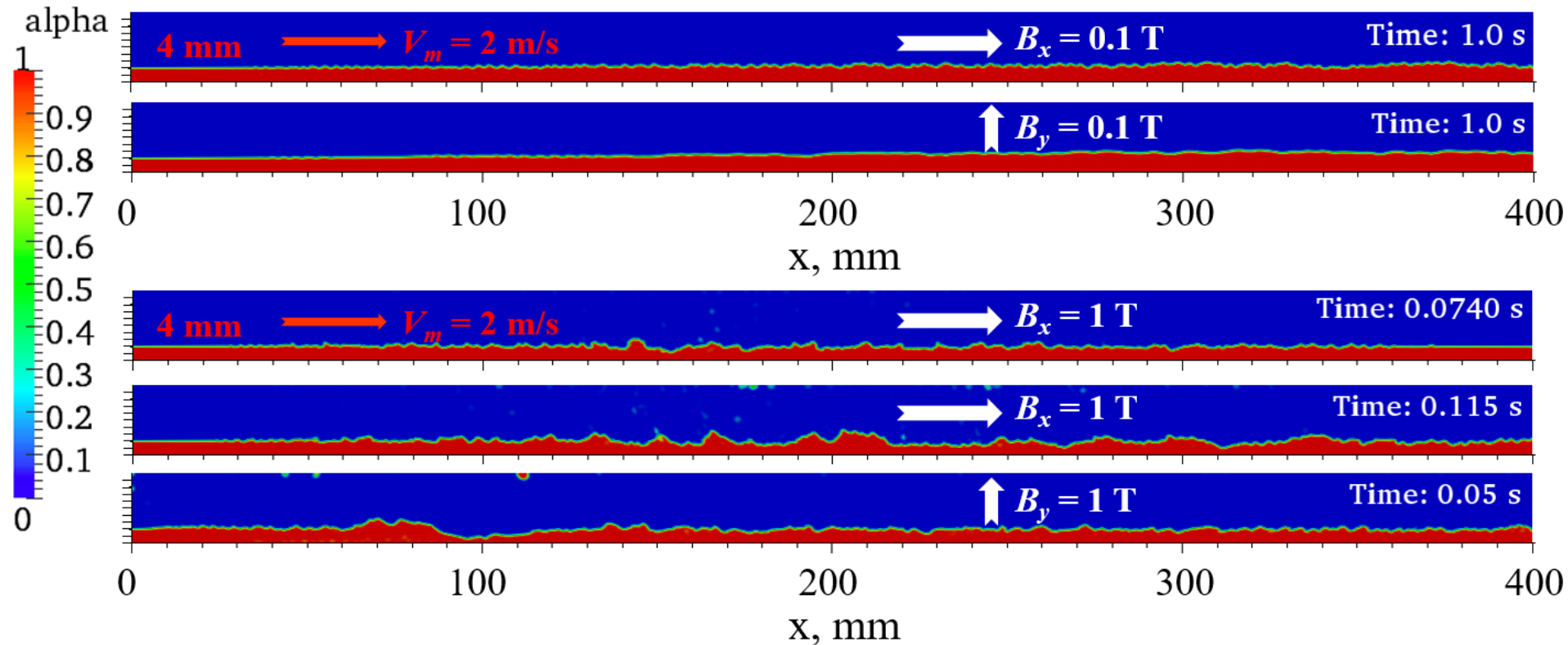
# OpenFOAM VOF-MHD Modelling

Snapshots of 4 mm *Li* flow with 1 & 2 m/s in porous media at various times



# OpenFOAM VOF-MHD Modelling

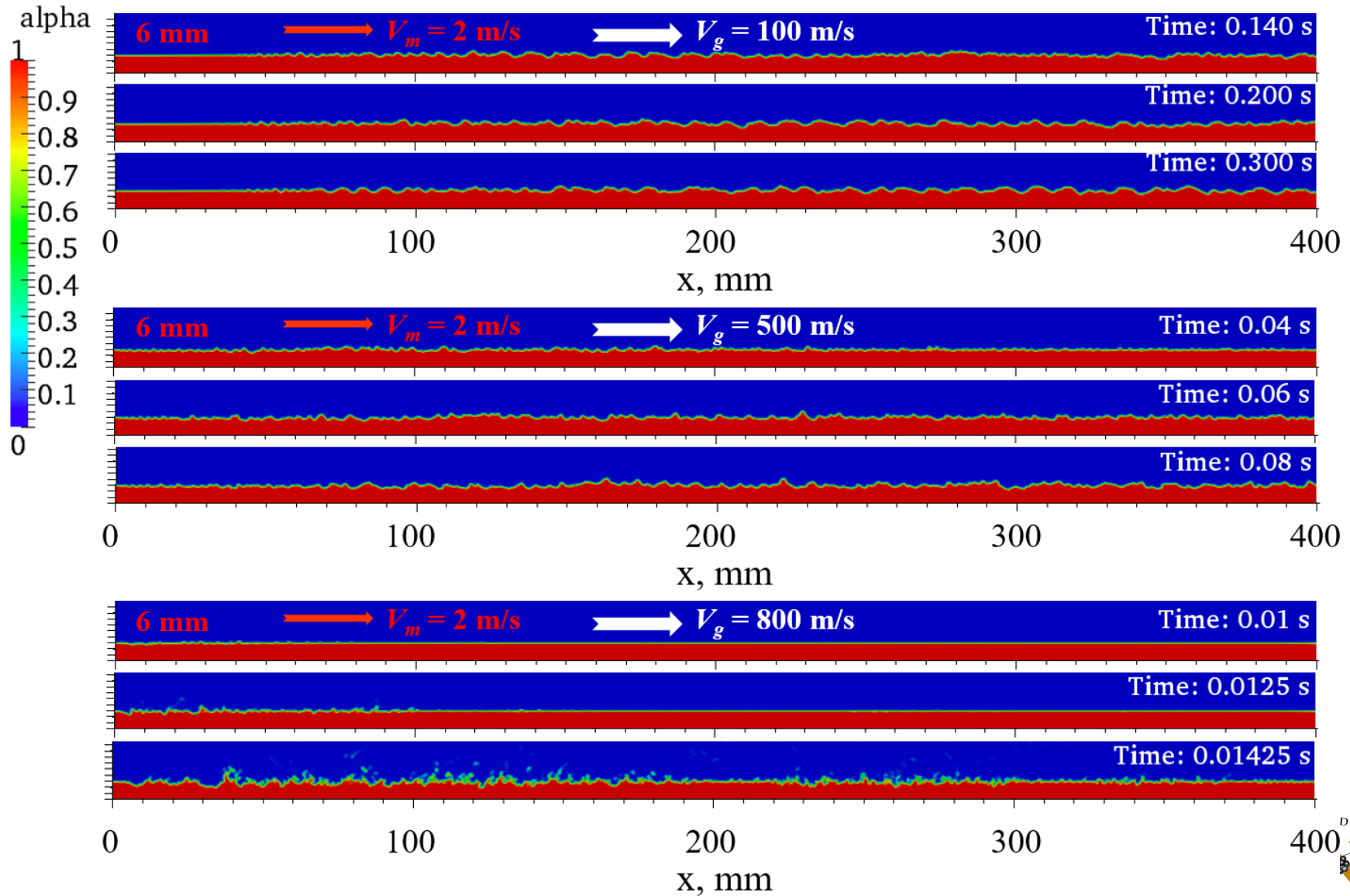
Snapshots of 4 mm *Li* flow with 2 m/s in the magnetic field of 0.1 & 1 T



- $B_x = 0.1$  T: *short-length waves* similar to those in the absence of B
- $B_y = 0.1$  T: *long-length waves* with *small amplitude*
- $B_x = 1$  T: *short-length ripples* with *ejection of droplets* from peaks
- $B_y = 1$  T: *droplet ejection* at early stage of wave development

# OpenFOAM VOF-MHD Modelling

Snapshots of *Ar* gas flow with various speed over 6 mm Li flowing with 2 m/s



# OpenFOAM VOF-MHD Modelling

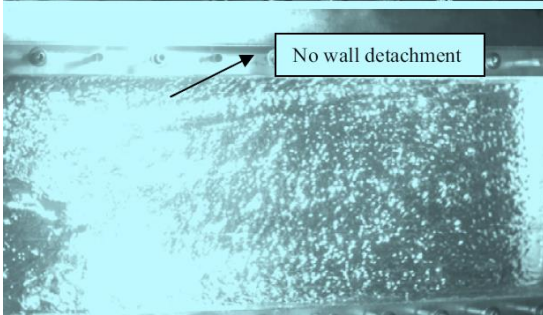
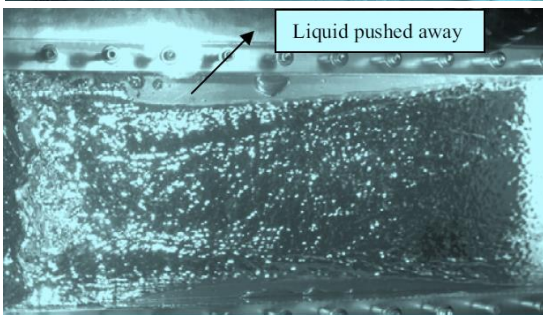
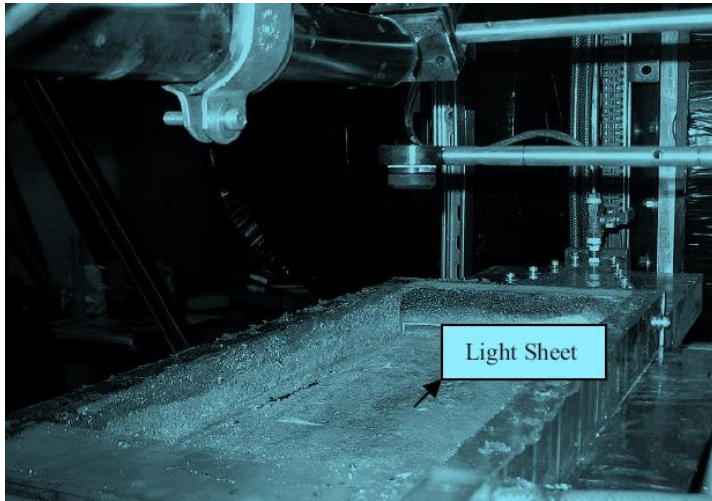
## ■ Summary

- Li flow in *the absence of gas (plasma) stream*
  - flat Li surface near inlet: *extending with increasing Li flow speed*
  - regime of stable (no waves) Li flow: *at a certain Li thickness*
  - Li flow in porous media: *dependent on porosity & permeability*
  - **B** effects: development of *short-wavelength ripples with ejection of droplets* at a certain threshold of **B**
- Li flow in *the presence of gas (plasma) stream*
  - faster development of surface waves: *with increasing gas speed*
  - wavelength decrease: *with increasing speed of gas stream*
  - at very high gas (plasma) speed: *growth of small ripples and their disintegration into droplets*

# reserved slides



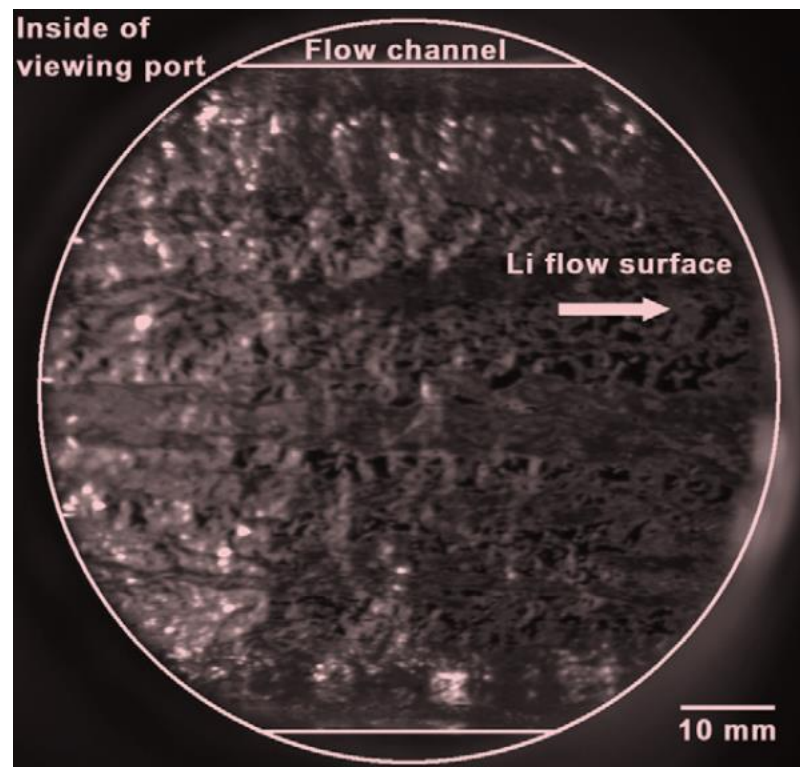
# Introduction: LM Flow in Magnetic Field (Exp.)



- SS channel:  $L \times W \times H \rightarrow 400 \times 200 \times 2$  mm
- fusion relevant magnetic field
- LM: Ga-67%, In-20.5% & Sn-12.5%
- hydraulic jump of LM emerging from nozzle: at a particular downstream location
- higher initial velocity (used range 1.0 m/s - 3.0 m/s): farther location of jump from inlet
- wall normal magnetic field: caused LM stream to pinch inward
- formation of separation spots: due to pinching
- increase in LM thickness at 160 mm downstream at high velocity:  $\sim 1$ -2 mm

Narula et al., A Study of Liquid Metal Film Flow, Under Fusion Relevant Magnetic Fields, Fusion Sci. Technol. **47**, 564 (2005) [UCLA]

# Introduction: Liquid Li flow in IFMIF (Exp.)



- **IFMIF**: generation of 14 MeV neutrons by 40 MeV deuteron beam injection into high-speed liquid Li plane jet
- **goal**: to study Li surface fluctuations in order to predict the neutron flux
- **flat plane jet**: thickness of 25 mm, width of 70 mm, guided along a concave wall with a radius of 250 mm
- **Li velocity range**: from 1 to 15 m/s
- **magnitude of Li surface oscillation**: from 2.2 to 2.9 mm at 175 mm downstream from nozzle at velocity of 15 m/s
- **wavelength of Li surface waves**: ~2-3 mm

*Kanemura et al., Investigation of free-surface fluctuations of liquid lithium flow for IFMIF lithium target by using an electro-contact probe, Fusion Eng. Des. 82, 2550 (2007) [Osaka University]*