MODELING DOPAMINE USING TD-LEARNING

Reinforcement Learning

Quantitative Life Sciences

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Outline

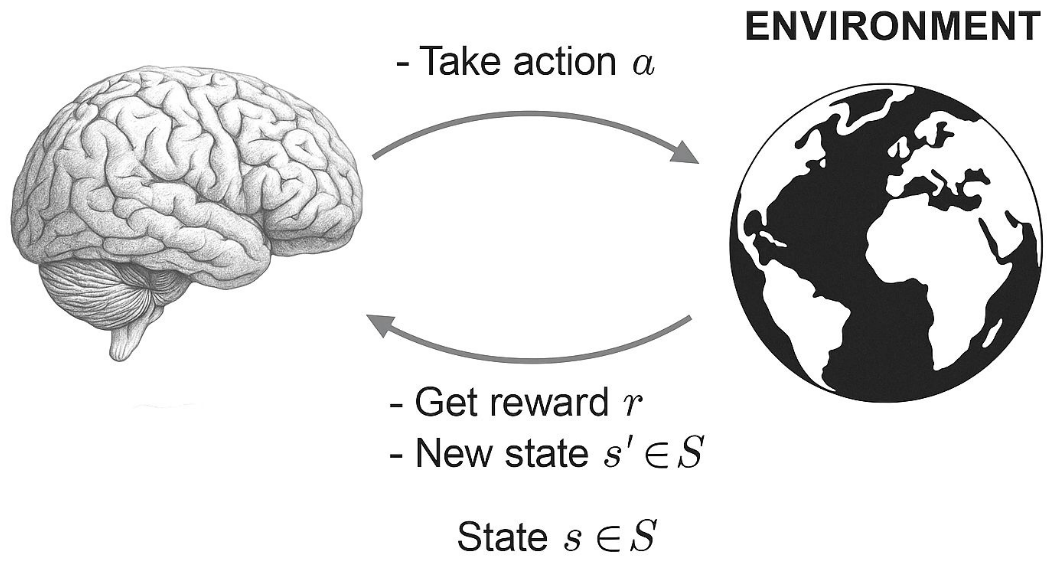
"What we observe is not nature itself, but nature exposed to our method of questioning."
- Werner Heisenberg

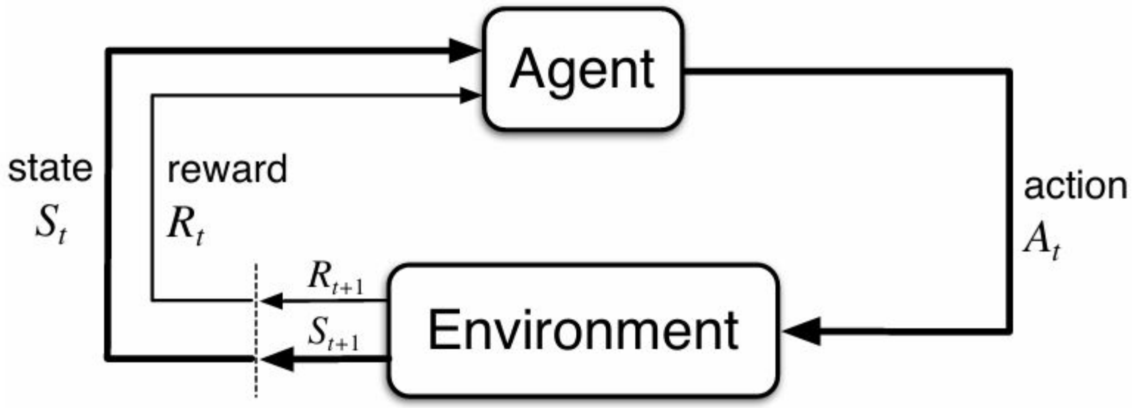
1. Motivation: Brain & Reinforcement Learning
2. Classical TD Learning
3. Observations
4. Distributional TD Learning
5. Neuroscience Validation

Neuromatch Academy. (2020). Available at: https://compneuro.neuromatch.io, The Universe Is a Machine That Keeps Learning, Scientists Say

Motivation: **Brain & Reinforcement Learning**

- The connection between RL and Brain is intuitive.

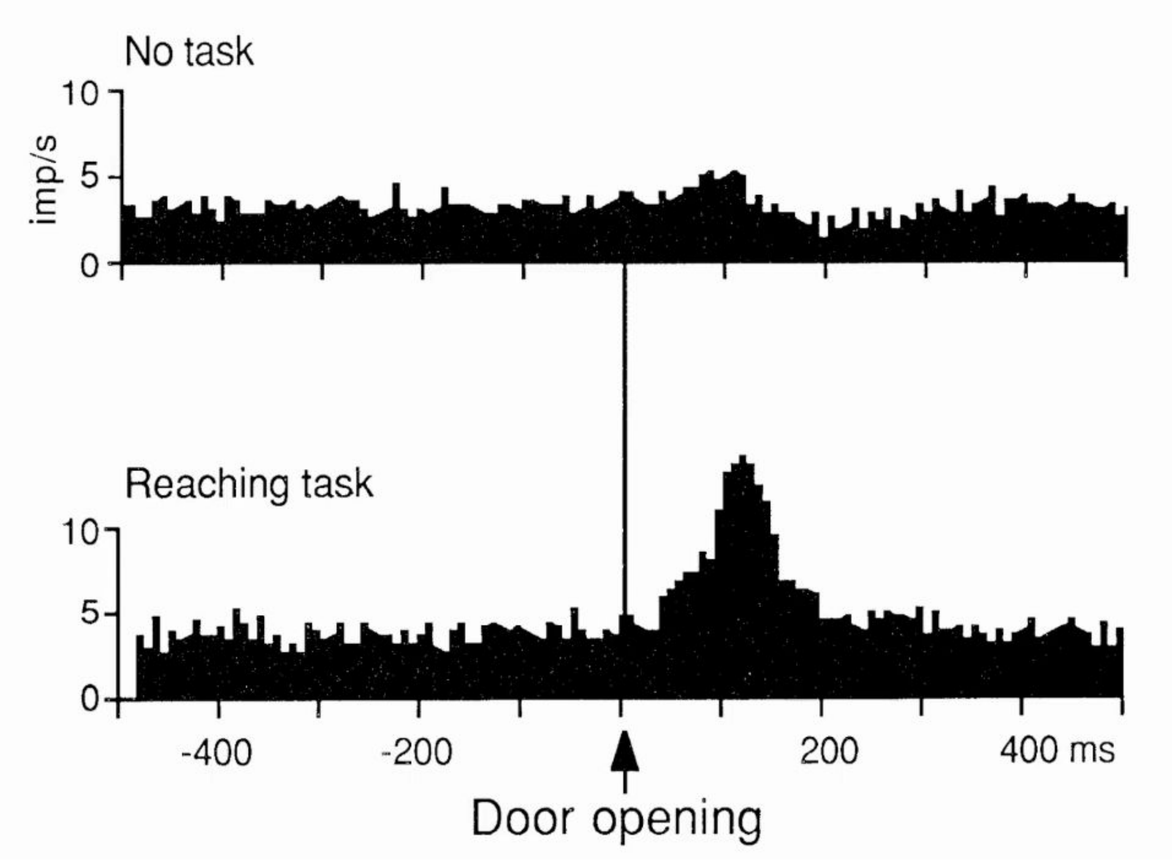


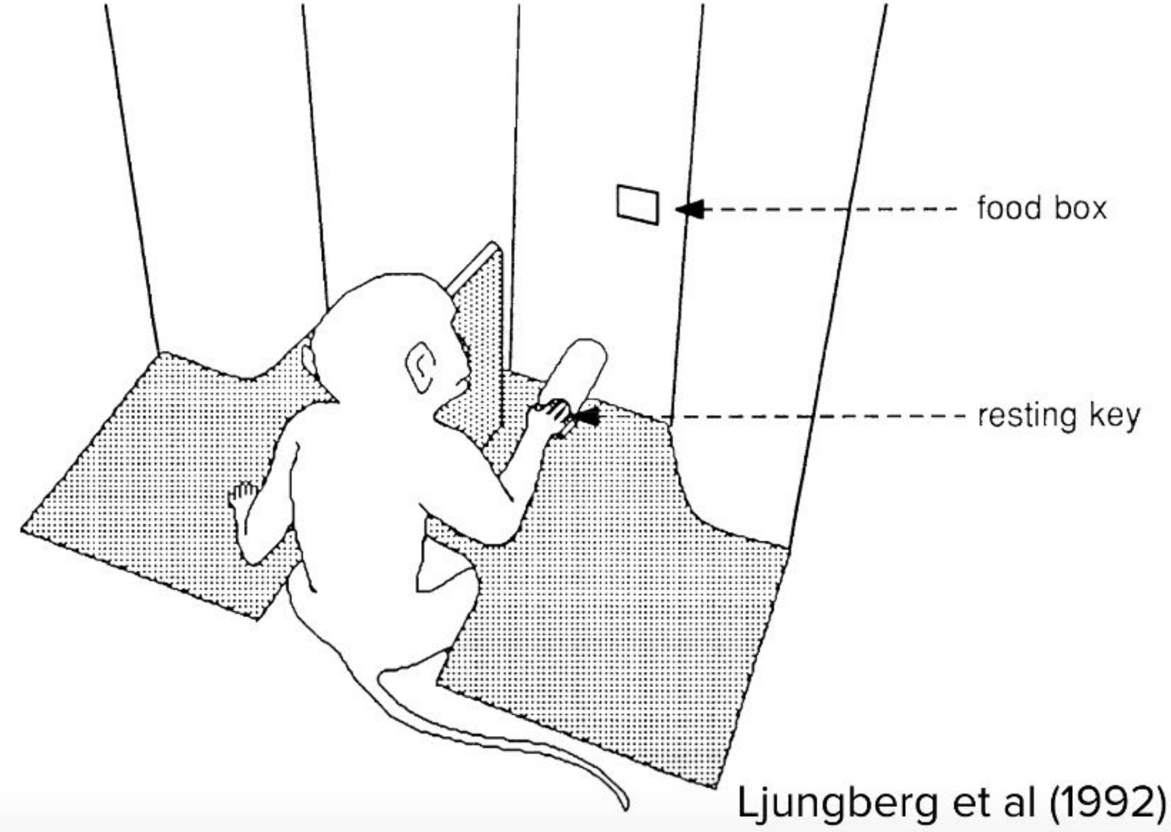


- This project replicates two foundational studies:
  - **Schultz et al. (1997):** Showed that dopamine neurons encode reward prediction errors.
  - **Dabney et al. (2020):** Revealed that dopamine neurons encode not just a single expected value, but a *distribution* of possible rewards.
- There are more ideas, model-based & model-free. **Hypothesis, our hippocampus develops models that the cortex trains while we are sleeping.**

Neuromatch Academy. (2020). Available at: https://compneuro.neuromatch.io

1) The agent goes through episodes with no need to take actions.
2) Each episode is a fixed sequence of states: S0→S1→S2→···→Sn. In each episode: A **Conditioned Stimulus (CS)** is shown early and an **Unconditioned Stimulus (US)** or reward.
3) The agent's task is to **learn to predict future rewards** based on the current state.





Ljungberg et al (1992)

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Classical TD-Learning: Framework

Return:

$$G_t = \sum_{k=0}^{\infty} \gamma^k r_{t+k+1}$$

Value Function:

$$V_\pi(s) = E[G_t \mid s_t = s] \qquad V_\pi(s) = E[r_{t+1} + \gamma V_\pi(s_{t+1}) \mid s_t = s]$$

$$V_\pi(s) = \sum_a \pi(a \mid s) \sum_{s',r} p(s', r \mid s)\,[r + \gamma V_\pi(s')]$$

TD-error & Update:

$$\delta_t = R_{t+1} + \gamma V(s_{t+1}) - V(s_t)$$

$$V(s_t) \leftarrow V(s_t) + \alpha\delta_t$$

In essence, first δ_t measures TD-error measures the discrepancy between values at time t and t+1, then we update V.

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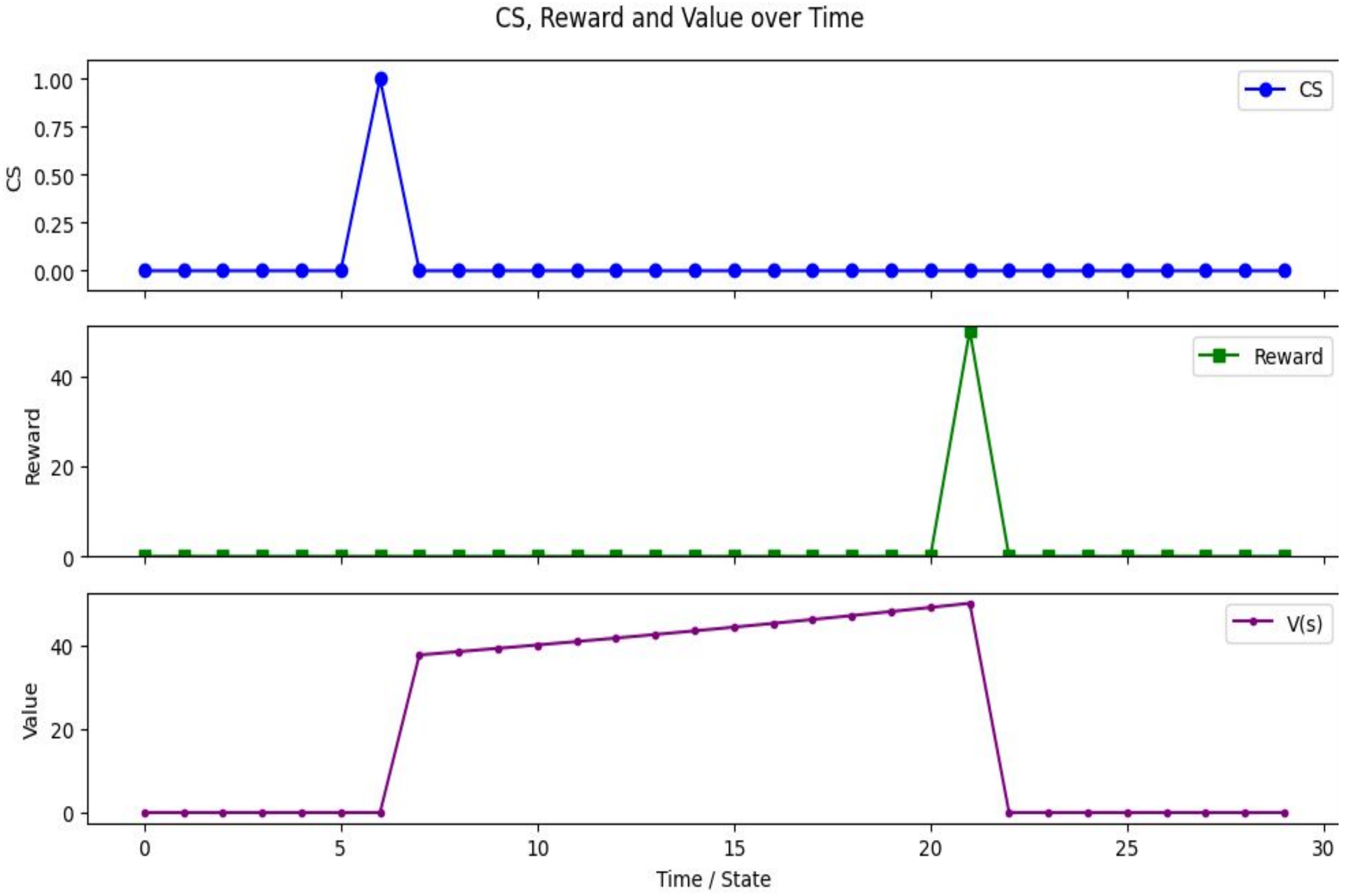
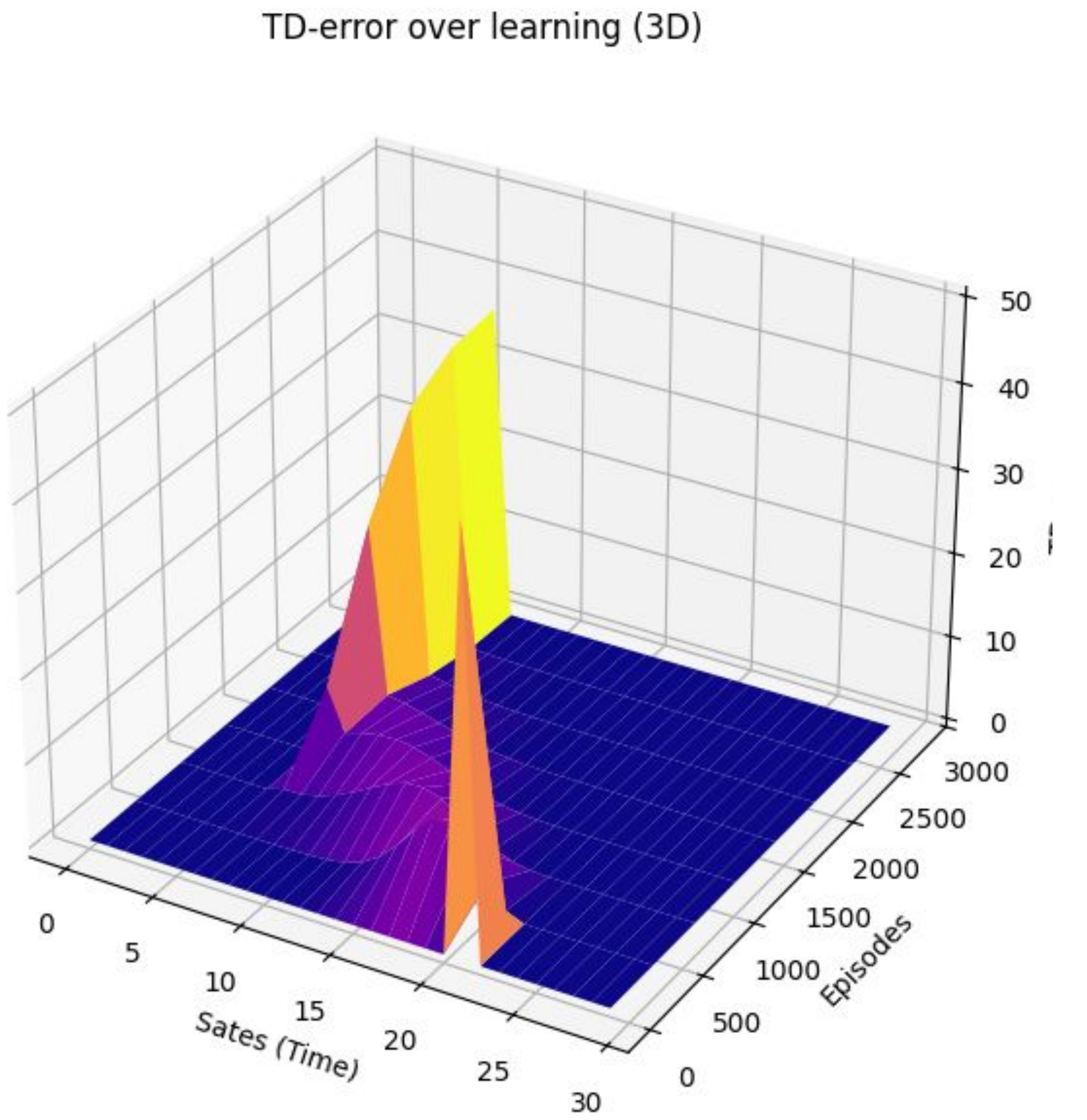
TD-Learning: Case 1 guaranteed rewards





After many episodes, CS starts to predict the future reward, so when the reward arrives you were already expecting the reward and there is no prediction error.

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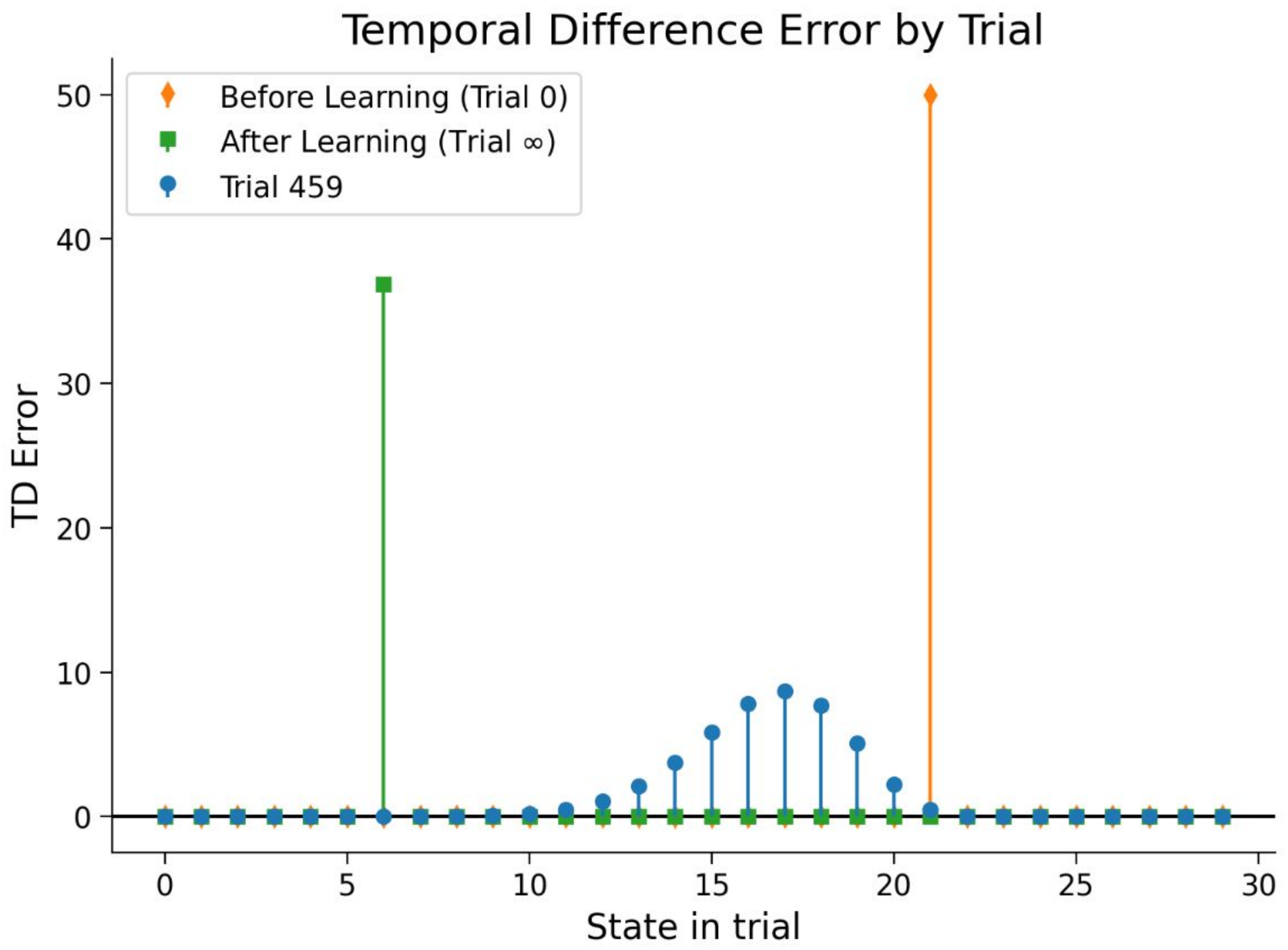
On TD-learning,

- α Controls learning rate: How much to trust each TD-error.
- γ Discounts future rewards: Higher values mean more future-oriented learning.

$$\delta_t = R_{t+1} + \gamma V(s_{t+1}) - V(s_t)$$

$$V(s_t) \leftarrow V(s_t) + \alpha \delta_t$$



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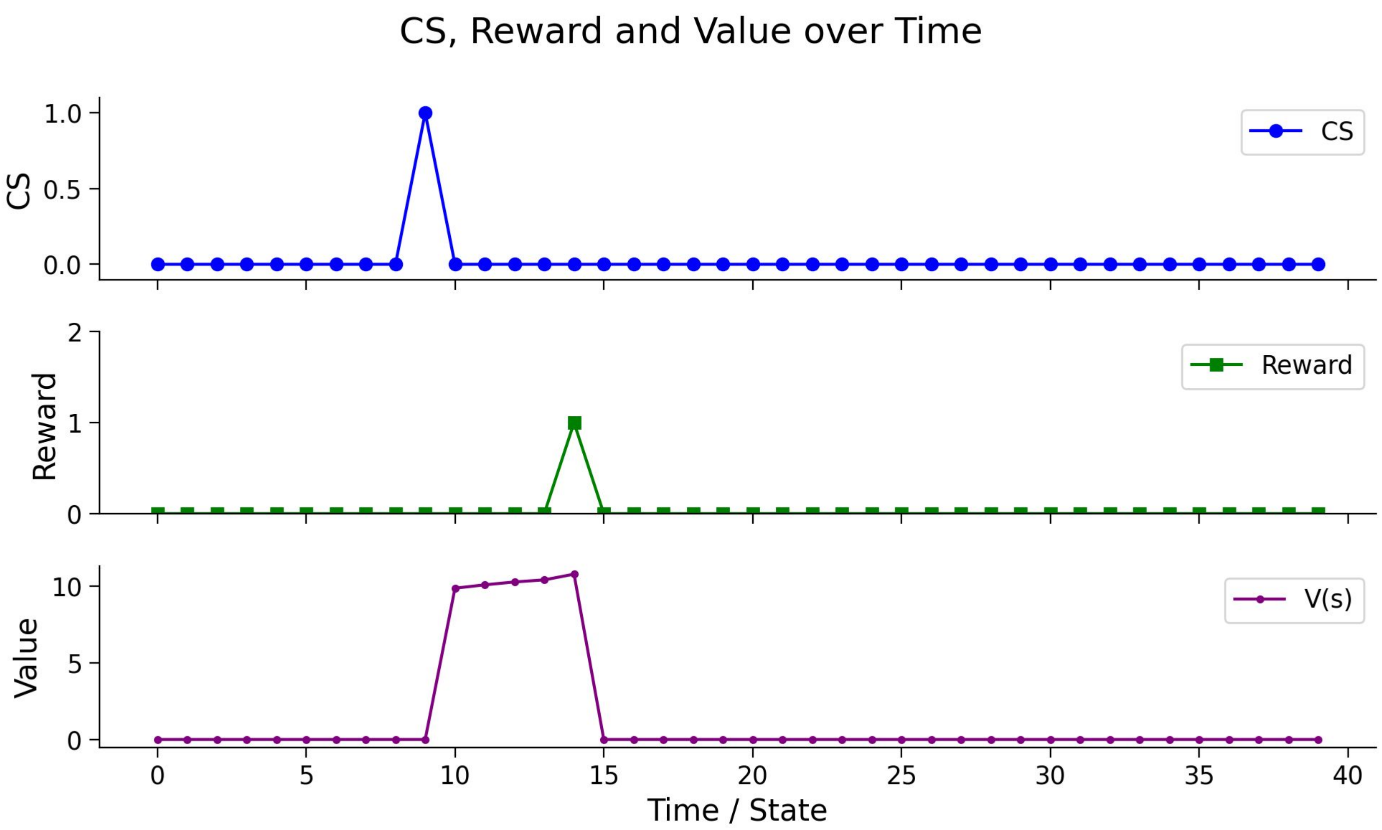
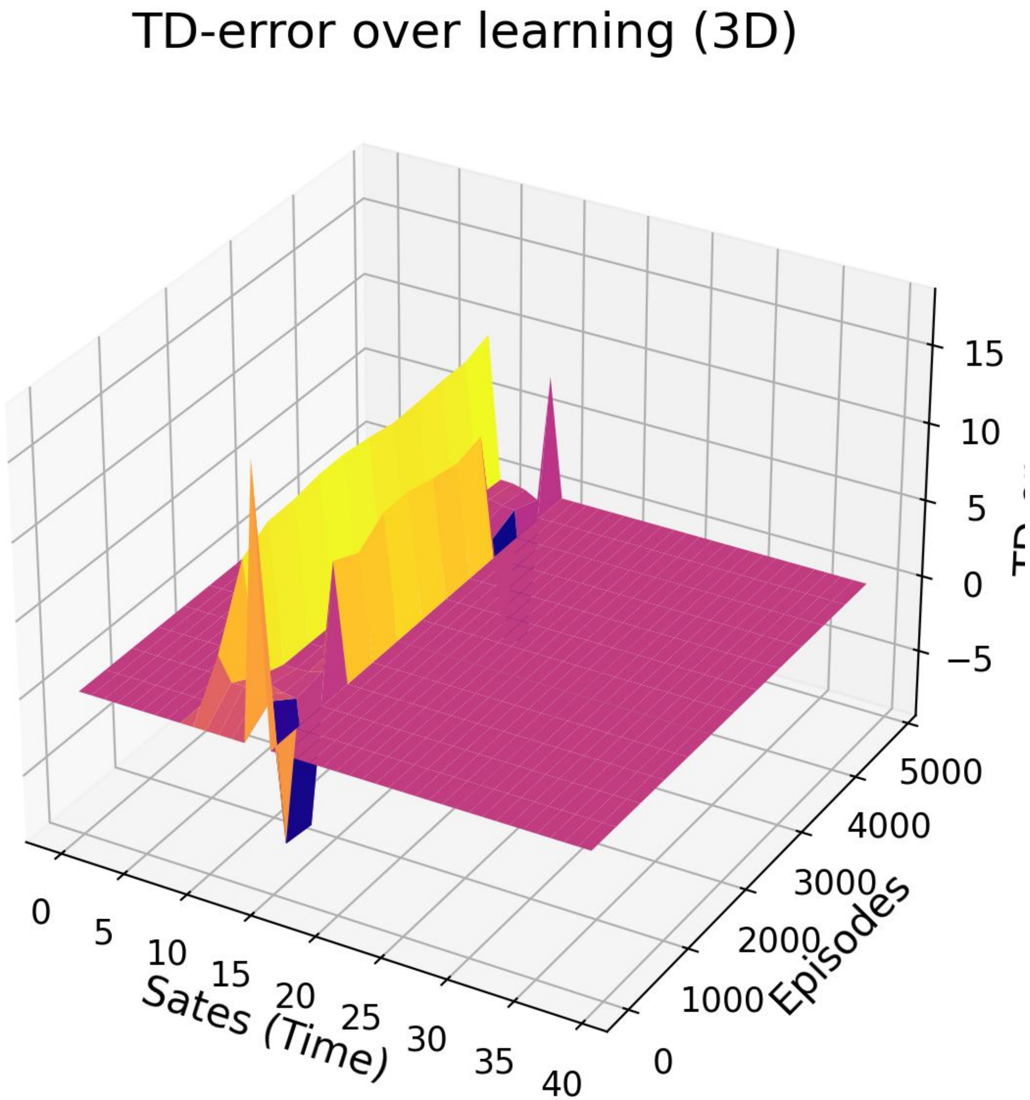
TD-Learning: Case 2 Multiple Reward Magnitudes





New Phenomena: Negative TD-Prediction with Variable Rewards:

$$\delta_t = 6 + \gamma \cdot 0 - 14 = -8$$

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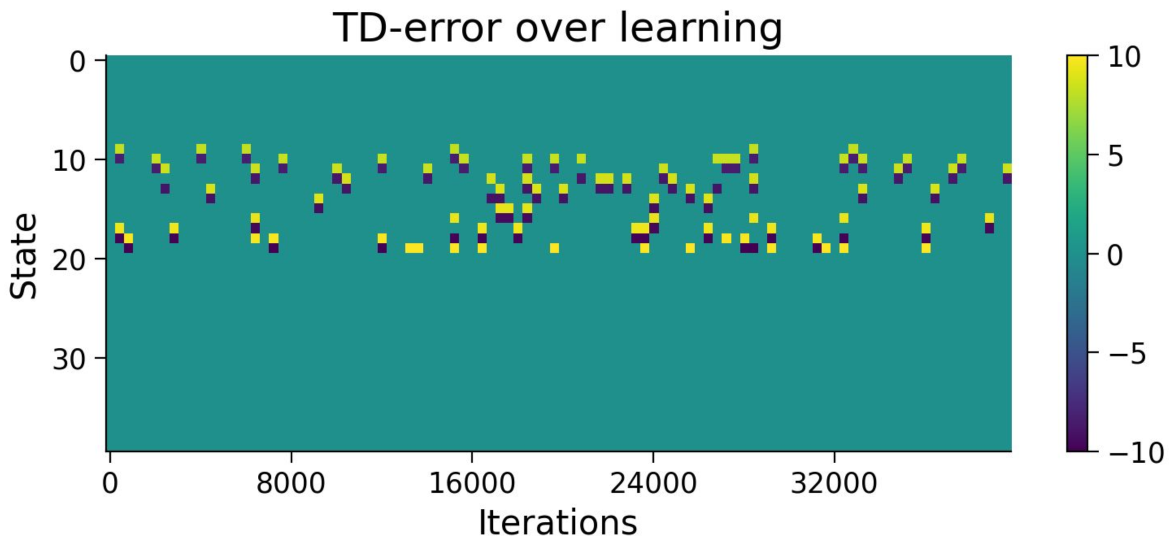
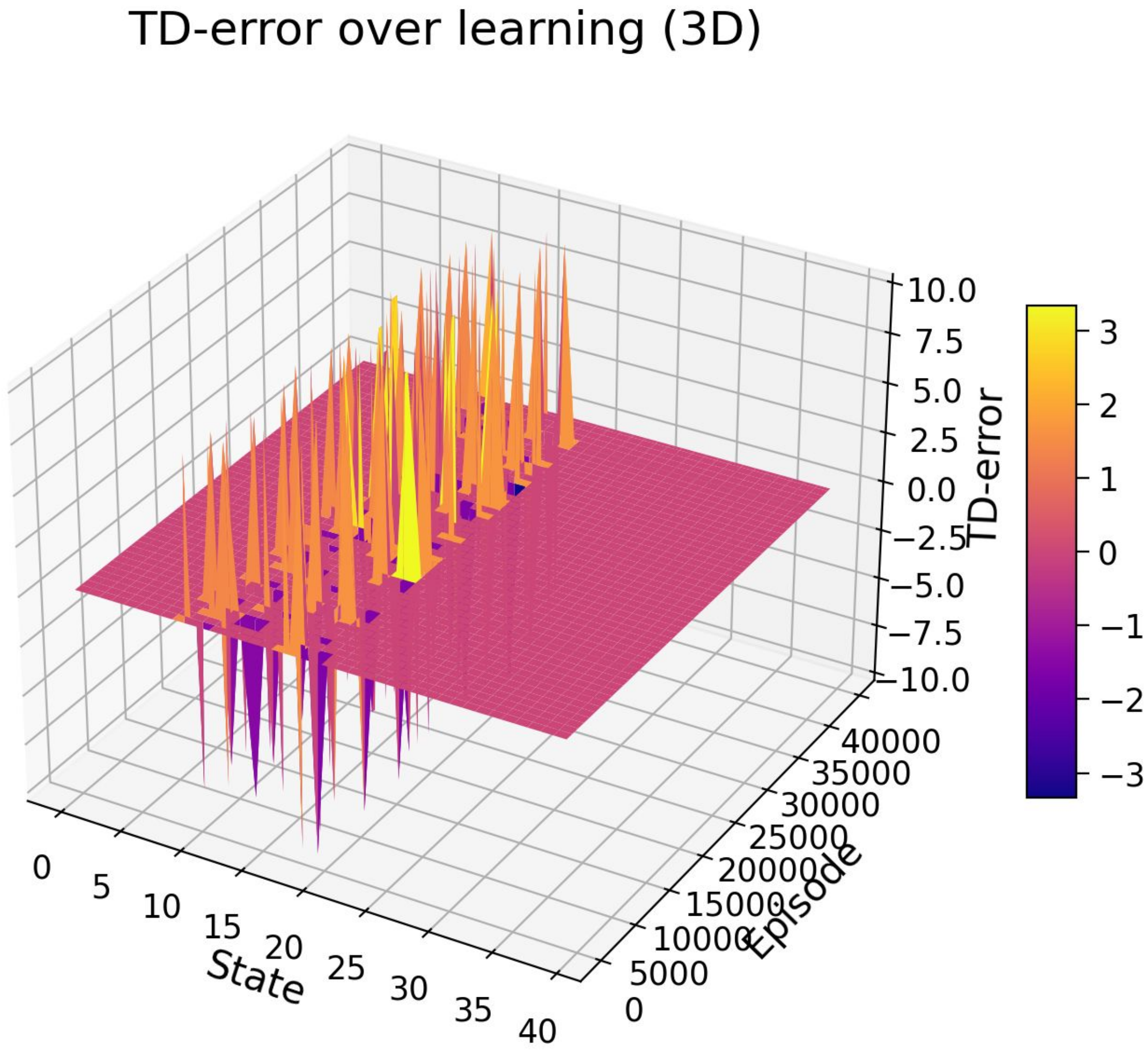




Curiously, (p, α) are highly relevant for the learning process.
Let's consider a macroscopic measures that captures the total variation in learning.

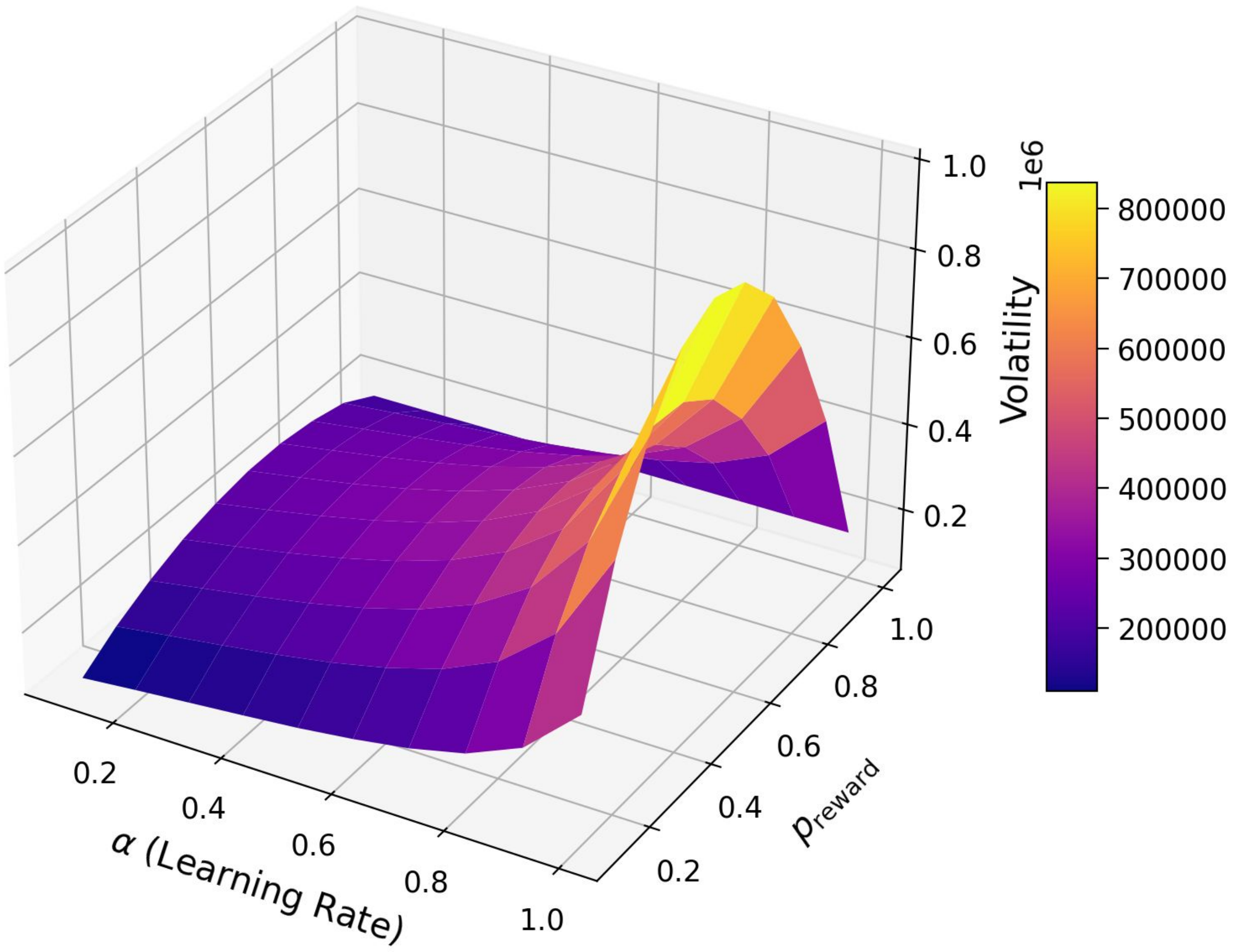
TD-Learning: Case 3 Probabilistic Rewards

$$\textbf{Cumulative Volatility} = \sum_e \sum_t \left|\delta_t^{(e)}\right|$$

🧠 Conclusions:

- Stochastic and deterministic rewards with the same expectation lead to equivalent TD updates.
- Cumulative volatility peaks when learning is fast (α high) and the environment is uncertain (p≈0.5).
- A population with heterogeneous (α,p) can model learning society.
- There may exist a nonlinear function p(α) reflecting how our rewards society reacts to your learning ability.

3D Surface: Cumulative TD-error Volatility Across α and p_{reward}



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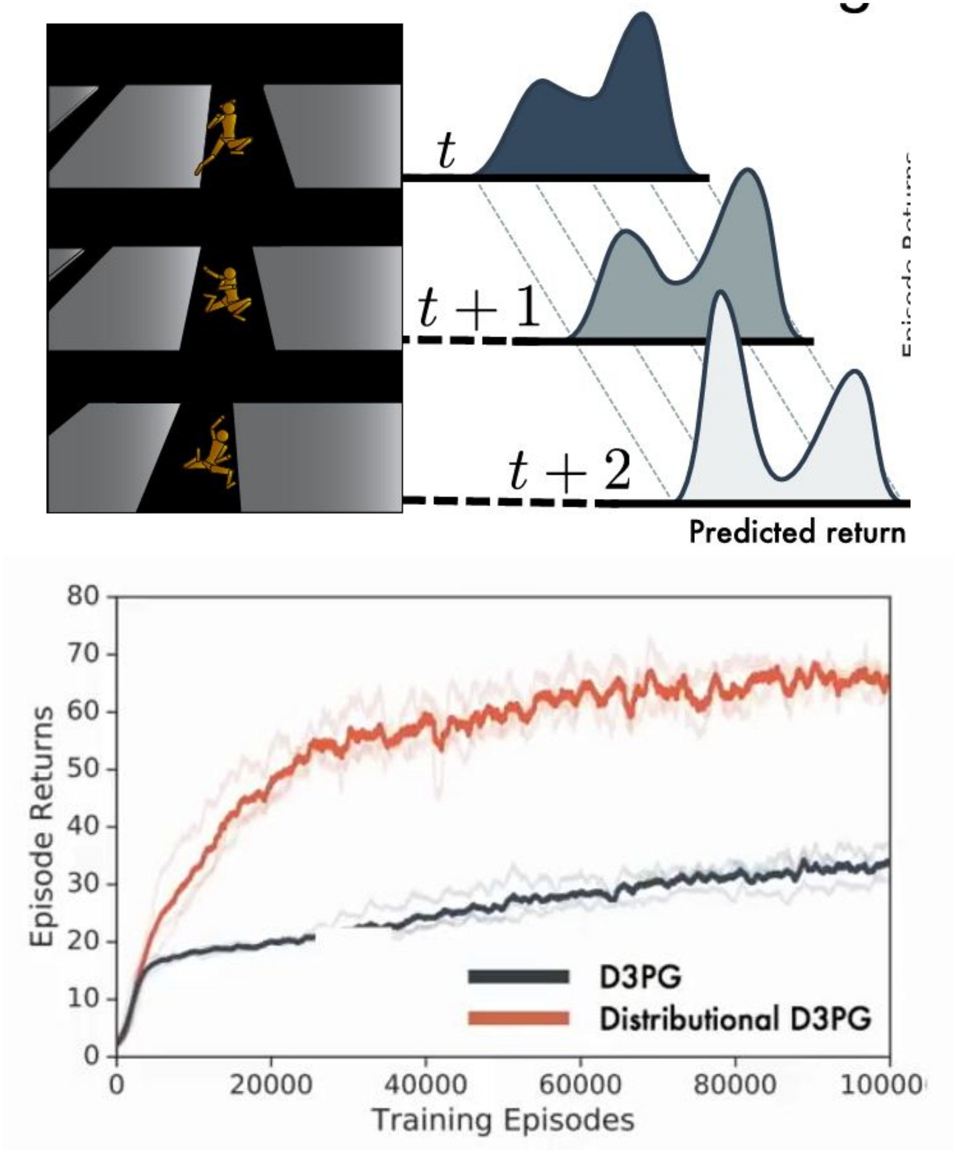
Is Learning Just a Single Value?

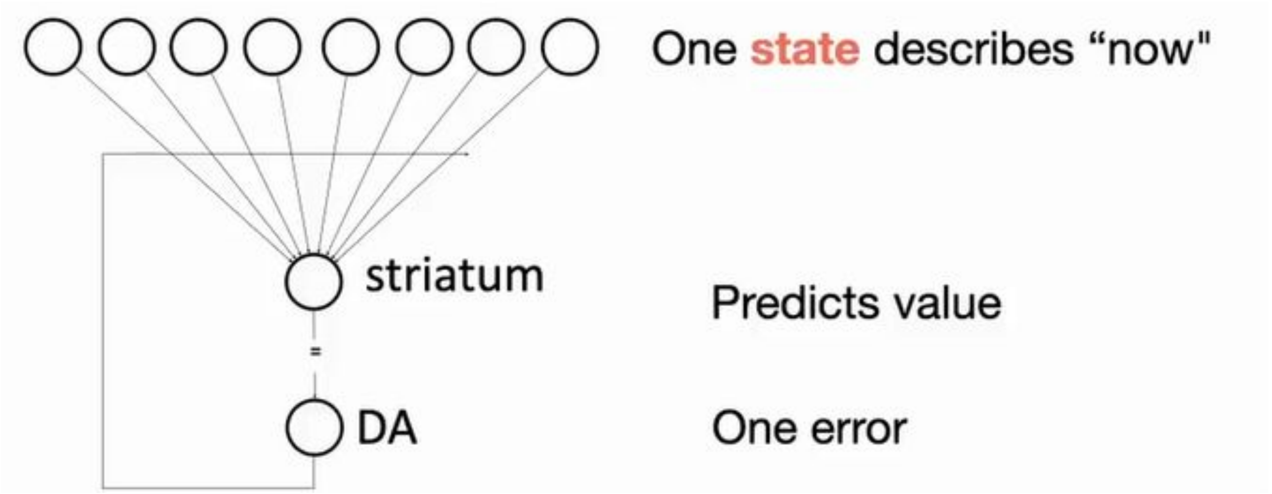
- In standard TD-learning, learning is reduced to estimating a **single expected value** of future rewards. However, in many real-world scenarios, rewards are variable and uncertain—better represented by a **distribution of predicted outcomes** rather than a single average. This idea is illustrated in the figure, adapted from [2].

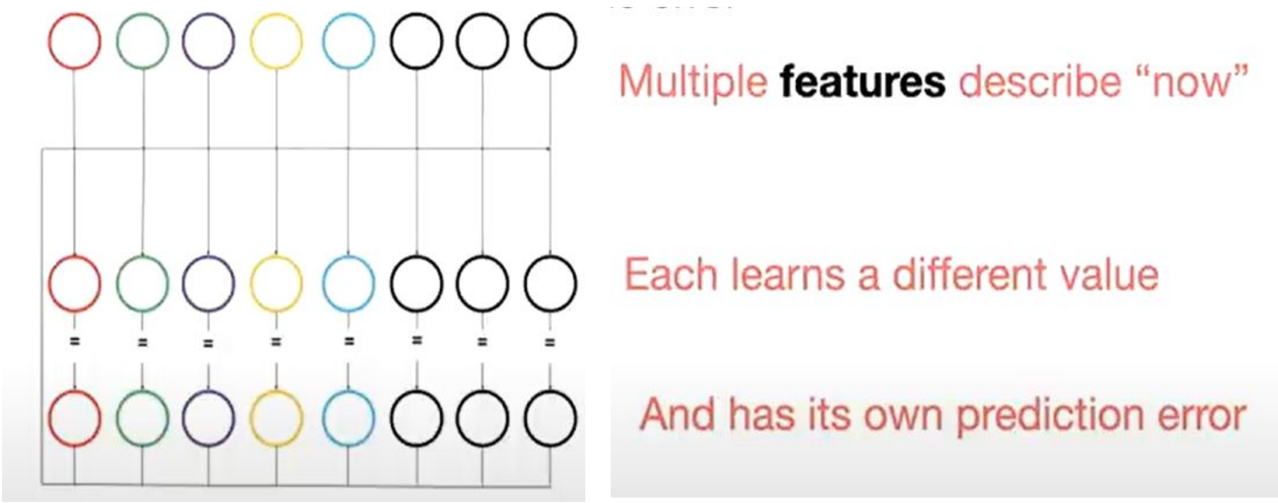


- Notably, distributional RL can increase performance in deep learning systems.

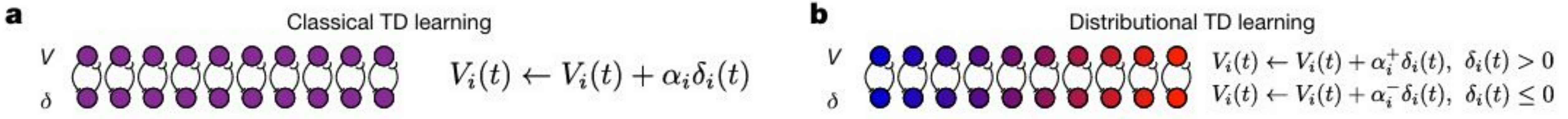
[2] A distributional code for value in dopamine-based RL. Dabney et al., Nature, 2020.

Standard TD-Learning vs Distribution TD-Learning





Insight: In essence each cell predicts different state



[2] A distributional code for value in dopamine-based RL. Dabney et al., Nature, 2020.

Distribution TD-Learning: Framework

🧠 **Standard TD Learning with Utility Function**

$$U(r) = \frac{f_{\max} \cdot \mathrm{sign}(r) \cdot |r|^{0.5}}{|r|^{0.5} + \sigma^{0.5}}$$

🧠 **TD-error & Update**

$$\delta = U(r) - V_i$$

$$V_i \leftarrow V_i + \alpha_i \cdot \eta \cdot \delta$$

🧠 **Spiking Reward prediction error**

$$R_{i,j} = \alpha_i \cdot (U(r_j) - V_i)$$

$$R_{i,j}^{\text{norm}} = \frac{R_{i,j}}{\mathrm{std}(R_{i,:})}$$

🧠 **Distribution TD Learning, Utility Function:**

$$U(r) = \frac{f_{\max} \cdot \mathrm{sign}(r) \cdot |r|^{0.5}}{|r|^{0.5} + \sigma^{0.5}}$$

🧠 **TD-error & Update using "Quantile":**

$$\delta_i = U(r) - Z_i$$

$$Z_i \leftarrow Z_i + \eta \cdot \left(\text{valence}_i \cdot \alpha_i^- + (1 - \text{valence}_i) \cdot \alpha_i^+\right) \cdot \delta_i$$

$$\text{valence}_i = \begin{cases} 1 & \text{if } \delta_i \leq 0 \quad \text{(negative error)} \\ 0 & \text{if } \delta_i > 0 \quad \text{(positive error)} \end{cases}$$

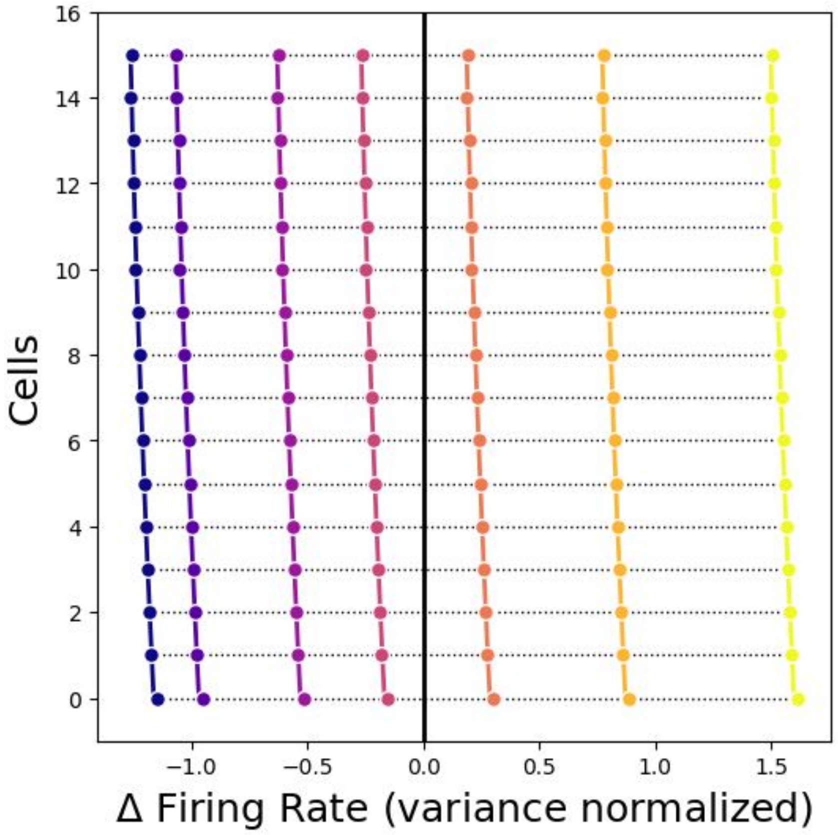
🧠 **Spiking Reward prediction error**

$$\delta_{i,j} = U(r_j) - Z_i$$

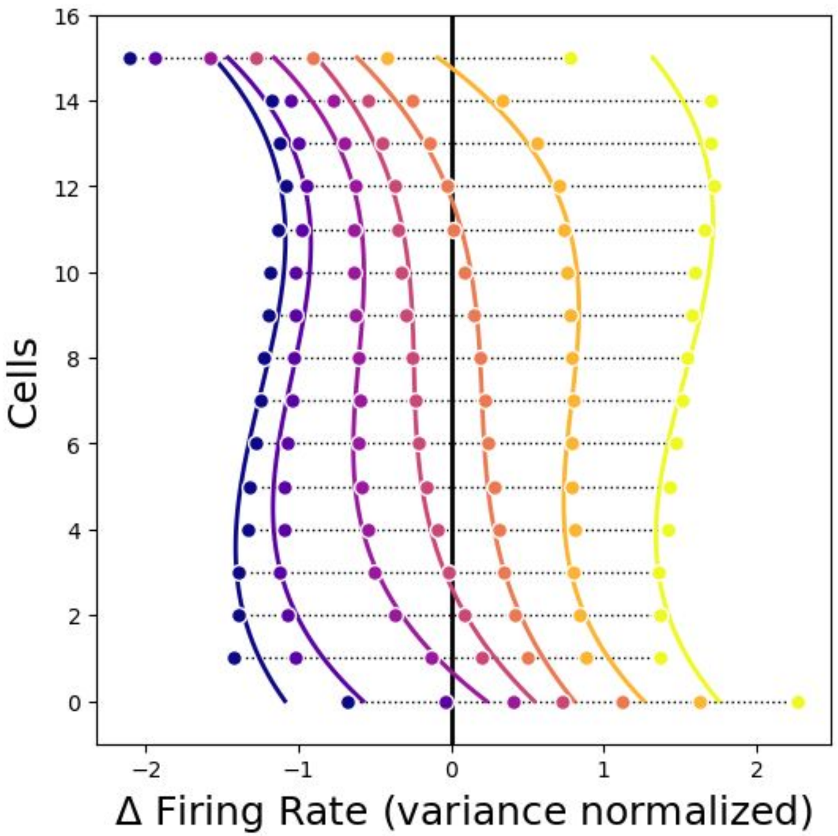
$$R_{i,j} = \left(\alpha_i^- \cdot 1[\delta_{i,j} \leq 0] + \alpha_i^+ \cdot 1[\delta_{i,j} > 0]\right) \cdot \delta_{i,j}$$

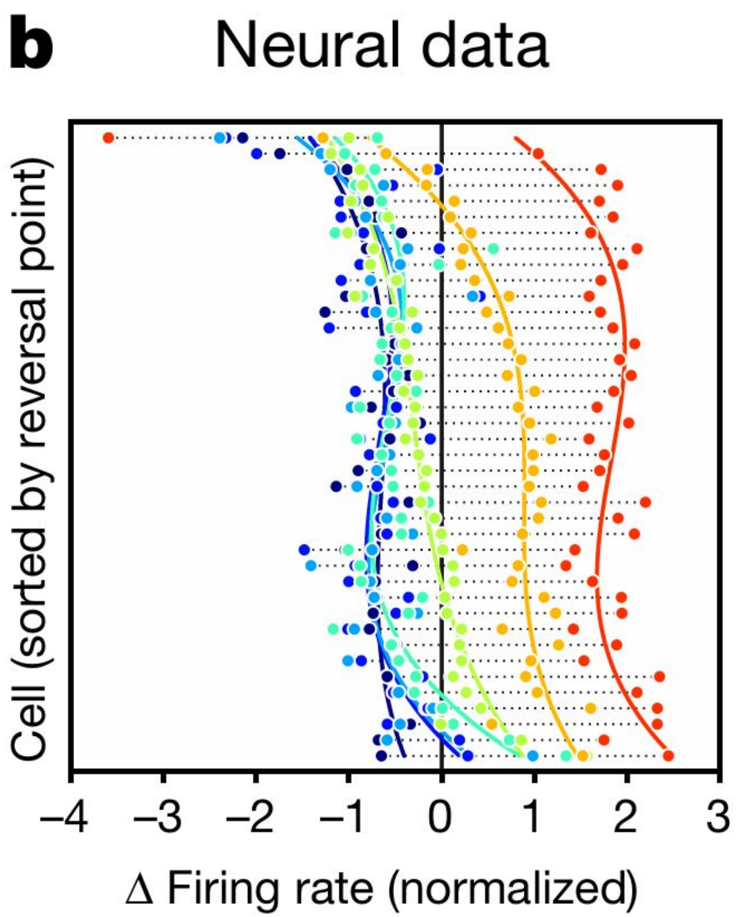
[2] A distributional code for value in dopamine-based RL. Dabney et al., Nature, 2020.

Standard TD-Learning



Distribution TD-Learning





Measured dopaminergic neurons in the Ventral Tegmental Area (VTA)

[2] A distributional code for value in dopamine-based RL. Dabney et al., Nature, 2020.

Conclusions :

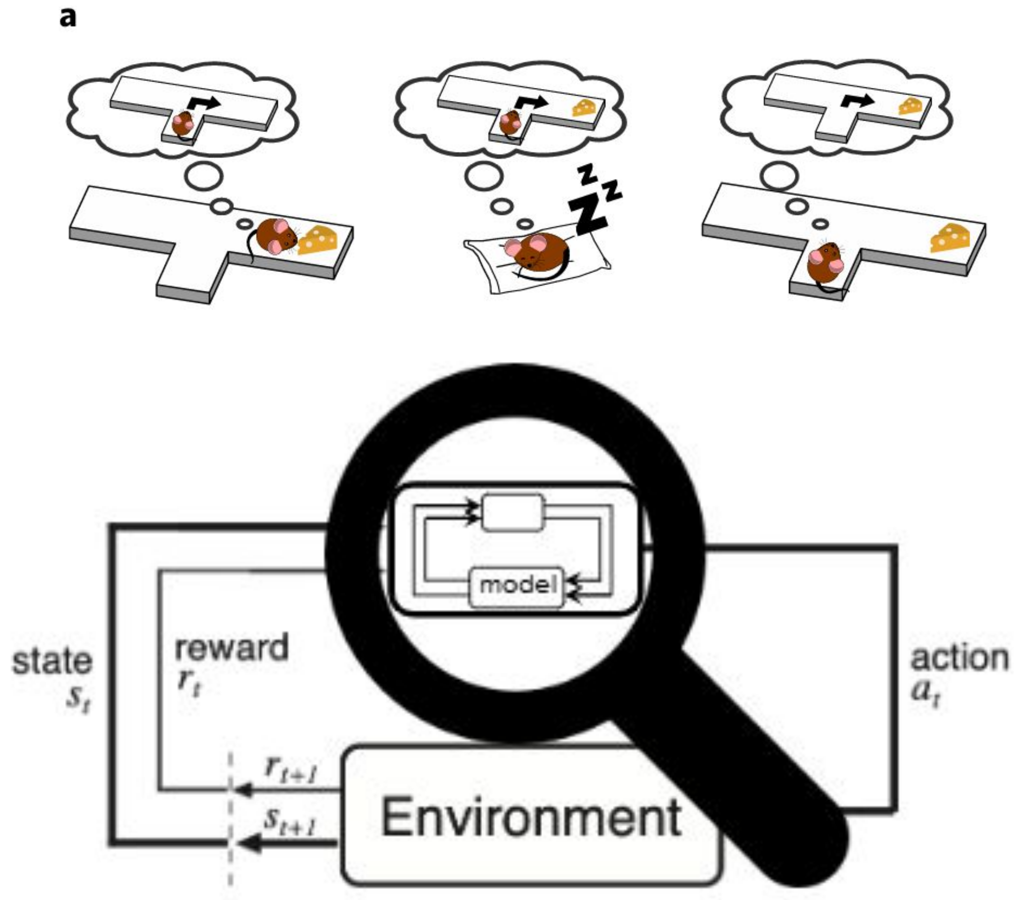
- **Classic TD model**:
  All neurons converge to similar responses; small variations are just noise.

- **Distributional TD model**:
  Neurons specialize — some are **optimistic**, some **pessimistic**, and others **neutral**.

- **Neural data (VTA dopamine neurons)**:
  This diversity is real and matching distributional predictions.

Other ways to see RL & Brain:

- **Model Free & Model Based.**



Prioritized memory access explains planning and hippocampal replay, Marcelo G. Mattar \& Nathaniel D. Daw, Nature Neuroscience, 2018.


THANKS