SPR Assignment 4

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Contents

1 Perceptron Algorithm

- Let $\mathcal{D} = \{x_i, y_i\}_{i=1}^n$ be the dataset given. Where input feature vector $x_i \in \mathbb{R}^d$ and the corresponding output is $y_i \in \{0, 1\}$.
- When given data is linearly separable, perceptron algorithm tries to find the hyper plane that give zero mis-classification error.
- The linear discriminant function used in perceptron is given by,

$$g(x) = w^T x + w_0$$

  Here, w is the weigh vector and w_0 is the bias term (threshold).
- Hence given an input feature vector x, if $g(x) > 0$ then x is assigned to $\mathbb{C}_1$ (class 1) else x is assigned to $\mathbb{C}_0$ (class 0).
- The geometric illustration of this for a 3-dimensional input given below helps in what perceptron algorithm does,

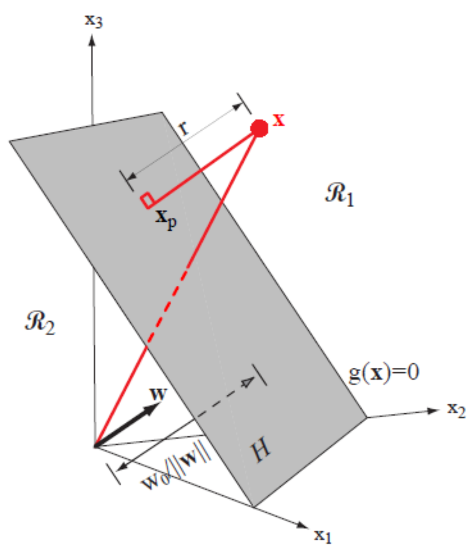


Figure 1: The linear decision boundary H, where $g(x) = w^T x + w_0 = 0$, separates the feature space into two half-spaces R_1 (where $g(x) > 0$) and R_2 (where $g(x) < 0$) [1]

- The objective function that we are trying to minimize in perceptron algorithm is,

$$J(w) = \sum_{i \in S} (-w^T x_i)$$

  where $S = \{i | sign(w^T x_i \neq y_i\}$, here $y_i \in \{-1, 1\}$

- We use gradient descent method to update the weights. The gradient of $J(w)$ is given by,
$$\nabla J(w) = \sum_{i \in S} (-x_i)$$

- The weight update rule at time instant t (Weight vector is initialised randomly at time instant $t = 0$) is,
$$w_{t+1} = w_t + \eta \sum_{i \in S} (-x_i)$$
where η is the learning rate.

- Since we are using all the misclassified inputs to update the weights we call this update rule as batch update rule.

- The pocket algorithm is a variant of perceptron algorithm in which we keep the best weight vector seen so far **in the pocket**. Here best weight vector is the one which results in least misclassification error so far, i.e., pocket algorithm keeps the best weight vector in the pocket instead of weight vector obtained in the last epoch. Hence this is best suited for non-separable data sets, where our goal is to find a weight vector with small number of misclassifications.

2 Linear Least Squares For classification

- Let $\mathcal{D} = \{x_i, y_i\}_{i=1}^{n}$ be the dataset given. Where input feature vector $x_i \in \mathbb{R}^d$ and the corresponding output is $y_i \in \{0, 1\}$.
- The objective function we are trying to minimise is,

$$J(w) = \sum_{i=1}^{n} (w^T x_i - y_i)^2$$

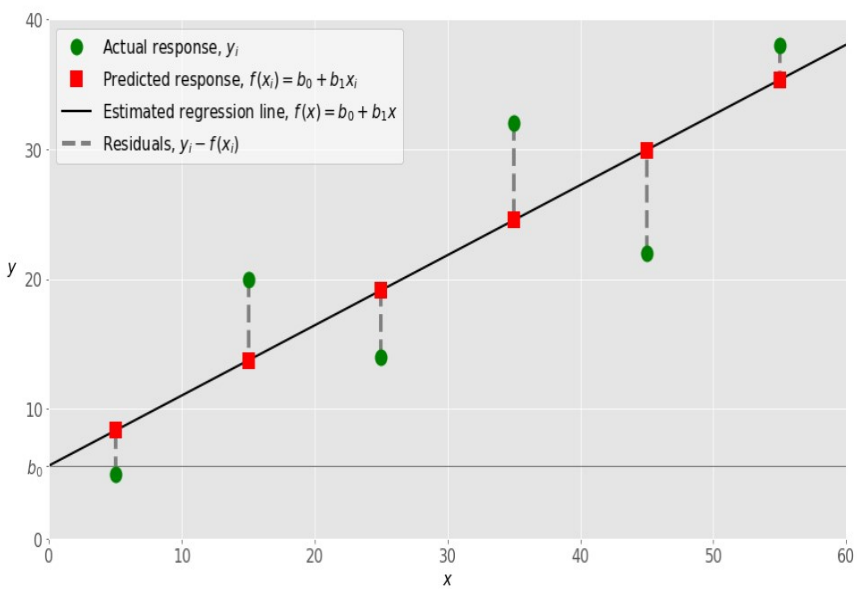
where, w is the weight vector.

- Finding the $\nabla J(w)$ w.r.t w and equating it to zero, we get the optimal weight vector as,

$$w = (X^T X)^{-1} X^T \mathbf{y}$$

where, i^{th} row of X consists of i^{th} augmented sample feature vector and $\mathbf{y} = [y_1, y_2, \ldots, y_n]$

- Here, $(X^T X)^{-1} X^T$ is called *pseudo-inverse* of the matrix X, denoted by $X^{\dagger}$.
- Here, the dimension of w is $((d+1)\text{x}1)$ dimension of X is $(n\text{x}(d+1))$ and the dimension of y is $\mathbf{y}$ is $(n\text{x}1)$



Example of simple linear regression

- For 2-dimensional case, given data points $(x_1, y_1), \ldots, (x_n, y_n)$ if we want to fit a straight line $f(x) = b_0 + b_1 x$ we use least squares method to find *parameters* b_0, b_1 which minimizes fitting error or which makes $f(x_i) = b_0 + b_1 x_i \approx y_i \ \forall i$.

- Now, the linear regression explained above can also be used for classification. This is because, for any input feature vector output y is either 0 or 1 which are also real numbers.

- The decision rule for classification is, for any augmented input feature vector x, if $w^T x > 0$ then we say $x \in \mathbb{C}_1$(class 1) else we say $x \in \mathbb{C}_0$(class 0).

3 Logistic Regression

- Logistic Regression is a supervised learning which tries to approximate the posterior probability of the dependant variable. (For classification dependant variable is either 0 or 1)
- We know that linear regression gives continuous output number values whereas in logistic regression these continuous values are given as input to logistic sigmoid function which gives us a value between 0 to 1.
- The expression for sigmoid function is,

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

- Let $\mathcal{D} = \{x_i, y_i\}_{i=1}^{n}$ be the dataset given. Where input feature vector $x_i \in \mathbb{R}^d$ and the corresponding output is $y_i \in \{0, 1\}$.
- In logistic regression, $z = w^T x$. Here we try to approximate, $log\left(\frac{p_1 f_1(x)}{p_0 f_0(x)}\right)$ using $w^T x$. Where, p_k : prior probability of class k and $f_k(x)$ is class conditional density of class k, for $k = 0, 1$
- The objective function that we are trying to minimize in logistic regression is cross-entropy loss function which is given by,

$$J(w) = \frac{1}{n} \sum_{i}^{n} \Big(y_i log(h(x_i)) + (1 - y_i) log(1 - h(x_i)) \Big)$$

  where $h(x_i) = \sigma(w^T x)$
- The advantage of using cross-entropy loss function can be explained using figure 2.

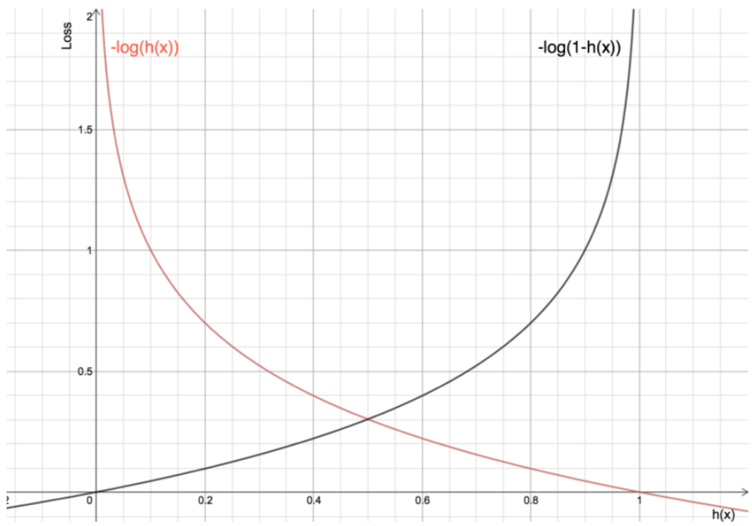


Figure 2: The wrong predictions are penalised more. When $y = 1$ the 2^{nd} term is zero. so objective function varies with respect to h(x) as shown by curve in red color and when $y = 0$ the 1^{st} term is zero so objective function varies with respect to h(x) as shown by curve in black color [2]

- As we can see from the above figure the objective function is a smooth function, hence it is easier to calculate gradient.
- We use gradient descent method to update the weights. The gradient of $J(w)$ is given by,

$$\nabla J(w) = \frac{1}{n}\sum_{i}^{n}(h(x_i) - y_i)x_i$$

- The weight update rule is,

$$w_{t+1} = w_t - \eta \nabla J(w)$$

where η is the learning rate.

4 Fisher Linear Discriminant

- Let $\mathcal{D} = \{x_i, y_i\}_{i=1}^{n}$ be the dataset given. Where input feature vector $x_i \in \mathbb{R}^d$ and the corresponding output is $y_i \in \{0, 1\}$
- In Fisher linear discriminant analysis (FLDA) we project input feature vector x to one dimension as

$$y = w^T x$$

  and classify x as sample from $\mathbb{C}_1$ if $y \geq -b$ else we classify x as sample from $\mathbb{C}_0$, where b is the threshold for classification.
- FLDA tries to find the weight vector whose direction is such that, it maximises the class separation.
- Let n_0 be the number of training feature vectors in $\mathbb{C}_0$ and n_1 be the number of training feature vectors in $\mathbb{C}_1$. Let $\mathbf{m}_0$ and $\mathbf{m}_1$ be the mean vectors of $\mathbb{C}_0$ and $\mathbb{C}_1$ respectively, i.e.,

$$\mathbf{m}_0 = \frac{1}{n_0} \sum_{i \in C_0} x_i$$

  and

$$\mathbf{m}_1 = \frac{1}{n_1} \sum_{i \in \mathbf{C}_1} x_i$$

- For separation between the classes, we can try to maximise the separation of the means of projected class i.e, maximise $(m_1 - m_0)$ where, $m_1 = w^T \mathbf{m}_1$ and $m_0 = w^T \mathbf{m}_0$
- We can maximise $m_1 - m_0$ by increasing the magnitude of w. Also maximising $(m_1 - m_0)$ does not mean they are well separated in the projected space.
- Hence in FLDA, we try to find weight vector such that it maximises projected class means and also minimises within class variance. This results in minimising class overlap in the projected space.
- Therefore, the objective function we are trying to maximise is,

$$J(w) = \frac{(m_1 - m_0)^2}{s_0^2 + s_1^2}$$

  where $s_k^2 = \sum_{x_i \in C_k} (x_i - \mathbf{m}_k)^2$ and $k = 0, 1$
- The above objective function can be written as,

$$J(w) = \frac{w^T S_B w}{w^T S_W w}$$

where S_B is called between class covariance matrix and is given by,

$$S_B = (\mathbf{m}_1 - \mathbf{m}_0)(\mathbf{m}_1 - \mathbf{m}_0)^T$$

and S_W is called with-in class covariance matrix and is given by,

$$S_W = \sum_{x_i \in \mathbb{C}_1} (x_i - \mathbf{m}_1)(x_i - \mathbf{m}_1)^T + \sum_{x_i \in \mathbb{C}_0} (x_i - \mathbf{m}_0)(x_i - \mathbf{m}_0)^T$$

- Differentiating $J(w)$ w.r.t w and equating it to zero, we get the w^* that maximises $J(w)$. At w^*,

  $$({w^*}^T S_B w^*) S_W w^* = ({w^*}^T S_W w^*) S_B w^*$$

  We can show that, $S_B w$ is always in the direction of $(\mathbf{m}_1 - \mathbf{m}_0)$.

- It turns out optimal weight vector is,

  $$w^* \propto S_W^{-1}(\mathbf{m}_1 - \mathbf{m}_0)$$

- Since S_W is sum of unit rank matrices, most of the times S_W will be a full rank matrix. If not w^* can be obtained using generalised eigen value decomposition.

- To find the threshold b based on which classification is made, we can find it either by trial and error method or use the below expression,

  $$b = -w^T \mathbf{m}$$

  here $\mathbf{m} = \frac{1}{n}(n_1 \mathbf{m}_1 + n_0 \mathbf{m}_0))$ (as given in 4.34 of [1]).

- Once w^* is found we classify given input x as, if $h(x) > 0$ then $x \in \mathbb{C}_1$ else $x \in \mathbb{C}_0$. Here $h(x) = {w^*}^T x + b$.

5 Observations

5.1 Question 1

- For 10-dimensional Gaussian with mean vector for class 1 all identically one and the mean vector for class 0 all identically zero, identity covariance matrix for both the classes and prior probabilities for classes to be 0.5, perceptron(pocket) algorithm (with learning rate of 0.9 and number of epochs 30) and linear regression gave us the maximum test (95.17%) and train accuracy (94.77%)

- For 2-dimensional Gaussian with mean vector for class 1 all identically one and the mean vector for class 0 all identically zero, identity covariance matrix for both the classes and prior probabilities for classes to be 0.5, linear and logistic regression gave better accuracy than FLDA and pocket algorithm

- For linearly non-separable data set, since the error for all the classifiers was close to 50%, we transformed the data to higher dimension using polynomial transformation which gave us better accuracy. Linear regression gave better test and train accuracy compared to other algorithms when the classifiers were applied to transformed data.

- For different mean and different covariance matrices ($\mu_0 = [3, 6]$ and $\mu_1 = [3, -2]$ and $\Sigma_0 = [0.5, 0; 0, 2]$),$\Sigma_1 = [2, 0; 0, 2]$. Since data was welly seperated we got test and train accuracy close to 99% for all the classifiers except when classes are equiprobable but Accuracy of FLD classifier decreases and data become imbalanced i.e for $p_0 = 0.8, p_1 = 0.2$ we got train and test accuracy around 90%.

5.2 Question 2

- We were not able to use the file given in the link hence we used the"German data credit card risk dataset" from the following link, https://online.stat.psu.edu/stat857/node/215/

- The German Credit Data contains data on 20 variables and the classification whether an applicant is considered a Good or a Bad credit risk for 1000 loan applicants. Here is a link to the German Credit data (right-click and "save as"). A predictive model developed on this data is expected to provide a bank manager guidance for making a decision whether to approve a loan to a prospective applicant based on his/her profiles.

- Linear regression gave better train accuracy of 77.14% and test accuracy of 73%.

5.3 Question 3

- For Porto - Seguro Safe driver prediction, we need to build a model that predicts whether a driver will initiate an auto insurance claim in the next year or not.
- The given data set is highly imbalanced data set ($p_0 = 0.96, p_1 = 0.04$), hence if we apply linear classifiers on this dataset the test and train accuracy will be around 50%.
- Hence, we can performed some pre-processing on the dataset. We used inbuilt under sampling method, it takes imbalanced data as input, under samples it such that classes will be equiprobable in the new data. After this pre-processing was performed the test and train accuracy was increased to 58% but it was not a significant increase in test and train accuracy.
- We can also transform the data set to higher dimension and then learn the linear classifier in higher dimensional space. This classifier will be a non-linear classifier in original dimension space.(here original dimension is 54 therefore it requires lot of time to transform it to higher dimensional space)

References

[1] Bishop, Christopher M. Pattern Recognition and Machine Learning. New York :Springer, 2006

[2] https://www.analyticsvidhya.com/blog/2020/11/binary-cross-entropy-aka-log-loss-the-cost-function-used-in-logistic-regression/

[3] Richard O. Duda, Peter E. Hart, and David G. Stork. 2000. ¡i¿Pattern Classification (2nd Edition)¡/i¿. Wiley-Interscience, USA.

SPR_Assignment_4

April 3, 2021

1 Statistical Pattern Recognition: Assignment 4

Title : Linear classifiers

Members : Jayanth S, Praveen Kumar N, Rishabh Roy

2 Importing

[ ]:
```
from sklearn.linear_model import Perceptron
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
from sklearn.linear_model import LinearRegression
from sklearn.linear_model import LogisticRegression
from sklearn import svm
from sklearn.model_selection import train_test_split
from sklearn.metrics import␣
 ↪f1_score,accuracy_score,confusion_matrix,plot_confusion_matrix

from imblearn.under_sampling import RandomUnderSampler
from sklearn.preprocessing import StandardScaler
from sklearn.preprocessing import PolynomialFeatures

import numpy as np
import pandas as pd
import math
from   numpy.random import rand

import matplotlib.pyplot as plt
import seaborn as sns

from IPython.display import Javascript
import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)
```

3 Linear classifiers built from scratch:

3.1 Perceptron:

[ ]:
```
class MyPerceptron():

    def __init__(self):
        self.w_est = None     # Estimated weight vector
        self.eta   = None     # Learning rate
        self.N_epochs = None # Number of epochs for Stochastic gradient descent
        self.miss_class = None  # List to store number of missclassifications in␣
↪each epoch
        self.n     = None     # Number of Training samples
        self.d     = None     # Dimension of feature vector


    def Perceptron_estimate(self, X_aug, y):

        '''
            Inputs          -> Augmented Matrix(X) and true labels vector(y)
            Outputs         -> Estimated weight vector using Stochastic gradient␣
↪descent(w_est)
                               List of missclassifications at each epoch
            Perceptron update rule -> w_(t+1) = w_(t) + eta * X_aug^T(y - y_est)/
↪2*n
                                      w_(t+1) = w_(t) - eta * X_aug^T(y_est - y)/
↪2*n
                                    This update rule ensures that the weight␣
↪vector obtained will give
                                    zero missclassifications if the classes are␣
↪"linearly separable"
        '''

        miss_class  = list()      # List to store the number of␣
↪missclassifications at each epoch
        np.random.seed(0)
        w_est = rand(self.d+1,1)  # Initialization of weight vector to random␣
↪values

        for _ in range(self.N_epochs):

            # Estimated outputs using weight vector "w_est"
            y_est    = np.sign(X_aug @ w_est) # (n X d+1).(d+1 X 1) -> (n X 1)
            # Error in the Estimated outputs
            error    = y_est - y                   # (n X 1)

            # Weight vector update
```

```
            dw       = (X_aug.T @ error) /(2*self.n)
            w_est    = w_est - self.eta*dw    # (d+1 X 1)

            # Storing the Binary Cross-Entropy Loss value for the current weight␣
↪vector(current epoch)
            miss_class.append(self.Num_miss_class(X_aug, y, w_est))

            # Stopping criteria
            if np.linalg.norm(dw) < 10**(-5):
                break

        return w_est,miss_class

    def Num_miss_class(self,X_aug,y,w_est):

        '''
           Output -> Num_miss_class : Number of samples missclassifeid by the␣
↪estimated weight vector "w_est"
        '''

        y_est  = X_aug @ w_est  # (n X d+1).(d+1 X 1) -> (n X 1)
        N_miss_class = np.sum(y_est!=y)

        return N_miss_class

    def fit(self, X, y,eta,N_epochs):

        '''
            X : Matrix containing training feature vectors in each row
              size of matrix -> (n X d) where n -> number of training samples, d␣
↪-> feature vector dimension
            y : Vector containing true labels of the n training samples (labels␣
↪-> -1(class 0) , +1(class 1))
              size of vector -> (n X 1)
          eta : Learning rate
          N_epochs : Number of epochs
        '''

        self.eta      = eta
        self.N_epochs = N_epochs
        self.n        = np.shape(X)[0]
        self.d        = np.shape(X)[1]
        X_aug         = np.concatenate((np.ones((self.n,1), dtype=int),X),␣
↪axis=1) # (n X d+1)

        w_est,miss_class = self.Perceptron_estimate(X_aug, y)
        self.w_est       = w_est
```

```
        self.miss_class  = miss_class

    def predict(self, X):

        '''
            Input  -> X      : Matrix containing testing feature vectors in each␣
↪row
                               size of matrix -> (n_test X d) where n_test ->␣
↪number of testing samples, d -> feature vector dimension
            Output -> y_pred : Vector containing predicted labels of the n_test␣
↪testing samples
                               size of vector -> (n_test X 1)
        '''

        n_test = np.shape(X)[0]
        X_aug  = np.concatenate((np.ones((n_test,1), dtype=int),X), axis=1)

        # Perceptron outputs obtained using estimated weight vector
        y_pred  = np.sign(X_aug@self.w_est) # (n_test X d+1) . (d+1 X 1) ->␣
↪(n_test X 1)
        y_pred[y_pred == 0] = 1             # when the predicted output is 0␣
↪classify it as class 1

        return y_pred

    def Accuracy(self,X,y):

        '''
            Inputs  -> X : Matrix containing testing feature vectors in each␣
↪row, size of matrix -> (n X d)
                       y : Vector containing the true class labels of the␣
↪feature vectors, size of the vector -> (n x 1)
            Output  -> Accuracy of the estimated weight vector "w_est" on the␣
↪given samples
        '''

        # Predict the class labels using "w_est" (estimated weight vector)
        y_pred   = self.predict(X)

        accuracy = accuracy_score(y,y_pred)*100

        return accuracy
```

3.2 Fischer linear discriminant:

```
[ ]: class FLD():

      def __init__(self):
            self.w_est = None     # Estimated weight vector
            self.n     = None     # Number of Training samples
            self.d     = None     # Dimension of feature vector

      def estimate_weight_vector(self, X, y):

            '''
                Inputs -> training Matrix(X) and true labels vector(y)
                Output -> w_est: estimated weight vector according to fischer linear␣
↪discriminant analysis
            '''

            C0_samples = X[np.where(y == 0)[0],:].T # Class 0 samples in training␣
↪data
            C1_samples = X[np.where(y == 1)[0],:].T # Class 1 samples in training␣
↪data

            #find the mean of class 0(M0) and mean of class 1(M1)
            M0 = np.mean(C0_samples,axis=1,keepdims = 1)    # (d X 1)
            M1 = np.mean(C1_samples,axis=1,keepdims = 1)    # (d X 1)

            #difference between class means
            mu_diff = M1-M0   # (d X 1)

            #within class matrix
            SW =  (C0_samples - M0) @ (C0_samples - M0).T + (C1_samples - M1) @␣
↪(C1_samples - M1).T # (d X d)

            #Optimal weight vector : inv(SW) x (M1 - M0)
            w = np.linalg.inv(SW+ np.identity(self.d)) @ mu_diff # (d X 1)

            #Optimal offset for the hyperplane
            M = ((C0_samples.shape[1]*M0 + C1_samples.shape[1]*M1)/(self.n)) # Mean␣
↪of all the training samples
            b = -w.T@M

            return b,w

      def fit(self, X, y):

            '''
               X : Matrix containing training feature vectors in each row
```

```
                size of matrix -> (n X d) where n -> number of training samples,␣
↪d -> feature vector dimension
            y : Vector containing true labels of the n training samples (labels␣
↪-> 0(class 0) , 1(class 1))
                size of vector -> (n X 1)
        '''
        self.d = np.shape(X)[1]
        self.n = np.shape(X)[0]

        b,w = self.estimate_weight_vector(X, y)  # (d+1 X 1)

        self.w_est = np.expand_dims(np.append(b,w),axis=-1)


    def predict(self, X):

        '''
           Input  -> X      : Matrix containing testing feature vectors in each␣
↪row
                              size of matrix -> (n_test X d) where n_test ->␣
↪number of testing samples, d -> feature vector dimension
           Output -> y_pred : Vector containing predicted labels of the n_test␣
↪testing samples
                              size of vector -> (n_test X 1)
        '''

        n_test = np.shape(X)[0]
        X_aug  = np.concatenate((np.ones((n_test,1), dtype=int),X), axis=1)

        # Threshold FLD outputs obtained using estimated weight vector
        y_tmp  = X_aug @ self.w_est    # (n_test X d+1) . (d+1 X 1) -> (n_test X␣
↪1)
        y_pred = np.where(y_tmp<0,0,1)

        return y_pred

    def Accuracy(self,X,y):

        '''
            Inputs  -> X : Matrix containing testing feature vectors in each␣
↪row, size of matrix -> (n X d)
                       y : Vector containing the true class labels of the␣
↪feature vectors, size of the vector -> (n x 1)
            Output  -> Accuracy of the estimated weight vector "w_est" on the␣
↪given samples
        '''
```

```
        # Predict the class labels using "w_est" (estimated weight vector)
        y_pred   = self.predict(X)

        accuracy = accuracy_score(y,y_pred)*100

        return accuracy
```

3.3 Linear regression:

```
[ ]: class MultipleLinearRegression():

        def __init__(self):
            self.w_est = None  # Estimated weight vector
            self.n     = None  # Number of Training samples
            self.d     = None  # Dimension of feature vector

        def estimate_weight_vector(self, X_aug, y):

            '''
                Inputs -> Augmented training Matrix(X_aug) and true labels vector(y)
                Output -> w_est = (X_aug^TX_aug)^-1 X_aug^Ty(Linear least squares␣
    ↪solution)
            '''

            inv    = np.linalg.inv( X_aug.T @ X_aug + np.identity(self.d+1)) # a@b␣
    ↪-> a.dot(b) -> dot(a,b)
            w_est  = inv @ X_aug.T @ y                # (d+1 X 1)

            return w_est

        def fit(self, X, y):

            '''
              X : Matrix containing training feature vectors in each row
                  size of matrix -> (n X d) where n -> number of training samples,␣
    ↪d -> feature vector dimension
              y : Vector containing true labels of the n training samples (labels␣
    ↪-> -1(class 0) , +1(class 1))
                  size of vector -> (n X 1)
            '''

            self.n = np.shape(X)[0]
            self.d = np.shape(X)[1]
            X_aug  = np.concatenate((np.ones((self.n,1), dtype=int),X), axis=1)  # (n␣
    ↪X d+1)
```

```
        self.w_est = self.estimate_weight_vector(X_aug, y)                #␣
↪(d+1 X 1)

    def predict(self, X):

        '''
            Input  -> X      : Matrix containing testing feature vectors in each␣
↪row
                               size of matrix -> (n_test X d) where n_test ->␣
↪number of testing samples, d -> feature vector dimension
            Output -> y_pred : Vector containing predicted labels of the n_test␣
↪testing samples
                               size of vector -> (n_test X 1)
        '''

        n_test = np.shape(X)[0]
        X_aug  = np.concatenate((np.ones((n_test,1), dtype=int),X), axis=1)

        # Regression outputs obtained using estimated weight vector
        y_tmp  = X_aug@self.w_est    # (n_test X d+1) . (d+1 X 1) -> (n_test X 1)

        # Threshold the regression outputs to get the classifier outputs
        y_pred = np.sign(y_tmp)
        y_pred[y_pred == 0] = 1 # when the regression output is 0 classify it as␣
↪class 1

        return y_pred

    def Accuracy(self,X,y):

        '''
            Inputs  -> X : Matrix containing testing feature vectors in each␣
↪row, size of matrix -> (n X d)
                       y : Vector containing the true class labels of the␣
↪feature vectors, size of the vector -> (n x 1)
            Output  -> Accuracy of the estimated weight vector "w_est" on the␣
↪given samples
        '''

        # Predict the class labels using "w_est" (estimated weight vector)
        y_pred   = self.predict(X)

        accuracy = accuracy_score(y,y_pred)*100

        return accuracy
```

3.4 Logistic regression:

[ ]:
```
class MyLogisticRegression():

    def __init__(self):
        self.w_est = None     # Estimated weight vector
        self.eta    = None     # Learning rate
        self.N_epochs = None # Number of epochs for Stochastic gradient descent
        self.loss   = None     # List to store loss in each epoch
        self.n      = None     # Number of Training samples
        self.d      = None     # Dimension of feature vector

    def sigmoid(self, x):
        return 1 / (1 + np.e**(-x))

    def SGD_estimate(self, X_aug, y):

        '''
            Inputs          -> Augmented Matrix(X) and true labels vector(y)
            Outputs         -> Estimated weight vector using Stochastic gradient␣
↪descent(w_est)
                                List of loss incurred at each epoch
            SGD update rule -> w_(t+1) = w_(t) - eta * X_aug^T(y_est - y)/n
                              This update rule ensures that we minimize the␣
↪Binary Cross-Entropy Loss Function
        '''

        loss  = list()             # List to store the loss incurred at each epoch
        np.random.seed(0)
        w_est = rand(self.d+1,1)  # Initialization of weight vector to random␣
↪values

        for _ in range(self.N_epochs):

            # Estimated outputs using weight vector "w_est"
            y_est    = self.sigmoid(X_aug @ w_est)  # (n X d+1).(d+1 X 1) -> (n␣
↪X 1)
            # Error in the Estimated outputs
            error    = y_est - y                    # (n X 1)

            # Weight vector update
            w_est    = w_est - self.eta*(X_aug.T @ error) / self.n  # (d+1 X 1) ␣
↪

            # Storing the Binary Cross-Entropy Loss value for the current weight␣
↪vector(current epoch)
            loss.append(self.Cross_entropy_cost(X_aug, y, w_est))
```

```
        return w_est,loss

    def Cross_entropy_cost(self,X_aug,y,w_est):

        '''
           Output -> loss : Binary Cross-Entropy Loss corresponding to the␣
↪estimated weight vector "w_est"
        '''

        z      = X_aug @ w_est                           # (n X d+1).(d+1 X 1) ->␣
↪(n X 1)
        loss_1 = y * np.log(self.sigmoid(z)+10**(-5))   # (n X 1)*(n X 1) -> (n X␣
↪1)[element wise multiplication]
        loss_0 = (1 - y) * np.log(1 - self.sigmoid(z)+10**(-5)) # (n X 1)*(n X␣
↪1) -> (n X 1)[element wise multiplication]
        loss   = -sum(loss_1 + loss_0) /self.n           # Binary Cross-Entropy␣
↪Loss

        return loss

    def fit(self, X, y,eta,N_epochs):

        '''
            X   : Matrix containing training feature vectors in each row
                size of matrix -> (n X d) where n -> number of training samples,␣
↪d -> feature vector dimension
            y   : Vector containing true labels of the n training samples␣
↪(labels -> 0(class 0) , 1(class 1))
                size of vector -> (n X 1)
            eta : Learning rate
            N_epochs : Number of epochs
        '''

        self.eta      = eta
        self.N_epochs = N_epochs
        self.n        = np.shape(X)[0]
        self.d        = np.shape(X)[1]
        X_aug         = np.concatenate((np.ones((self.n,1), dtype=int),X),␣
↪axis=1) # (n X d+1)

        w_est,loss = self.SGD_estimate(X_aug, y)
        self.w_est = w_est
        self.loss  = loss

    def predict(self, X):
```

```
        '''
            Input  -> X       : Matrix containing testing feature vectors in each␣
↪row
                              size of matrix -> (n_test X d) where n_test ->␣
↪number of testing samples, d -> feature vector dimension
            Output -> y_pred : Vector containing predicted labels of the n_test␣
↪testing samples
                              size of vector -> (n_test X 1)
        '''

        n_test   = np.shape(X)[0]
        X_aug    = np.concatenate((np.ones((n_test,1), dtype=int),X), axis=1) #␣
↪(n X d+1)

        # Computing regression outputs using estimated weight vector
        z      = X_aug @ self.w_est  # (n_test X d+1).(d+1 X 1) -> (n_test X 1)
        y_tmp  = self.sigmoid(z)

        # Threshold the obtained outputs to get predicted class labels(0 or 1)
        y_pred = np.where(y_tmp<0.5,0,1)

        return y_pred

    def Accuracy(self,X,y):

        '''
            Inputs  -> X : Matrix containing testing feature vectors in each␣
↪row, size of matrix -> (n X d)
                       y : Vector containing the true class labels of the␣
↪feature vectors, size of the vector -> (n x 1)
            Output  -> Accuracy of the estimated weight vector "w_est" on the␣
↪given samples
        '''

        # Predict the class labels using "w_est" (estimated weight vector)
        y_pred   = self.predict(X)

        accuracy = accuracy_score(y,y_pred)*100

        return accuracy
```

4 Supporting functions:

4.1 Multivariate gaussian data generation:

```
[ ]: def generate_data(N_train,N_test,mu,covariance,p):

         '''
          Inputs:
          N_train, N_test : Number of training and testing samples required
          mu              : Matrix containing mean vectors for each class
                              size of mu -> (c X d), where d is the dimension of␣
     ↪feature vector, c is number of classes(2 here)
          covariance      : list of covariance matrices of different classes
                              size of each covariance matrix -> (d x d), length of␣
     ↪covariance list -> c(2 here)
          p               : list containing prior probabilities of each class
                              length of list p -> c(2 here)
          Outputs:
          train_samples : Matrix containing feature vectors from different classes␣
     ↪according to prior probability 'p' for training
                          size of Matrix -> (N_train X d)
          train_labels  : Vector of corresponding labels for the train_samples
                          size of vector -> (N_train X 1)
          test_samples  : Matrix containing feature vectors from different classes␣
     ↪according to prior probability 'p' for testing
                          size of Matrix -> (N_test X d)
          test_labels   : Vector of corresponding labels for the test_samples
                          size of vector -> (N_test X 1)                                ␣
     ↪
         '''

        train_samples = list()
        train_labels =  list()

        for i in range(N_train):

            class_selected = np.random.choice([0,1],p=p)
            mu_sel         = mu[class_selected,:]
            temp_sample    = np.random.
     ↪multivariate_normal(mu_sel,covariance[class_selected])
            train_samples.append(temp_sample)
            train_labels.append(class_selected)

        train_samples = np.array(train_samples)
        train_labels  = np.expand_dims(np.array(train_labels),axis=-1)

        test_samples = list()
```

```
    test_labels  = list()

    for i in range(N_test):

        class_selected = np.random.choice([0,1],p=p)
        mu_sel         = mu[class_selected,:]
        temp_sample    = np.random.
↪multivariate_normal(mu_sel,covariance[class_selected])
        test_samples.append(temp_sample)
        test_labels.append(class_selected)

    test_samples = np.array(test_samples)
    test_labels  = np.expand_dims(np.array(test_labels),axis=-1)

    N = N_train+N_test
    print(f"train-test split : {int((N_train/N)*100)}:{int((N_test/N)*100)}\n")
    print(f"Number of training data points : {train_samples.shape[0]}")
    print(f"Number of test data points   : {test_samples.shape[0]}")
    print(f"Dimension of feature vectors : {train_samples.shape[1]}\n")

    return train_samples,train_labels,test_samples,test_labels
```

4.2 Data preprocessing:

4.2.1 German credit card data:

[153]:
```
def German_data_numeric_preprocessing(testdata_per):

    '''
      Inputs:
      testdata_per  : proprtion of test data for splitting data into train and␣
↪test data

      Outputs:
      train_samples : Matrix containing feature vectors from different classes
                      size of Matrix -> (N_train X d)
      train_labels  : Vector of corresponding labels for the train_samples
                      size of vector -> (N_train X 1)
      test_samples  : Matrix containing feature vectors from different classes
                      size of Matrix -> (N_test X d)
      test_labels   : Vector of corresponding labels for the test_samples
                      size of vector -> (N_test X 1)
      Here N_train -> Number training data points(N*(1-testdata_per)) , N_test␣
↪-> Number of test data points(N*testdata_per)
           N_train + N_test = N = 1000 for german credit data
           d -> dimension of each feature vector = 20 for german credit data    ␣
↪
```

```
    '''

    german_data = pd.read_csv(r"/content/drive/MyDrive/Colab Notebooks/
↪German_credit_data/german_credit_data.csv")

    labels      = german_data['Creditability']  #obtain the values(labels) from␣
↪the 'Creditability' column

    german_data = german_data.drop(['Creditability'],axis = 1)   # remove the␣
↪'Creditability' column from the data

    scaler = StandardScaler()
    scaler.fit(german_data)
    german_data = scaler.transform(german_data)

    train_samples, test_samples, train_labels, test_labels =␣
↪train_test_split(german_data, labels, test_size = testdata_per)
    train_labels = np.expand_dims(train_labels,axis=-1) # (1000*(1-testdata_per)␣
↪X 1)
    test_labels  = np.expand_dims(test_labels,axis=-1)  # (1000*testdata_per X 1)

    prob_cat = [np.count_nonzero(labels == cat)/len(labels) for cat in np.
↪unique(labels)]

    print(f"class probabilities: p0 = {prob_cat[0]},p1 = {prob_cat[1]}")
    print(f"For train-test split : {int((1-testdata_per)*100)}:
↪{int(testdata_per*100)}\n")
    print(f"Number of training data points : {train_samples.shape[0]}")
    print(f"Number of test data points   : {test_samples.shape[0]}")
    print(f"Dimension of feature vectors : {train_samples.shape[1]}\n")

    return train_samples,train_labels,test_samples,test_labels
```

4.2.2 Porto-seguro Safe driver prediction data:

[154]:
```
def Safe_driver_prediction_preprocessing(testdata_per):

    '''
      Inputs:
      testdata_per   : proprtion of test data for splitting data into train and␣
↪test data

      Outputs:
      train_samples : Matrix containing feature vectors from different classes
                      size of Matrix -> (N_train X d)
      train_labels   : Vector of corresponding labels for the train_samples
```

```
                   size of vector -> (N_train X 1)
     test_samples  : Matrix containing feature vectors from different classes
                   size of Matrix -> (N_test X d)
     test_labels   : Vector of corresponding labels for the test_samples
                   size of vector -> (N_test X 1)
     Here N_train -> Number training data points(N*(1-testdata_per)) , N_test␣
↪-> Number of test data points(N*testdata_per)
          N = 595212 (before sampling) , N = 43388(after down sampling)
          d -> dimension of each feature vector = 54 safe driver prediction␣
↪data
    '''

   driver_data = pd.read_csv(r"/content/drive/MyDrive/Colab Notebooks/
↪Safe_driver_prediction/train.csv")

   labels      = driver_data['target']  #obtain the values(labels) from the␣
↪'target' column

   # remove the 'target' column  and other unwanted columns from the data
   # unwanted columns : which if of no use in classification

   unwanted    = ['target','id','ps_ind_07_bin','ps_ind_08_bin','ps_ind_09_bin']
   driver_data = driver_data.drop(unwanted,axis = 1)

   rus = RandomUnderSampler(random_state=1)
   rus.fit(driver_data, labels)
   resampled_data, resampled_labels = rus.fit_sample(driver_data, labels)

   scaler = StandardScaler()
   scaler.fit(resampled_data)
   resampled_data = scaler.transform(resampled_data)

   train_samples, test_samples, train_labels, test_labels =␣
↪train_test_split(resampled_data, resampled_labels, test_size = testdata_per)
   train_labels = np.expand_dims(train_labels,axis=-1)  #␣
↪(43388*(1-testdata_per) X 1)
   test_labels  = np.expand_dims(test_labels,axis=-1)   # (43388*testdata_per X␣
↪1)

   prob_cat = [np.count_nonzero(labels == cat)/len(labels) for cat in np.
↪unique(labels)]
   prob_cat_resam = [np.count_nonzero(resampled_labels == cat)/
↪len(resampled_labels) for cat in np.unique(resampled_labels)]

   print(f"class probabilities : p0 = {prob_cat[0]},p1 = {prob_cat[1]}")
```

```
    print(f"class probabilities after under sampling: p0 =␣
↪{prob_cat_resam[0]},p1 = {prob_cat_resam[1]}")
    print(f"For train-test split : {int((1-testdata_per)*100)}:
↪{int(testdata_per*100)}\n")
    print(f"Number of training data points : {train_samples.shape[0]}")
    print(f"Number of test data points  : {test_samples.shape[0]}")
    print(f"Dimension of feature vectors : {train_samples.shape[1]}\n")

    return train_samples,train_labels,test_samples,test_labels
```

4.3 Perceptron testing function:

```
[ ]: def␣
↪Perceptron_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs):
↪

        # Dimension of the feature vector
        d = np.shape(train_samples)[1]
        # Converting class labels from 0 or 1 to -1 or +1
        ntrain_labels = 2*train_labels - 1
        ntest_labels  = 2*test_labels - 1

        print("*********** PERCEPTRON *************\n")
        ## Using IN-BUILT classifier (Fits linear model with Stochastic Gradient␣
↪Descent)

        # Creating class object
        per_cls_in    = Perceptron()

        # Estimate the weight vector
        per_cls_in.fit(train_samples,ntrain_labels.ravel())

        # Computing the accuracy of the estimated weight vector
        y_pred_train = per_cls_in.predict(train_samples)
        y_pred_test  = per_cls_in.predict(test_samples)
        train_accuracy_per_in = accuracy_score(ntrain_labels,y_pred_train.
↪ravel())*100
        test_accuracy_per_in  = accuracy_score(ntest_labels,y_pred_test.
↪ravel())*100

        # Number of iterations run by in-built classifier to get optimal weight␣
↪vector
        # N_epochs = per_cls_in.n_iter_

        ## Using classifier built from scratch
```

```
        # Creating class object
        per_cls     = MyPerceptron()

        # Estimate the weight vector
        per_cls.fit(train_samples,ntrain_labels,eta,N_epochs)

        # Computing the accuracy of the estimated weight vector
        train_accuracy_per = per_cls.Accuracy(train_samples,ntrain_labels)
        test_accuracy_per  = per_cls.Accuracy(test_samples,ntest_labels)

        N_train = np.shape(train_samples)[0]
        N_test  = np.shape(test_samples)[0]
        N = N_train+N_test
        print(f"train-test split : {math.ceil((N_train/N)*100)}:{math.
↪ceil((N_test/N)*100)}, Learning rate : {eta}, Number of epochs : {N_epochs}\n")
        print("Using classifier built from scratch")
        print(f"Train accuracy= {train_accuracy_per:.2f} %")
        print(f"Test accuracy = {test_accuracy_per:.2f}%")

        return per_cls_in,per_cls
```

4.4 FLD testing:

[ ]:
```
def FLD_testing(train_samples,train_labels,test_samples,test_labels):

        # Dimension of the feature vector
        d = np.shape(train_samples)[1]

        print("*********** FLD *************\n")
        ## Using classifier built from scratch

        # Creating class object
        fld_cls     = FLD()

        # Estimate the weight vector
        fld_cls.fit(train_samples,train_labels)

        # Computing the accuracy of the estimated weight vector
        train_accuracy_fld = fld_cls.Accuracy(train_samples,train_labels)
        test_accuracy_fld  = fld_cls.Accuracy(test_samples,test_labels)

        N_train = np.shape(train_samples)[0]
        N_test  = np.shape(test_samples)[0]
        N = N_train+N_test
```

```
        print(f"train-test split : {math.ceil((N_train/N)*100)}:{math.
↪ceil((N_test/N)*100)}\n")
        print("Using classifier built from scratch")
        print(f"Train accuracy= {train_accuracy_fld:.2f} %")
        print(f"Test accuracy = {test_accuracy_fld:.2f}%")

        return fld_cls
```

4.5 Linear regression testing function:

```
[ ]: def Linreg_testing(train_samples,train_labels,test_samples,test_labels):

        # Dimension of the feature vector
        d = np.shape(train_samples)[1]
        # Converting class labels from 0 or 1 to -1 or +1
        ntrain_labels = 2*train_labels - 1
        ntest_labels  = 2*test_labels - 1

        print("*********** LINEAR REGRESSION *************\n")
        ## Using IN-BUILT classifier

        # Creating class object
        linreg_cls_in    = LinearRegression()

        # Estimate the weight vector
        linreg_cls_in.fit(train_samples,ntrain_labels.ravel())

        # Computing the accuracy of the estimated weight vector
        y_pred_train = linreg_cls_in.predict(train_samples)
        y_pred_train = np.sign(y_pred_train)
        y_pred_train[y_pred_train == 0] = 1
        y_pred_test  = linreg_cls_in.predict(test_samples)
        y_pred_test = np.sign(y_pred_test)
        y_pred_test[y_pred_test == 0] = 1
        train_accuracy_linreg_in = accuracy_score(ntrain_labels,y_pred_train.
↪ravel())*100
        test_accuracy_linreg_in  = accuracy_score(ntest_labels,y_pred_test.
↪ravel())*100

        ## Using classifier built from scratch

        # Creating class object
        linreg_cls    = MultipleLinearRegression()

        # Estimate the weight vector
        linreg_cls.fit(train_samples,ntrain_labels)
```

```
        # Computing the accuracy of the estimated weight vector
        train_accuracy_linreg = linreg_cls.Accuracy(train_samples,ntrain_labels)
        test_accuracy_linreg  = linreg_cls.Accuracy(test_samples,ntest_labels)

        N_train = np.shape(train_samples)[0]
        N_test  = np.shape(test_samples)[0]
        N = N_train+N_test
        print(f"train-test split : {math.ceil((N_train/N)*100)}:{math.
↪ceil((N_test/N)*100)}\n")
        print("Using IN-BUILT classifier")
        print(f"Train accuracy= {train_accuracy_linreg_in:.2f} %")
        print(f"Test accuracy = {test_accuracy_linreg_in:.2f} %")
        print("Using classifier built from scratch")
        print(f"Train accuracy= {train_accuracy_linreg:.2f} %")
        print(f"Test accuracy = {test_accuracy_linreg:.2f} %")

        return linreg_cls_in,linreg_cls
```

4.6 Logistic regression testing function:

```
[ ]: def␣
↪Logreg_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs,loss_plot␣
↪= True):

        # Dimension of the feature vector
        d = np.shape(train_samples)[1]
        print("*********** LOGISTIC REGRESSION *************\n")
        ## Using IN-BUILT classifier

        # Creating class object
        logreg_cls_in     = LogisticRegression()

        # Estimate the weight vector
        logreg_cls_in.fit(train_samples,train_labels.ravel())

        # Computing the accuracy of the estimated weight vector
        y_pred_train = logreg_cls_in.predict(train_samples)
        y_pred_test  = logreg_cls_in.predict(test_samples)
        train_accuracy_logreg_in = accuracy_score(train_labels,y_pred_train.
↪ravel())*100
        test_accuracy_logreg_in  = accuracy_score(test_labels,y_pred_test.
↪ravel())*100

        ## Using classifier built from scratch
```

```
        # Creating class object
        logreg_cls     = MyLogisticRegression()

        # Estimate the weight vector
        logreg_cls.fit(train_samples,train_labels,eta,N_epochs)

        # Computing the accuracy of the estimated weight vector
        train_accuracy_logreg = logreg_cls.Accuracy(train_samples,train_labels)
        test_accuracy_logreg  = logreg_cls.Accuracy(test_samples,test_labels)

        N_train = np.shape(train_samples)[0]
        N_test  = np.shape(test_samples)[0]
        N = N_train+N_test
        print(f"train-test split : {math.ceil((N_train/N)*100)}:{math.
 ↪ceil((N_test/N)*100)}, Learning rate : {eta}, Number of epochs : {N_epochs}\n")
        print("Using IN-BUILT classifier")
        print(f"Train accuracy= {train_accuracy_logreg_in:.2f} %")
        print(f"Test accuracy = {test_accuracy_logreg_in:.2f} %")
        print("Using classifier built from scratch")
        print(f"Train accuracy= {train_accuracy_logreg:.2f} %")
        print(f"Test accuracy = {test_accuracy_logreg:.2f} %")

        if (loss_plot == True):
            # Plotting loss values vs Epoch number
            loss = logreg_cls.loss
            plt.plot(loss)
            plt.title("Binary cross entropy loss values vs Epoch number")
            plt.xlabel("Epoch number")
            plt.ylabel("Loss value")

        return logreg_cls_in,logreg_cls
```

4.7 Results plotting function:

```
[ ]: def␣
 ↪plot_res(per_cls,fld_cls,linreg_cls,logreg_cls,per_cls_in,linreg_cls_in,logreg_cls_in,train_s
 ↪

        ntest_labels  = 2*test_labels - 1
        min_x     = np.min(test_samples[:,0])
        max_x     = np.max(test_samples[:,0])
        x         = np.arange(start = min_x,stop =max_x)

        w_est_per    = per_cls.w_est    # estimated weight vector from perceptron
        w_est_fld    = fld_cls.w_est    # estimated weight vector from FLDA
```

```
    w_est_linreg = linreg_cls.w_est # estimated weight vector from Linear␣
↪regression
    w_est_logreg = logreg_cls.w_est # estimated weight vector from Logistic␣
↪regression


    # Seperating hyperplanes obtained from classifiers built from scratch
    h_per    =-(w_est_per[0]+w_est_per[1]*x)/w_est_per[2]          #␣
↪Hyperplane obtained from Perceptron
    h_fld    =-(w_est_fld[0]+w_est_fld[1]*x)/w_est_fld[2]          #␣
↪Hyperplane obtained from FLDA
    h_linreg =-(w_est_linreg[0]+w_est_linreg[1]*x)/w_est_linreg[2] #␣
↪Hyperplane obtained from Linear regression
    h_logreg =-(w_est_logreg[0]+w_est_logreg[1]*x)/w_est_logreg[2] #␣
↪Hyperplane obtained from Logistic regression

    # estimated weight vector from perceptron(IN-BUILT)
    w   = per_cls_in.coef_
    w_0 = np.expand_dims(per_cls_in.intercept_,axis=-1)
    w_est_per_in = np.expand_dims(np.append(w_0,w),axis=-1)

    # estimated weight vector from Linear regression(IN-BUILT)
    w   = linreg_cls_in.coef_
    w_0 = np.expand_dims(linreg_cls_in.intercept_,axis=-1)
    w_est_linreg_in = np.expand_dims(np.append(w_0,w),axis=-1)

    # estimated weight vector from Logistic regression(IN-BUILT)
    w   = logreg_cls_in.coef_
    w_0 = np.expand_dims(logreg_cls_in.intercept_,axis=-1)
    w_est_logreg_in = np.expand_dims(np.append(w_0,w),axis=-1)

    # Seperating hyperplanes obtained from IN-BUILT classifiers
    h_per_in    =-(w_est_per_in[0]+w_est_per_in[1]*x)/w_est_per_in[2]        ␣
↪  # Hyperplane obtained from Perceptron(IN-BUILT)
    h_linreg_in =-(w_est_linreg_in[0]+w_est_linreg_in[1]*x)/
↪w_est_linreg_in[2] # Hyperplane obtained from Linear regression(IN-BUILT)
    h_logreg_in =-(w_est_logreg_in[0]+w_est_logreg_in[1]*x)/
↪w_est_logreg_in[2] # Hyperplane obtained from Logistic regression(IN-BUILT)

    avoid_scroll_output_window()

    ## Training data

    train_cls_0 = train_samples[np.where(train_labels == 0)[0],:] # Class 0␣
↪samples in training data
```

```
        train_cls_1 = train_samples[np.where(train_labels == 1)[0],:] # Class 1␣
↪samples in training data

        # Plotting obtained hyperplanes on training samples
        plt.figure(figsize=(8,8))
        plt.scatter(train_cls_0[:,0],train_cls_0[:,1], marker='+',label = 'class␣
↪0')
        plt.scatter(train_cls_1[:,0],train_cls_1[:,1], c= 'green',␣
↪marker='+',label = 'class 1')
        sns.lineplot(x=x,y=h_per,legend = 'brief', label ='Hyperplane obtained␣
↪from Perceptron',linewidth = 2,color='pink')
        sns.lineplot(x=x,y=h_fld,legend = 'brief', label ='Hyperplane obtained␣
↪from FLDA',linewidth = 2,color='purple')
        sns.lineplot(x=x,y=h_linreg,legend = 'brief', label ='Hyperplane␣
↪obtained from Linear regression',linewidth = 2,color='black')
        sns.lineplot(x=x,y=h_logreg,legend = 'brief', label ='Hyperplane␣
↪obtained from Logistic regression',linewidth = 2,color='orange')
        plt.title("Training data(p0:{}, p1:{})".format(p0,p1),fontsize=18,␣
↪fontweight='bold')
        plt.legend()
        plt.show()

        ## Test data

        test_cls_0 = test_samples[np.where(test_labels == 0)[0],:] # Class 0␣
↪samples in test data
        test_cls_1 = test_samples[np.where(test_labels == 1)[0],:] # Class 1␣
↪samples in test data

        # Plotting test data samples
        plt.figure(figsize=(8,8))
        plt.scatter(test_cls_0[:,0],test_cls_0[:,1], marker='+',label = 'class␣
↪0')
        plt.scatter(test_cls_1[:,0],test_cls_1[:,1], c= 'green',␣
↪marker='+',label = 'class 1')
        plt.title("Test data(p0:{}, p1:{})".format(p0,p1),fontsize=18,␣
↪fontweight='bold')
        plt.legend()
        plt.show()

        # Perceptron on test samples
        y_pred           = per_cls.predict(test_samples)
        pred_test_cls_0 = test_samples[np.where(y_pred == -1)[0],:] # Predicted␣
↪class 0 samples in test data
        pred_test_cls_1 = test_samples[np.where(y_pred == 1)[0],:]  # Predicted␣
↪class 1 samples in test data
```

```
    # Plotting hyperplane obtained from perceptron regression on test samples
    plt.figure(figsize=(8,8))
    plt.scatter(pred_test_cls_0[:,0],pred_test_cls_0[:,1], marker='+',label␣
↪= 'Predicted as class 0')
    plt.scatter(pred_test_cls_1[:,0],pred_test_cls_1[:,1], c= 'green',␣
↪marker='+',label = 'Predicted as class 1')
    sns.lineplot(x=x,y=h_per,legend = 'brief', label ='Hyperplane obtained␣
↪from Perceptron',linewidth = 3,color='pink')
    # Computing the accuracy of the estimated weight vector
    test_accuracy_per  = per_cls.Accuracy(test_samples,ntest_labels)
    plt.title("Perceptron classifier on test data(Accuracy:{:.2f} %)".
↪format(test_accuracy_per),fontsize=18, fontweight='bold')
    plt.legend()
    plt.show()

    # FLDA on test samples
    y_pred          = fld_cls.predict(test_samples)
    pred_test_cls_0 = test_samples[np.where(y_pred == 0)[0],:] # Predicted␣
↪class 0 samples in test data
    pred_test_cls_1 = test_samples[np.where(y_pred == 1)[0],:] # Predicted␣
↪class 1 samples in test data

    # Plotting hyperplane obtained from FLDA on test samples
    plt.figure(figsize=(8,8))
    plt.scatter(pred_test_cls_0[:,0],pred_test_cls_0[:,1], marker='+',label␣
↪= 'Predicted as class 0')
    plt.scatter(pred_test_cls_1[:,0],pred_test_cls_1[:,1], c= 'green',␣
↪marker='+',label = 'Predicted as class 1')
    sns.lineplot(x=x,y=h_fld,legend = 'brief', label ='Hyperplane obtained␣
↪from FLDA',linewidth = 3,color='purple')
    # Computing the accuracy of the estimated weight vector
    test_accuracy_fld  = fld_cls.Accuracy(test_samples,test_labels)
    plt.title("FLD classifier on test data(Accuracy:{:.2f} %)".
↪format(test_accuracy_fld),fontsize=18, fontweight='bold')
    plt.legend()
    plt.show()

    # Linear regression on test samples
    y_pred          = linreg_cls.predict(test_samples)
    pred_test_cls_0 = test_samples[np.where(y_pred == -1)[0],:] # Predicted␣
↪class 0 samples in test data
    pred_test_cls_1 = test_samples[np.where(y_pred == 1)[0],:]  # Predicted␣
↪class 1 samples in test data

    # Plotting hyperplane obtained from linear regression on test samples
```

```
    plt.figure(figsize=(8,8))
    plt.scatter(pred_test_cls_0[:,0],pred_test_cls_0[:,1], marker='+',label␣
↪= 'Predicted as class 0')
    plt.scatter(pred_test_cls_1[:,0],pred_test_cls_1[:,1], c= 'green',␣
↪marker='+',label = 'Predicted as class 1')
    sns.lineplot(x=x,y=h_linreg,legend = 'brief', label ='Hyperplane␣
↪obtained from Linear regression',linewidth = 3,color='black')
    # Computing the accuracy of the estimated weight vector
    test_accuracy_linreg  = linreg_cls.Accuracy(test_samples,ntest_labels)
    plt.title("Linear regression classifier on test data(Accuracy:{:.2f} %)".
↪format(test_accuracy_linreg),fontsize=18, fontweight='bold')
    plt.legend()
    plt.show()

    # Logistic regression on test samples
    y_pred          = logreg_cls.predict(test_samples)
    pred_test_cls_0 = test_samples[np.where(y_pred == 0)[0],:] # Predicted␣
↪class 0 samples in test data
    pred_test_cls_1 = test_samples[np.where(y_pred == 1)[0],:] # Predicted␣
↪class 1 samples in test data

    # Plotting hyperplane obtained from logistic regression on test samples
    plt.figure(figsize=(8,8))
    plt.scatter(pred_test_cls_0[:,0],pred_test_cls_0[:,1], marker='+',label␣
↪= 'Predicted as class 0')
    plt.scatter(pred_test_cls_1[:,0],pred_test_cls_1[:,1], c= 'green',␣
↪marker='+',label = 'Predicted as class 1')
    sns.lineplot(x=x,y=h_logreg,legend = 'brief', label ='Hyperplane␣
↪obtained from Logistic regression',linewidth = 3,color='orange')
    # Computing the accuracy of the estimated weight vector
    test_accuracy_logreg  = logreg_cls.Accuracy(test_samples,test_labels)
    plt.title("Logistic regression classifier on test data(Accuracy:{:.2f}␣
↪%)".format(test_accuracy_logreg),fontsize=18, fontweight='bold')
    plt.legend()
    plt.show()
```

4.8 To avoid the scroll in the ouput window of each cell:

[ ]:
```
def avoid_scroll_output_window():
  # To avoid the scroll in the ouput window of each cell
  display(Javascript('''google.colab.output.setIframeHeight(0, true, {maxHeight:␣
↪5000})'''))
```

5 Problem 1:

5.1 Part a: 10 Dimensional data

Different mean vectors, same covariance matrices

5.1.1 Data generation:

[ ]:
```
#dimension of the data
d=10

#mean vectors for class-0 and class-1
mu_0 = np.zeros((1,d))
mu_1 = np.ones((1,d))
mu = np.concatenate((mu_0,mu_1))

#covariance matrix of class-0 and class-1(same: Identity matrix)
covariance = [np.identity(d),np.identity(d)]

#prior probabilities for classes
p0 = 0.7
p1 = 0.3
p = [p0, p1]

# No. of training samples and testing samples
N_train = 7000
N_test  = 3000

#generate training and testing data
train_samples,train_labels,test_samples,test_labels =␣
 ↪generate_data(N_train,N_test,mu,covariance,p)
```

```
train-test split : 70:30

Number of training data points : 7000
Number of test data points  : 3000
Dimension of feature vectors : 10
```

5.1.2 Perceptron:

[ ]:
```
eta = 0.9
N_epochs = 30
_ =␣
 ↪Perceptron_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs)
```

```
*********** PERCEPTRON *************
```

```
train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30

Using classifier built from scratch
Train accuracy= 95.20 %
Test accuracy = 94.63%
```

5.1.3 FLDA:

```
[ ]: _ = FLD_testing(train_samples,train_labels,test_samples,test_labels)
```

```
*********** FLD *************

train-test split : 70:30

Using classifier built from scratch
Train accuracy= 87.21 %
Test accuracy = 86.90%
```

5.1.4 Linear regression:

```
[ ]: _ = Linreg_testing(train_samples,train_labels,test_samples,test_labels)
```

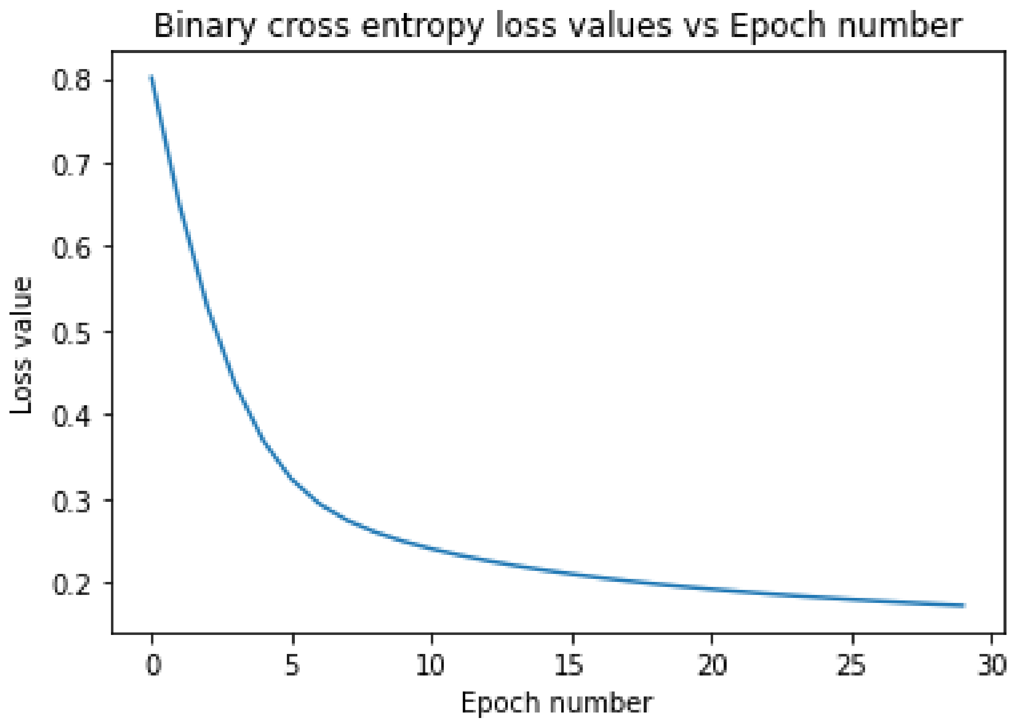
```
*********** LINEAR REGRESSION *************

train-test split : 70:30

Using IN-BUILT classifier
Train accuracy= 95.10 %
Test accuracy = 94.73 %
Using classifier built from scratch
Train accuracy= 95.10 %
Test accuracy = 94.73 %
```

5.1.5 Logistic regression:

```
[ ]: eta      = 0.9
     N_epochs = 30
     _ =␣
      ↪Logreg_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs)
```

```
*********** LOGISTIC REGRESSION *************

train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30

Using IN-BUILT classifier
Train accuracy= 95.19 %
Test accuracy = 94.60 %
Using classifier built from scratch
```

```
Train accuracy= 95.17 %
Test accuracy = 94.37 %
```



5.2 Part b: 2 Dimensional data

Different mean vectors, same covariance matrices

- $\mu_0 = [0, 0]$ and $\mu\_1 = [1, 1]$ and $\Sigma_0 = \Sigma_1$ = Identity Matrix.
- $p_0 = 0.7, p_1 = 0.3$

5.2.1 Data generation:

```
[ ]: #dimension of the data
     d=2

     #mean vectors for class-0 and class-1
     mu_0 = np.zeros((1,d))
     mu_1 = np.ones((1,d))
     mu = np.concatenate((mu_0,mu_1))

     #covariance matrix of class-0 and class-1(same: Identity matrix)
     covariance = [np.identity(d),np.identity(d)]

     #prior probabilities for classes
```

```
p0 = 0.7
p1 = 0.3
p = [p0, p1]

# No. of training samples and testing samples
N_train = 7000
N_test  = 3000

#generate training and testing data
train_samples,train_labels,test_samples,test_labels =␣
 ↪generate_data(N_train,N_test,mu,covariance,p)
```

```
train-test split : 70:30

Number of training data points : 7000
Number of test data points  : 3000
Dimension of feature vectors : 2
```

5.2.2 Perceptron:

```
[ ]: eta = 0.9
N_epochs = 30
per_cls_in,per_cls =␣
 ↪Perceptron_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs)
```

```
*********** PERCEPTRON *************

train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30

Using classifier built from scratch
Train accuracy= 78.11 %
Test accuracy = 76.53%
```

5.2.3 FLDA:

```
[ ]: fld_cls = FLD_testing(train_samples,train_labels,test_samples,test_labels)
```

```
*********** FLD *************

train-test split : 70:30

Using classifier built from scratch
Train accuracy= 72.30 %
Test accuracy = 71.97%
```

5.2.4 Linear regression:

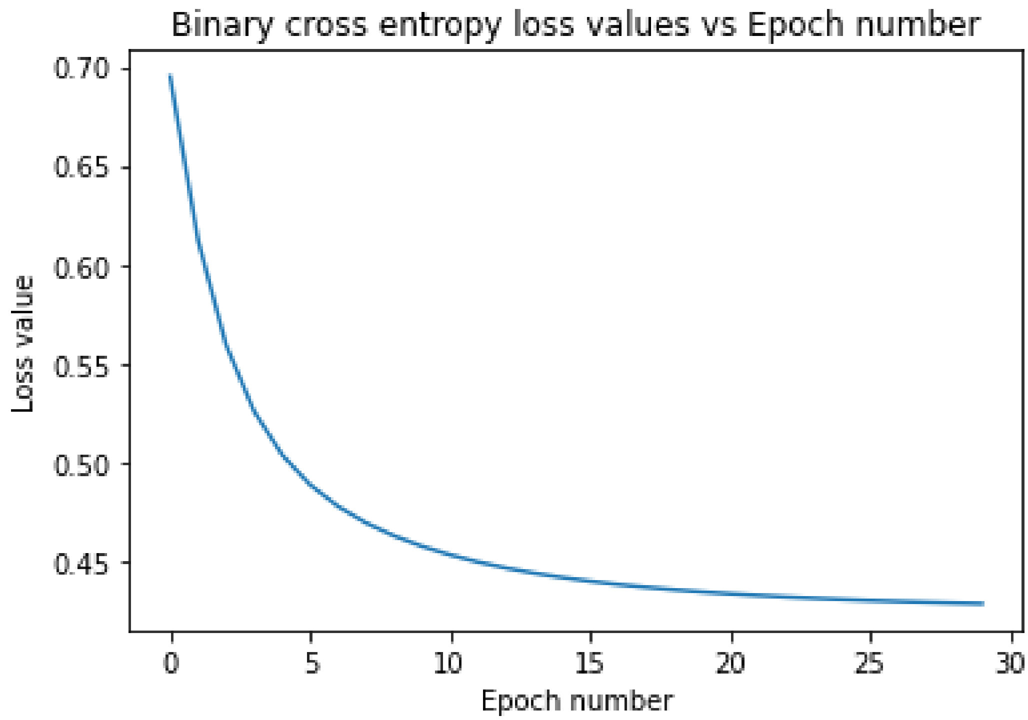
[ ]:
```
linreg_cls_in,linreg_cls =␣
↪Linreg_testing(train_samples,train_labels,test_samples,test_labels)
```

```
*********** LINEAR REGRESSION *************

train-test split : 70:30

Using IN-BUILT classifier
Train accuracy= 80.60 %
Test accuracy = 79.77 %
Using classifier built from scratch
Train accuracy= 80.60 %
Test accuracy = 79.77 %
```

5.2.5 Logistic regression:

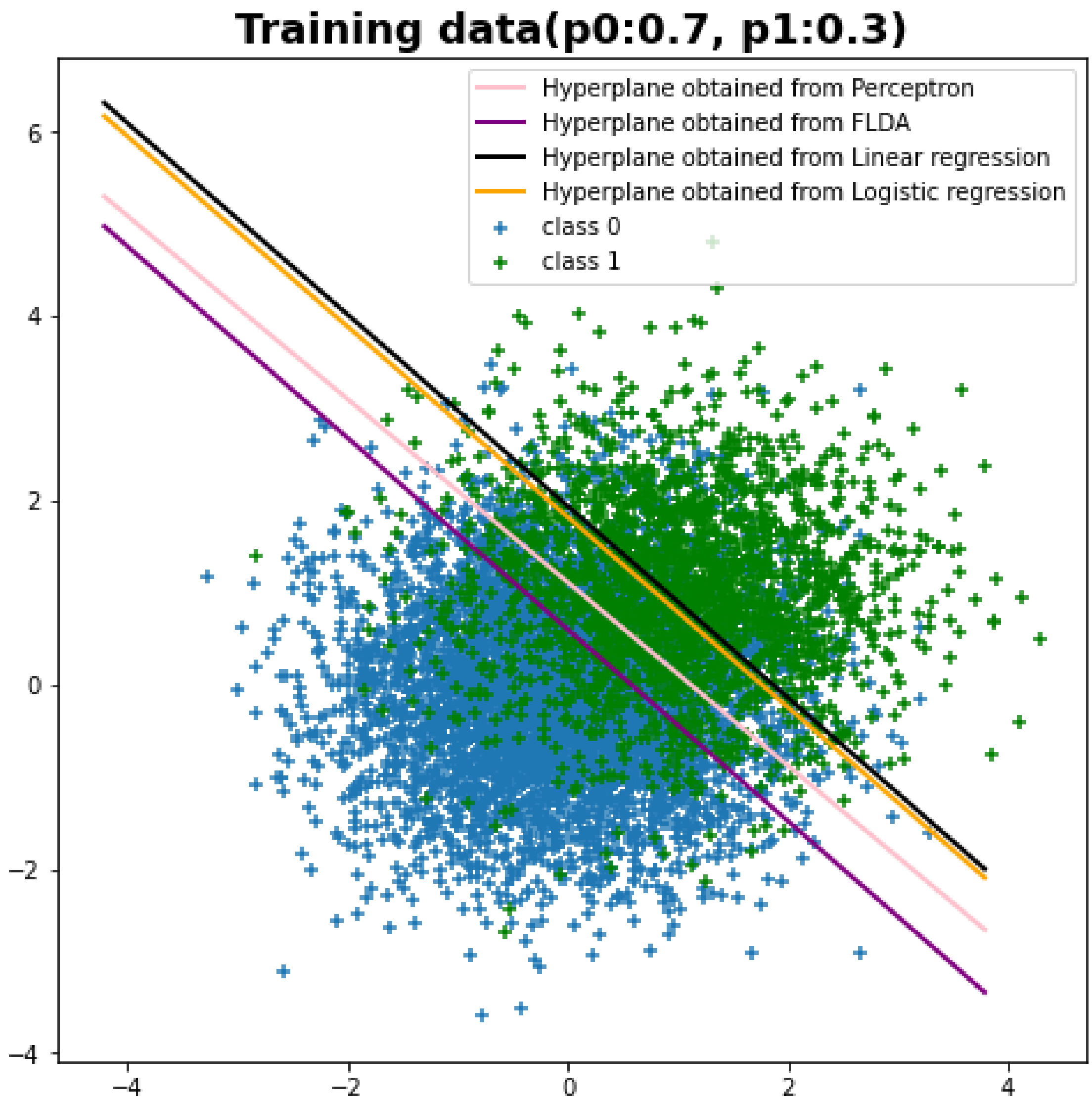
[ ]:
```
eta      = 0.9
N_epochs = 30
logreg_cls_in,logreg_cls =␣
↪Logreg_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs)
```

```
*********** LOGISTIC REGRESSION *************

train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30

Using IN-BUILT classifier
Train accuracy= 80.71 %
Test accuracy = 79.50 %
Using classifier built from scratch
Train accuracy= 80.73 %
Test accuracy = 79.43 %
```

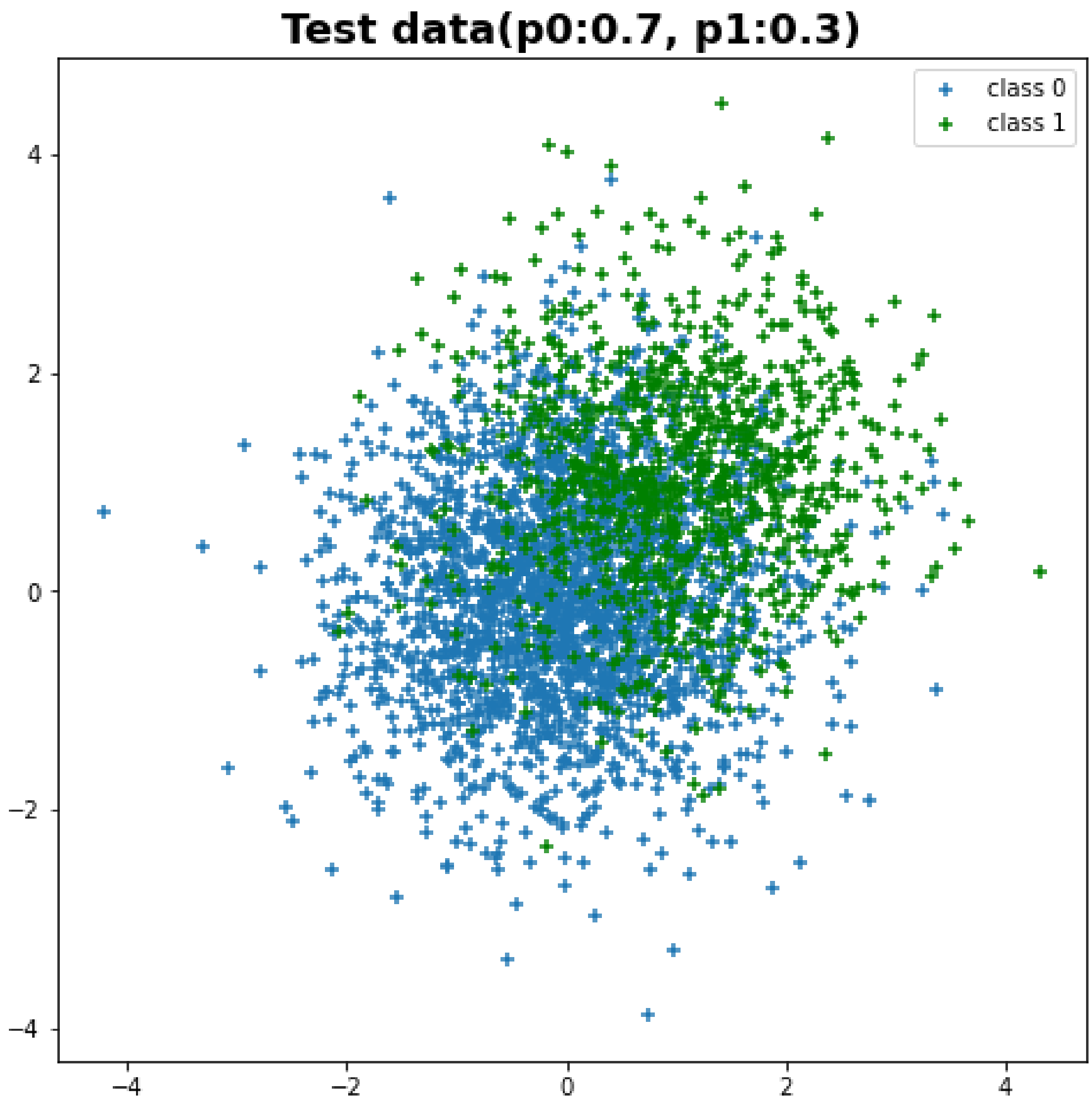


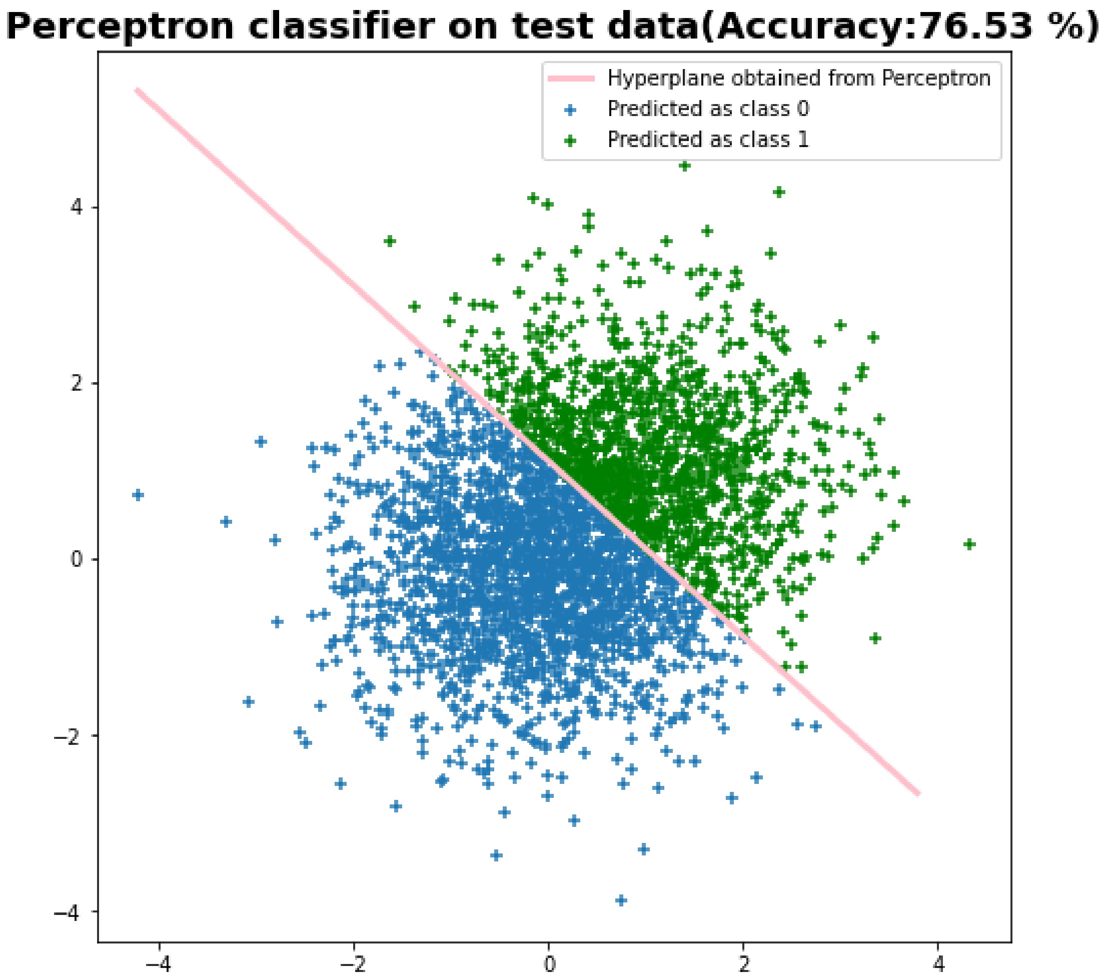
5.2.6 Plotting results:

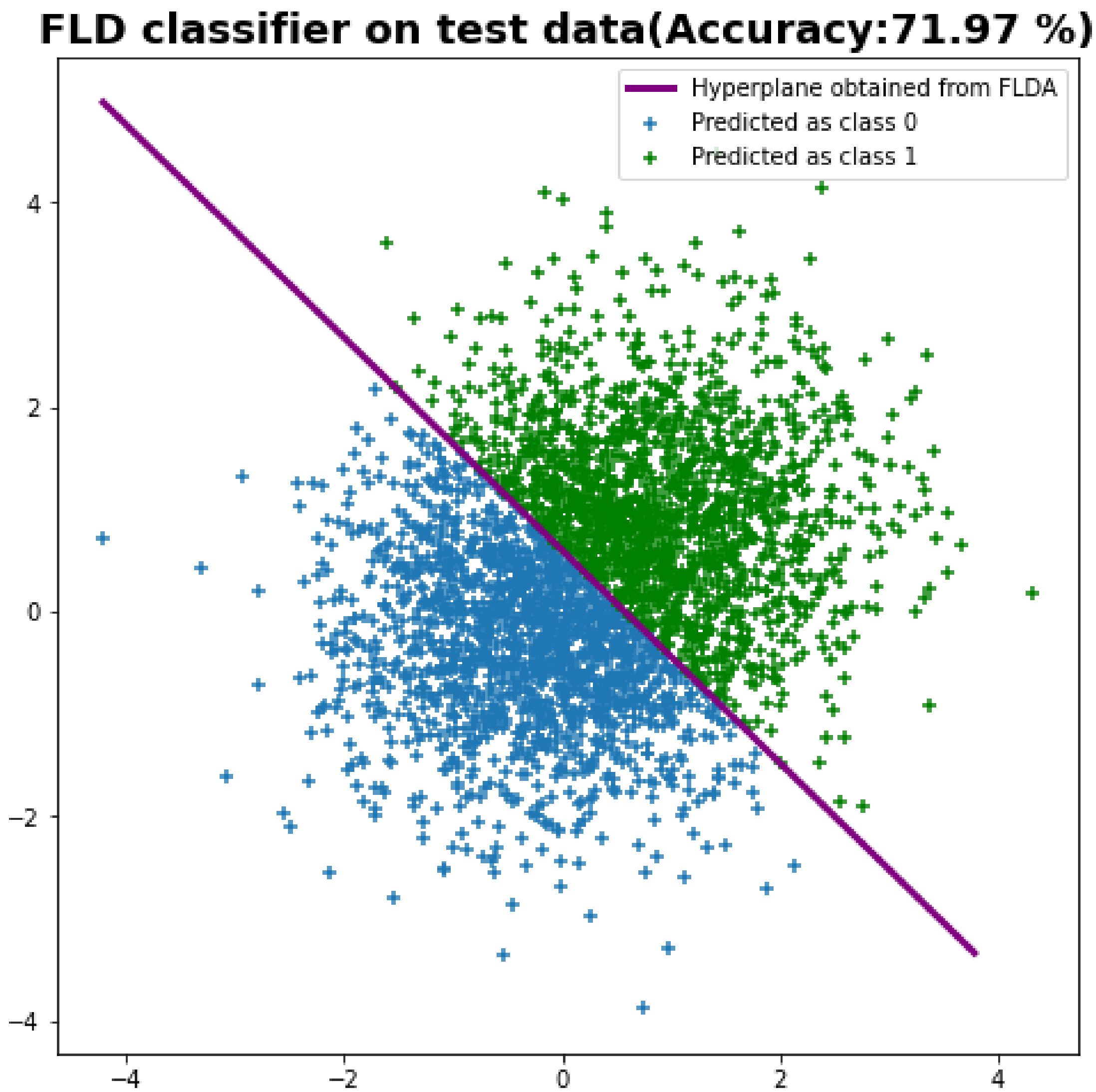
[ ]:
```
plot_res(per_cls,fld_cls,linreg_cls,logreg_cls,per_cls_in,linreg_cls_in,logreg_cls_in,train_samp
```



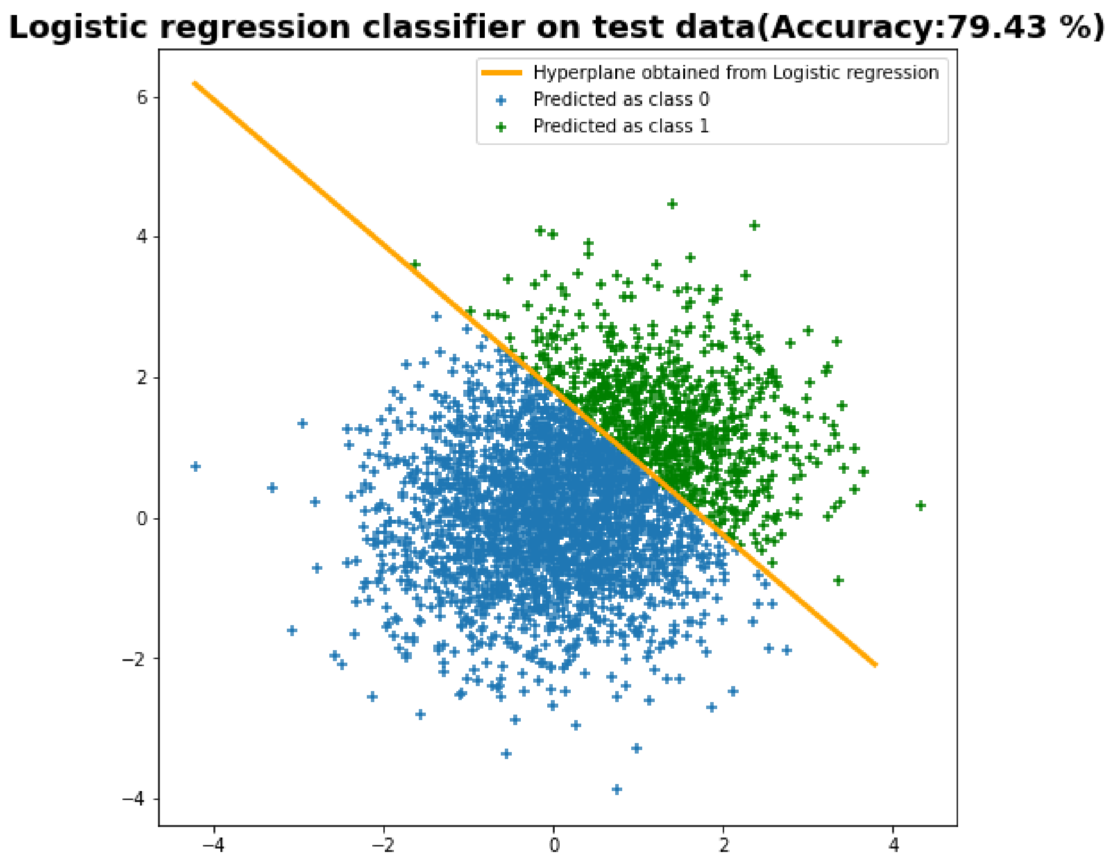
Training data(p0:0.7, p1:0.3)
Hyperplane obtained from Perceptron
Hyperplane obtained from FLDA
Hyperplane obtained from Linear regression
Hyperplane obtained from Logistic regression
class 0
class 1
−4
−2
0
2
4
6


Test data(p0:0.7, p1:0.3)
class 0
class 1
4
2
0
−2
−4
−4
−2
0
2
4


Perceptron classifier on test data(Accuracy:76.53 %)
Hyperplane obtained from Perceptron
Predicted as class 0
Predicted as class 1
4
2
0
−2
−4
−4
−2
0
2
4


FLD classifier on test data(Accuracy:71.97 %)
Hyperplane obtained from FLDA
Predicted as class 0
Predicted as class 1
4
2
0
−2
−4
−4
−2
0
2
4


Linear regression classifier on test data(Accuracy:79.77 %)
Hyperplane obtained from Linear regression
Predicted as class 0
Predicted as class 1
6
4
2
0
−2
−4
−4
−2
0
2
4



5.3 Part c: Linearly non separable data

Same mean vectors, different covariance matrices $\mu_0 = [0, 0]$ and $\mu\_1 = [1, 1]$ and Σ_0 = Identity Matrix, $\Sigma_1 = [1, 0.9; 0.9, 1]$, $p_0 = 0.5, p_1 = 0.5$

5.3.1 Data generation:

[ ]:
```
#mean vectors for class-0 and class-1(same: zero mean vector)
mu = np.concatenate((np.ones((1,d)),np.ones((1,d))))

#covariance matrix of class-0 and class-1
cov_0 = np.identity(d)
cov_1 = np.array([[1,0.9],[0.9,1]])
covariance = [cov_0,cov_1]

#prior probabilities for classes
p0 = 0.5
p1 = 0.5
p = [p0, p1]
```

```
# No. of training samples and testing samples
N_train = 7000
N_test  = 3000

#generate training and testing data
train_samples,train_labels,test_samples,test_labels =␣
 ↪generate_data(N_train,N_test,mu,covariance,p)

#Transform generated d dimensional data to higher dimension where it can be␣
 ↪linearly separable
#Transforming the data to higher dimension using polynomial transformation
poly = PolynomialFeatures(2, include_bias= 1)

train_samples_tr = poly.fit_transform(train_samples)
test_samples_tr  = poly.fit_transform(test_samples)
print(f"Dimension of transformed feature vectors : {np.
 ↪shape(train_samples_tr)[1]}")
```

```
train-test split : 70:30

Number of training data points : 7000
Number of test data points  : 3000
Dimension of feature vectors : 2

Dimension of transformed feature vectors : 6
```

5.3.2 Perceptron:

[ ]:
```
eta = 0.9
N_epochs = 30
per_cls_in,per_cls =␣
 ↪Perceptron_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs)
# Perceptron on transformed data
print("\nPerceptron on transformed data\n")
per_cls_in_tr,per_cls_tr =␣
 ↪Perceptron_testing(train_samples_tr,train_labels,test_samples_tr,test_labels,eta,N_epochs)
```

```
*********** PERCEPTRON *************

train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30

Using classifier built from scratch
Train accuracy= 52.37 %
Test accuracy = 51.23%

Perceptron on transformed data
```

```
*********** PERCEPTRON *************

train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30

Using classifier built from scratch
Train accuracy= 58.67 %
Test accuracy = 57.93%
```

5.3.3 FLDA:

```
[ ]: fld_cls = FLD_testing(train_samples,train_labels,test_samples,test_labels)
     # FLDA on transformed data
     print("\nFLDA on transformed data\n")
     fld_cls_tr =␣
      ↪FLD_testing(train_samples_tr,train_labels,test_samples_tr,test_labels)
```

```
*********** FLD *************

train-test split : 70:30

Using classifier built from scratch
Train accuracy= 50.71 %
Test accuracy = 49.77%

FLDA on transformed data

*********** FLD *************

train-test split : 70:30

Using classifier built from scratch
Train accuracy= 73.93 %
Test accuracy = 73.60%
```

5.3.4 Linear regression:

```
[ ]: linreg_cls_in,linreg_cls =␣
      ↪Linreg_testing(train_samples,train_labels,test_samples,test_labels)
     # Linear regression on transformed data
     print("\nLinear regression on transformed data\n")
     linreg_cls_in_tr,linreg_cls_tr =␣
      ↪Linreg_testing(train_samples_tr,train_labels,test_samples_tr,test_labels)
```

```
*********** LINEAR REGRESSION *************

train-test split : 70:30

Using IN-BUILT classifier
```

```
Train accuracy= 51.91 %
Test accuracy = 50.80 %
Using classifier built from scratch
Train accuracy= 51.91 %
Test accuracy = 50.80 %

Linear regression on transformed data

*********** LINEAR REGRESSION *************

train-test split : 70:30

Using IN-BUILT classifier
Train accuracy= 73.89 %
Test accuracy = 73.43 %
Using classifier built from scratch
Train accuracy= 73.89 %
Test accuracy = 73.43 %
```

5.3.5 Logistic regression:

[ ]:
```
eta      = 0.9
N_epochs = 30
logreg_cls_in,logreg_cls =␣
 ↪Logreg_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs,False)
# Logistic regression on transformed data
print("\nLogistic regression on transformed data\n")
logreg_cls_in_tr,logreg_cls_tr =␣
 ↪Logreg_testing(train_samples_tr,train_labels,test_samples_tr,test_labels,eta,N_epochs,False)
```

```
*********** LOGISTIC REGRESSION *************

train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30

Using IN-BUILT classifier
Train accuracy= 51.90 %
Test accuracy = 50.77 %
Using classifier built from scratch
Train accuracy= 55.19 %
Test accuracy = 55.10 %

Logistic regression on transformed data

*********** LOGISTIC REGRESSION *************

train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30
```

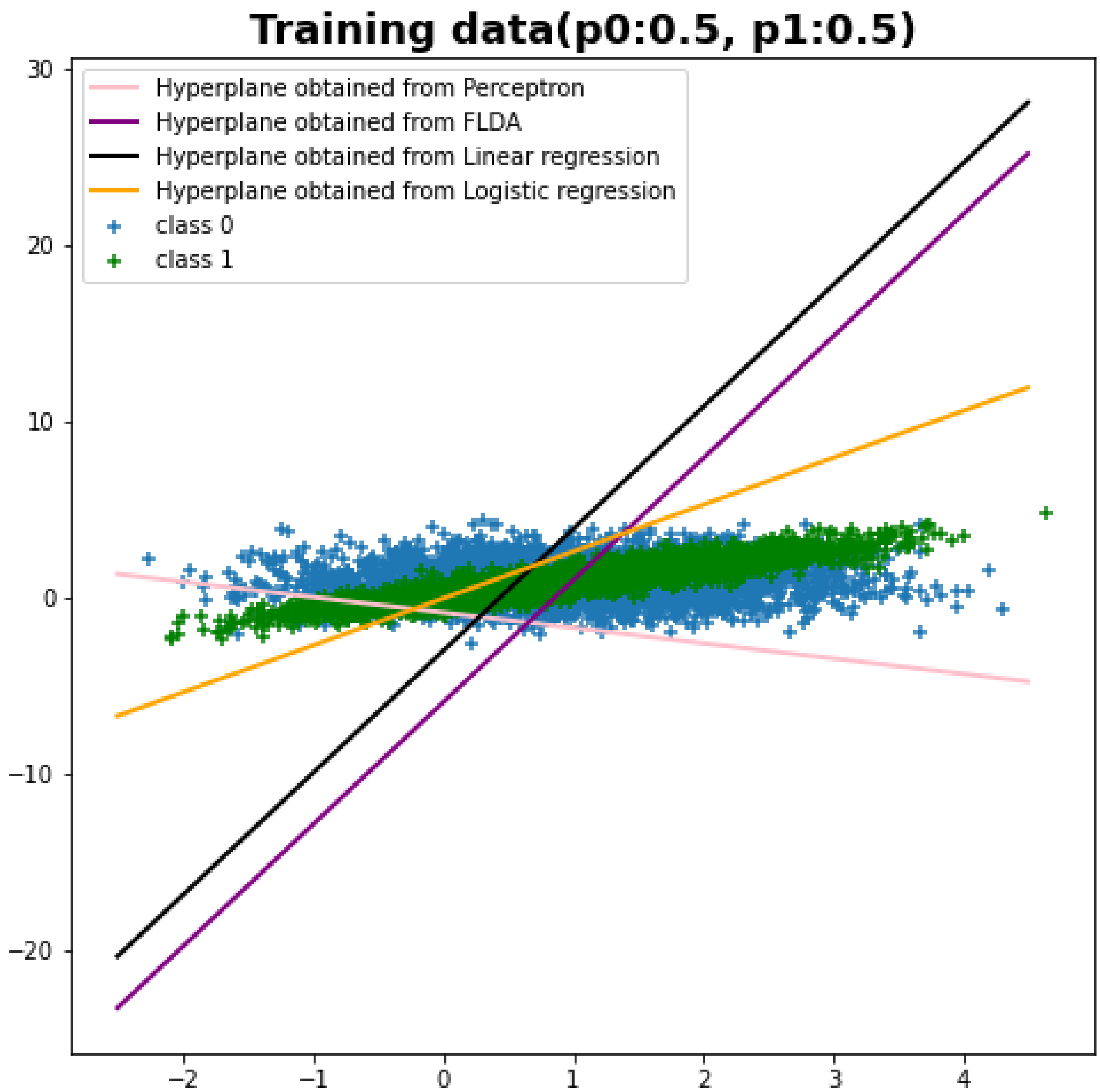
```
Using IN-BUILT classifier
Train accuracy= 76.03 %
Test accuracy = 74.80 %
Using classifier built from scratch
Train accuracy= 65.56 %
Test accuracy = 64.07 %
```

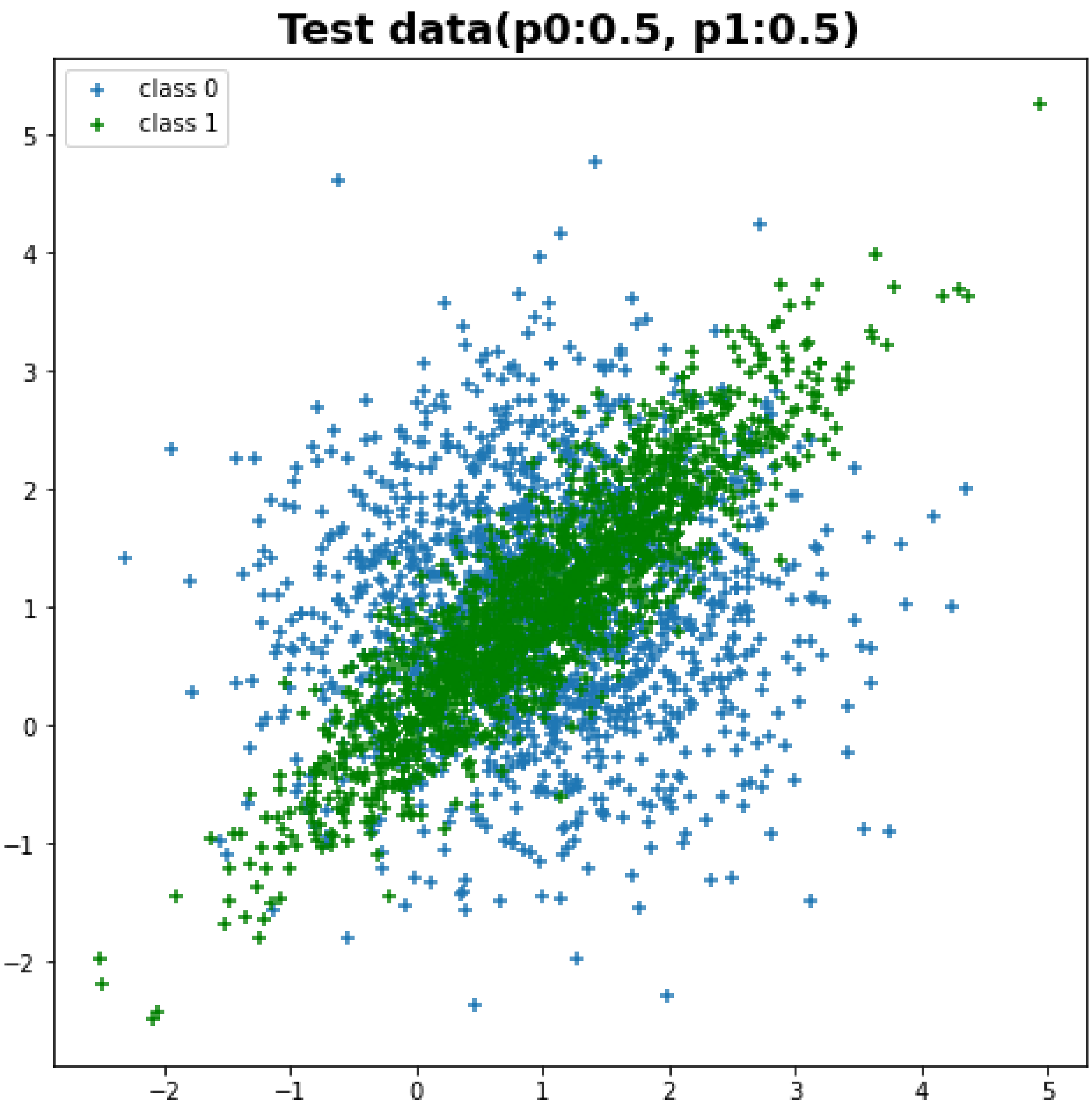
5.3.6 Plotting results:

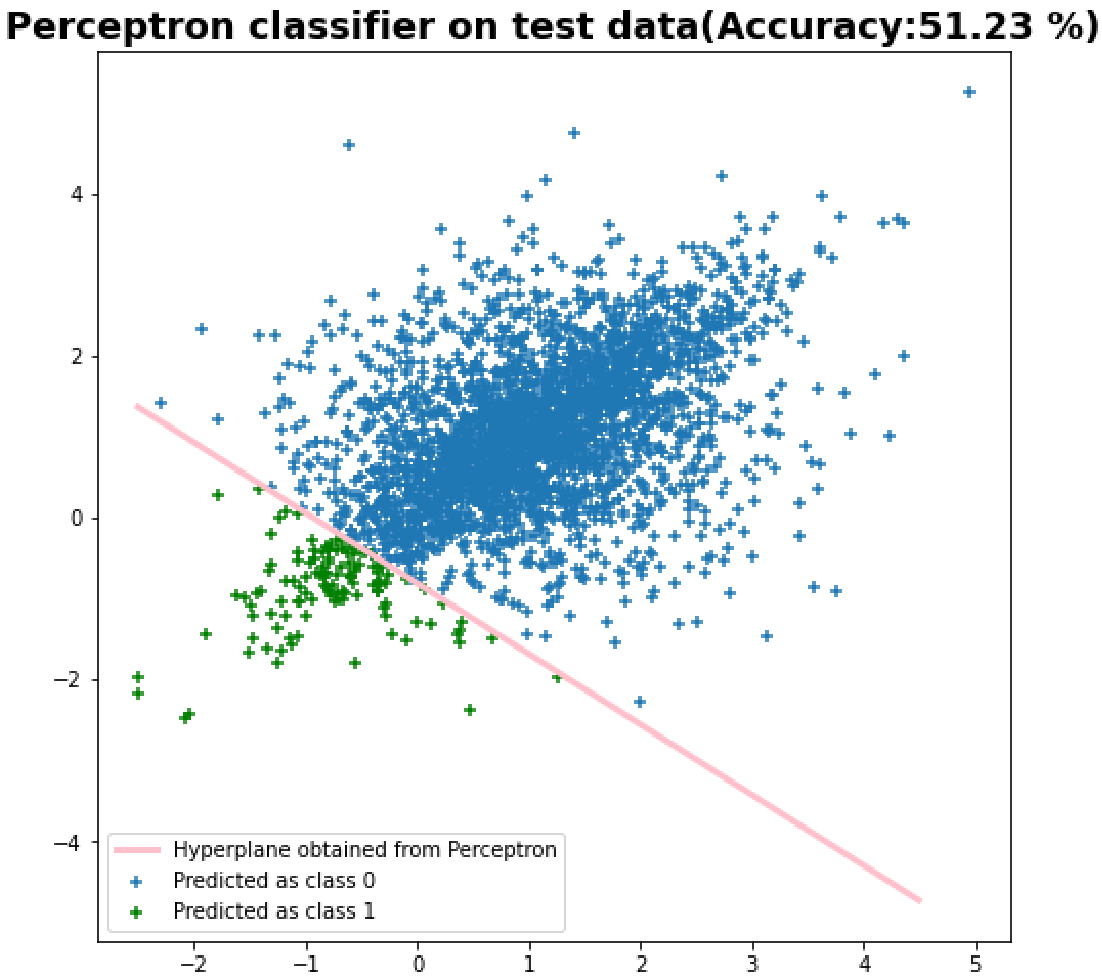
[ ]:
```
plot_res(per_cls,fld_cls,linreg_cls,logreg_cls,per_cls_in,linreg_cls_in,logreg_cls_in,train_samp
# Plot of classification on transformed data

ntest_labels  = 2*test_labels - 1
# Perceptron on transformed test samples
y_pred          = per_cls_tr.predict(test_samples_tr)
pred_test_cls_0 = test_samples[np.where(y_pred == -1)[0],:] # Predicted class 0␣
 ↪samples in transformed test data
pred_test_cls_1 = test_samples[np.where(y_pred == 1)[0],:]  # Predicted class 1␣
 ↪samples in transformed test data

plt.figure(figsize=(8,8))
plt.scatter(pred_test_cls_0[:,0],pred_test_cls_0[:,1], marker='+',label =␣
 ↪'Predicted as class 0')
plt.scatter(pred_test_cls_1[:,0],pred_test_cls_1[:,1], c= 'green',␣
 ↪marker='+',label = 'Predicted as class 1')
# Computing the accuracy of the estimated weight vector
test_accuracy_per  = per_cls_tr.Accuracy(test_samples_tr,ntest_labels)
plt.title("Perceptron classifier on transformed test data(Accuracy:{:.2f} %)".
 ↪format(test_accuracy_per),fontsize=18, fontweight='bold')
plt.legend()
plt.show()

# FLDA on transformed test samples
y_pred          = fld_cls_tr.predict(test_samples_tr)
pred_test_cls_0 = test_samples[np.where(y_pred == 0)[0],:] # Predicted class 0␣
 ↪samples in transformed test data
pred_test_cls_1 = test_samples[np.where(y_pred == 1)[0],:] # Predicted class 1␣
 ↪samples in transformed test data

plt.figure(figsize=(8,8))
plt.scatter(pred_test_cls_0[:,0],pred_test_cls_0[:,1], marker='+',label =␣
 ↪'Predicted as class 0')
plt.scatter(pred_test_cls_1[:,0],pred_test_cls_1[:,1], c= 'green',␣
 ↪marker='+',label = 'Predicted as class 1')
# Computing the accuracy of the estimated weight vector
test_accuracy_fld  = fld_cls_tr.Accuracy(test_samples_tr,test_labels)
```

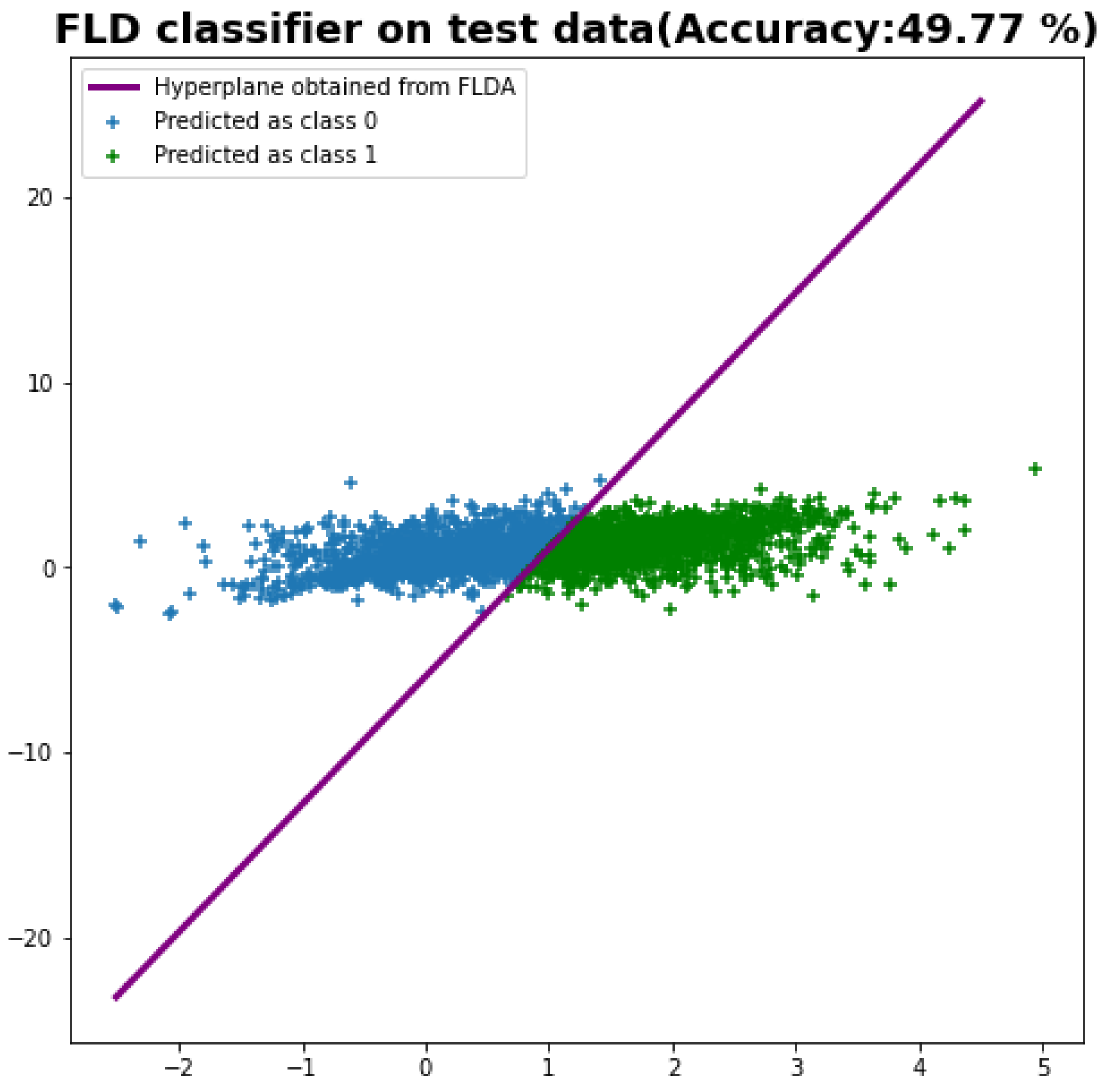
```
plt.title("FLD classifier on transformed test data(Accuracy:{:.2f} %)".
 ↪format(test_accuracy_fld),fontsize=18, fontweight='bold')
plt.legend()
plt.show()

# Linear regression on transformed test samples
y_pred          = linreg_cls_tr.predict(test_samples_tr)
pred_test_cls_0 = test_samples[np.where(y_pred == -1)[0],:] # Predicted class 0␣
 ↪samples in transformed test data
pred_test_cls_1 = test_samples[np.where(y_pred == 1)[0],:]  # Predicted class 1␣
 ↪samples in transformed test data

plt.figure(figsize=(8,8))
plt.scatter(pred_test_cls_0[:,0],pred_test_cls_0[:,1], marker='+',label =␣
 ↪'Predicted as class 0')
plt.scatter(pred_test_cls_1[:,0],pred_test_cls_1[:,1], c= 'green',␣
 ↪marker='+',label = 'Predicted as class 1')
# Computing the accuracy of the estimated weight vector
test_accuracy_linreg  = linreg_cls_tr.Accuracy(test_samples_tr,ntest_labels)
plt.title("Linear regression classifier on transformed test data(Accuracy:{:.2f}␣
 ↪%)".format(test_accuracy_linreg),fontsize=18, fontweight='bold')
plt.legend()
plt.show()

# Logistic regression on transformed test samples
y_pred          = logreg_cls_tr.predict(test_samples_tr)
pred_test_cls_0 = test_samples[np.where(y_pred == 0)[0],:] # Predicted class 0␣
 ↪samples in transformed test data
pred_test_cls_1 = test_samples[np.where(y_pred == 1)[0],:] # Predicted class 1␣
 ↪samples in transformed test data

plt.figure(figsize=(8,8))
plt.scatter(pred_test_cls_0[:,0],pred_test_cls_0[:,1], marker='+',label =␣
 ↪'Predicted as class 0')
plt.scatter(pred_test_cls_1[:,0],pred_test_cls_1[:,1], c= 'green',␣
 ↪marker='+',label = 'Predicted as class 1')
# Computing the accuracy of the estimated weight vector
test_accuracy_logreg  = logreg_cls_tr.Accuracy(test_samples_tr,test_labels)
plt.title("Logistic regression classifier on transformed test data(Accuracy:{:.
 ↪2f} %)".format(test_accuracy_logreg),fontsize=18, fontweight='bold')
plt.legend()
plt.show()
```

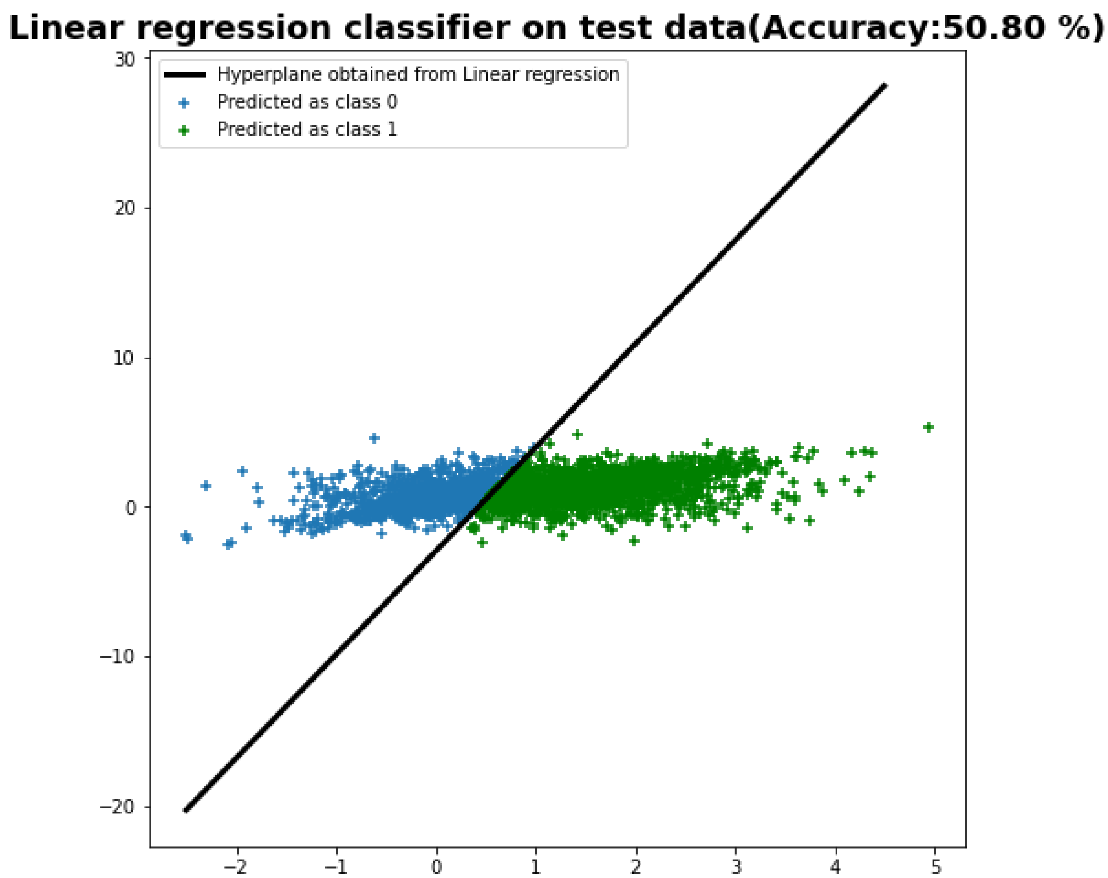
```
<IPython.core.display.Javascript object>
```

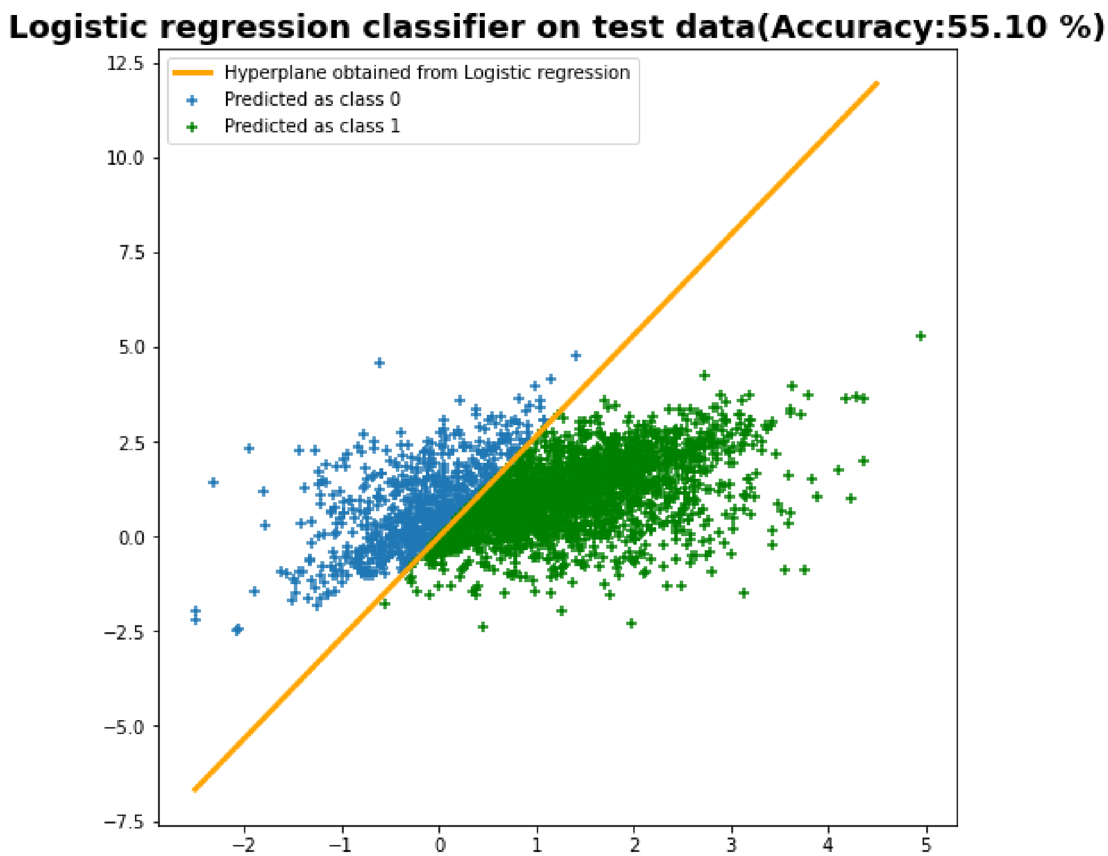

Training data(p0:0.5, p1:0.5)
Hyperplane obtained from Perceptron
Hyperplane obtained from FLDA
Hyperplane obtained from Linear regression
Hyperplane obtained from Logistic regression
class 0
class 1
30
20
10
0
−10
−20
−2
−1
0
1
2
3
4

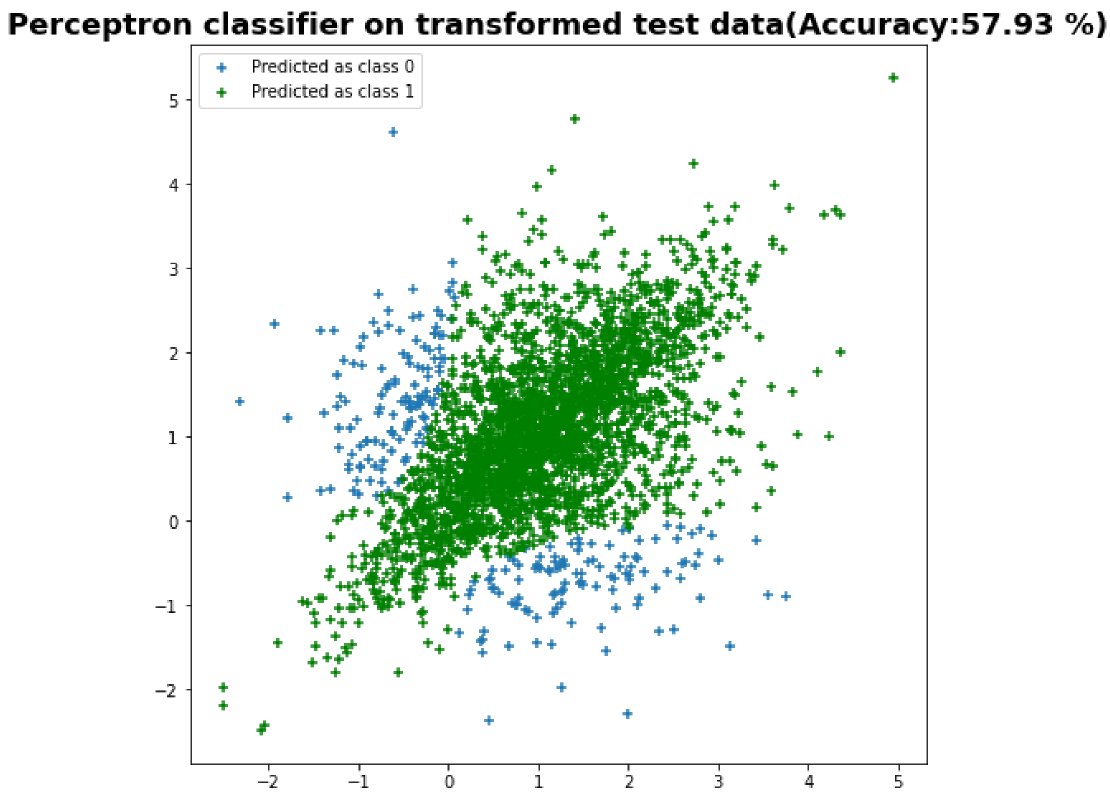

Test data(p0:0.5, p1:0.5)
class 0
class 1
5
4
3
2
1
0
−1
−2
−2
−1
0
1
2
3
4
5


Perceptron classifier on test data(Accuracy:51.23 %)
Hyperplane obtained from Perceptron
Predicted as class 0
Predicted as class 1
4
2
0
−2
−4
−2
−1
0
1
2
3
4
5

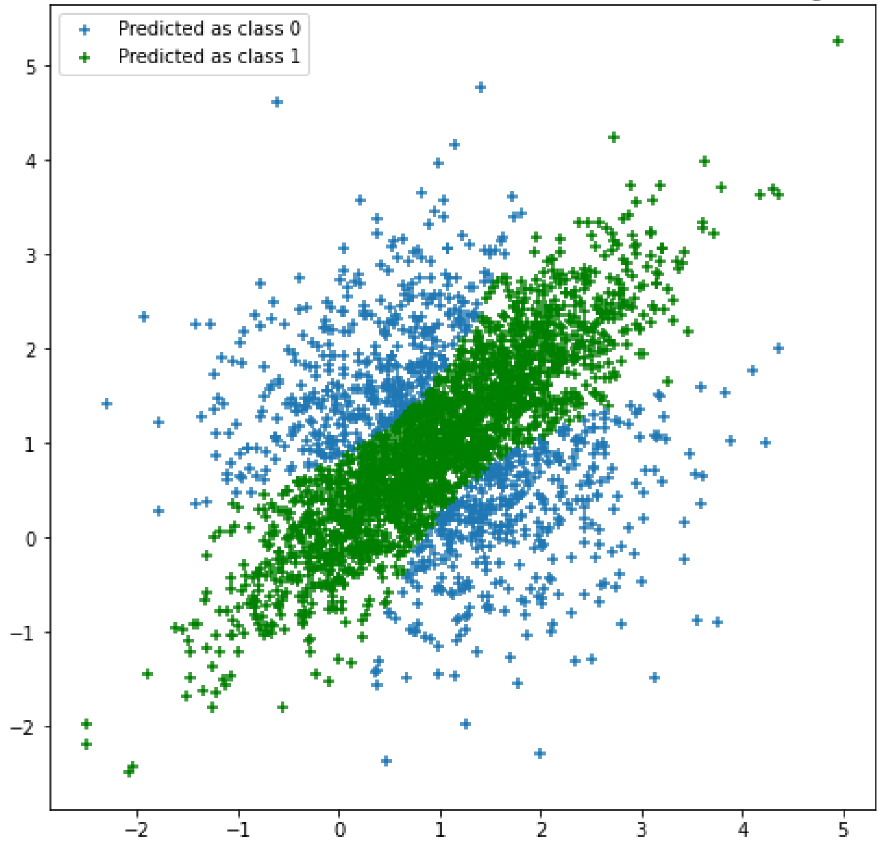

FLD classifier on test data(Accuracy:49.77 %)
Hyperplane obtained from FLDA
Predicted as class 0
Predicted as class 1
20
10
0
−10
−20
−2
−1
0
1
2
3
4
5


Linear regression classifier on test data(Accuracy:50.80 %)
Hyperplane obtained from Linear regression
Predicted as class 0
Predicted as class 1
30
20
10
0
−10
−20
−2
−1
0
1
2
3
4
5

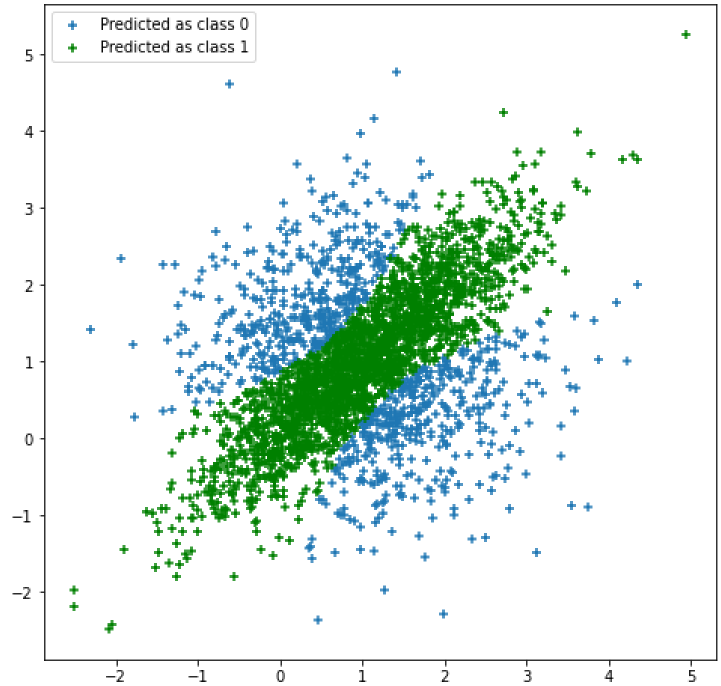

Logistic regression classifier on test data(Accuracy:55.10 %)
Hyperplane obtained from Logistic regression
Predicted as class 0
Predicted as class 1
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−2
−1
0
1
2
3
4
5


Perceptron classifier on transformed test data(Accuracy:57.93 %)
Predicted as class 0
Predicted as class 1
5
4
3
2
1
0
−1
−2
−2
−1
0
1
2
3
4
5

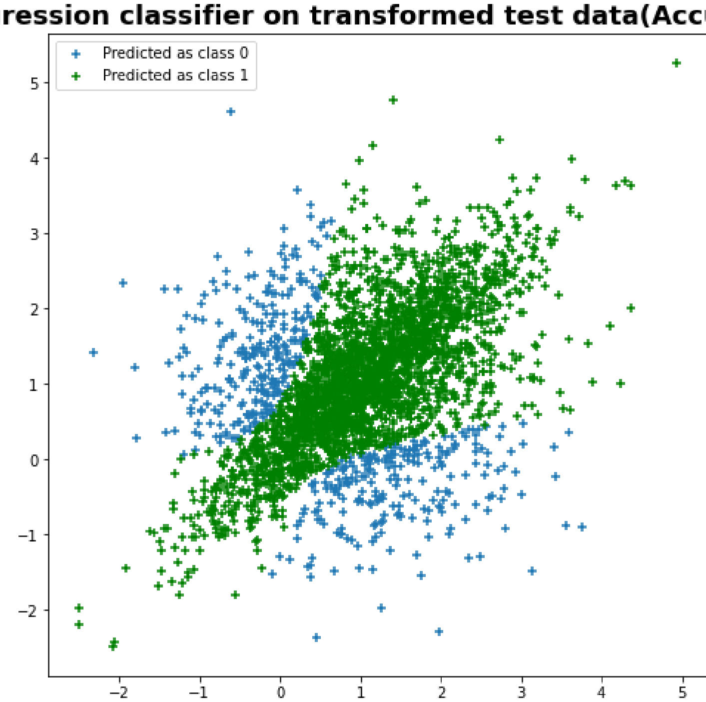
FLD classifier on transformed test data(Accuracy:73.60 %)



Linear regression classifier on transformed test data(Accuracy:73.43 %)



Logistic regression classifier on transformed test data(Accuracy:64.07 %)



5.4 Part d:

Different mean vectors & different covariance matrices $\mu_0 = [3, 6]$ and $\mu\_1 = [3, -2]$ and $\Sigma_0 = [0.5, 0; 0, 2]$, $\Sigma_1 = [2, 0; 0, 2]$. $p_0 = 0.8, p_1 = 0.2$

5.4.1 Data generation:

[ ]:
```
#mean vectors for class-0 and class-1(same: zero mean vector)
mu_0 = np.array([[3],[6]])
mu_1 = np.array([[3],[-2]])
mu   = np.concatenate((mu_0.T,mu_1.T))

#covariance matrix of class-0 and class-1
cov_0 = np.array([[0.5,0],[0,2]])
cov_1 = np.array([[2,0],[0,2]])
covariance = [cov_0,cov_1]

#prior probabilities for classes
p0 = 0.8
p1 = 0.2
p = [p0, p1]

# No. of training samples and testing samples
N_train = 7000
N_test  = 3000

#generate training and testing data
train_samples,train_labels,test_samples,test_labels =␣
 ↪generate_data(N_train,N_test,mu,covariance,p)
```

```
train-test split : 70:30

Number of training data points : 7000
Number of test data points   : 3000
Dimension of feature vectors : 2
```

5.4.2 Perceptron:

[ ]:
```
eta = 0.9
N_epochs = 30
per_cls_in,per_cls =␣
 ↪Perceptron_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs)
```

```
*********** PERCEPTRON *************

train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30
```

```
Using classifier built from scratch
Train accuracy= 99.04 %
Test accuracy = 99.00%
```

5.4.3 FLDA:

```
[ ]: fld_cls = FLD_testing(train_samples,train_labels,test_samples,test_labels)
```

```
*********** FLD *************

train-test split : 70:30

Using classifier built from scratch
Train accuracy= 90.60 %
Test accuracy = 90.23%
```

5.4.4 Linear regression:

```
[ ]: linreg_cls_in,linreg_cls =␣
      ↪Linreg_testing(train_samples,train_labels,test_samples,test_labels)
```

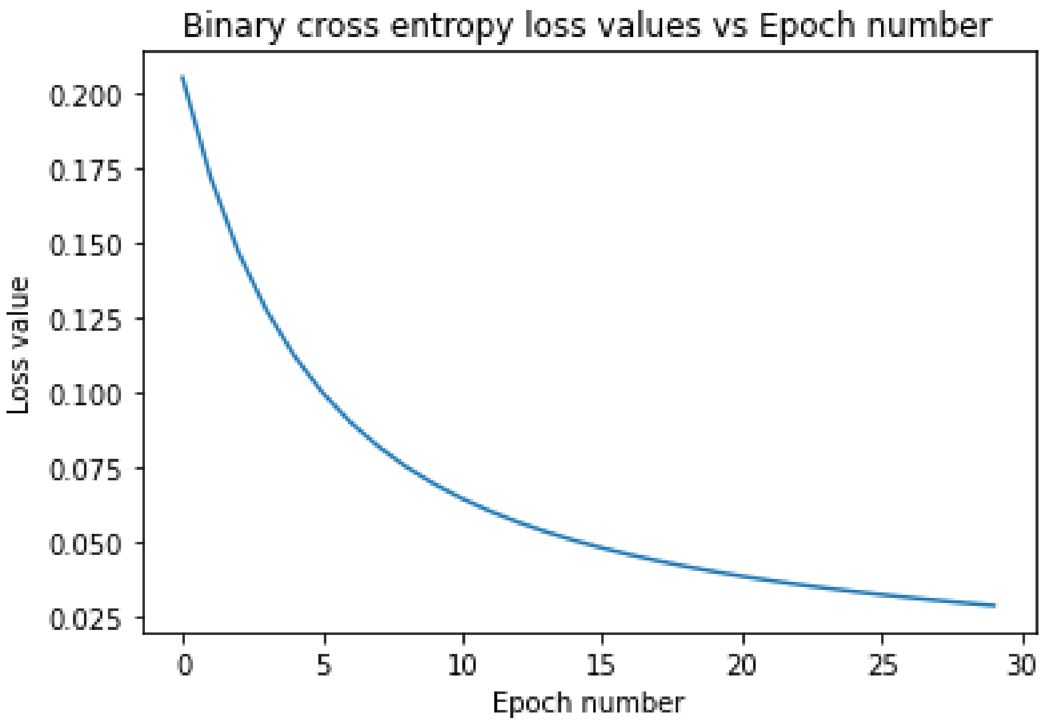
```
*********** LINEAR REGRESSION *************

train-test split : 70:30

Using IN-BUILT classifier
Train accuracy= 99.80 %
Test accuracy = 99.77 %
Using classifier built from scratch
Train accuracy= 99.80 %
Test accuracy = 99.77 %
```

5.4.5 Logistic regression:

```
[ ]: eta      = 0.9
     N_epochs = 30
     logreg_cls_in,logreg_cls =␣
      ↪Logreg_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs)
```

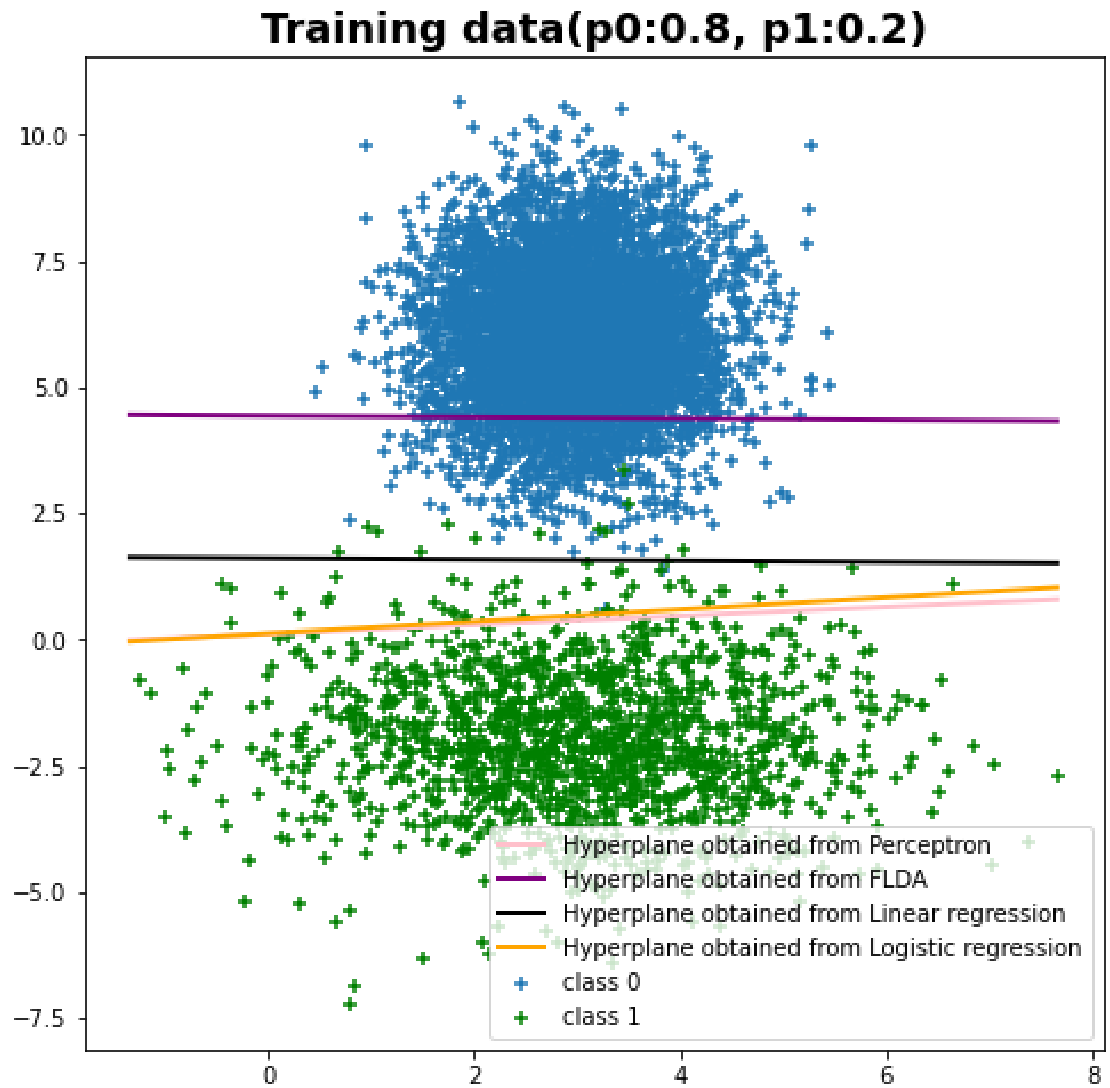
```
*********** LOGISTIC REGRESSION *************

train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30

Using IN-BUILT classifier
Train accuracy= 99.80 %
Test accuracy = 99.73 %
Using classifier built from scratch
```

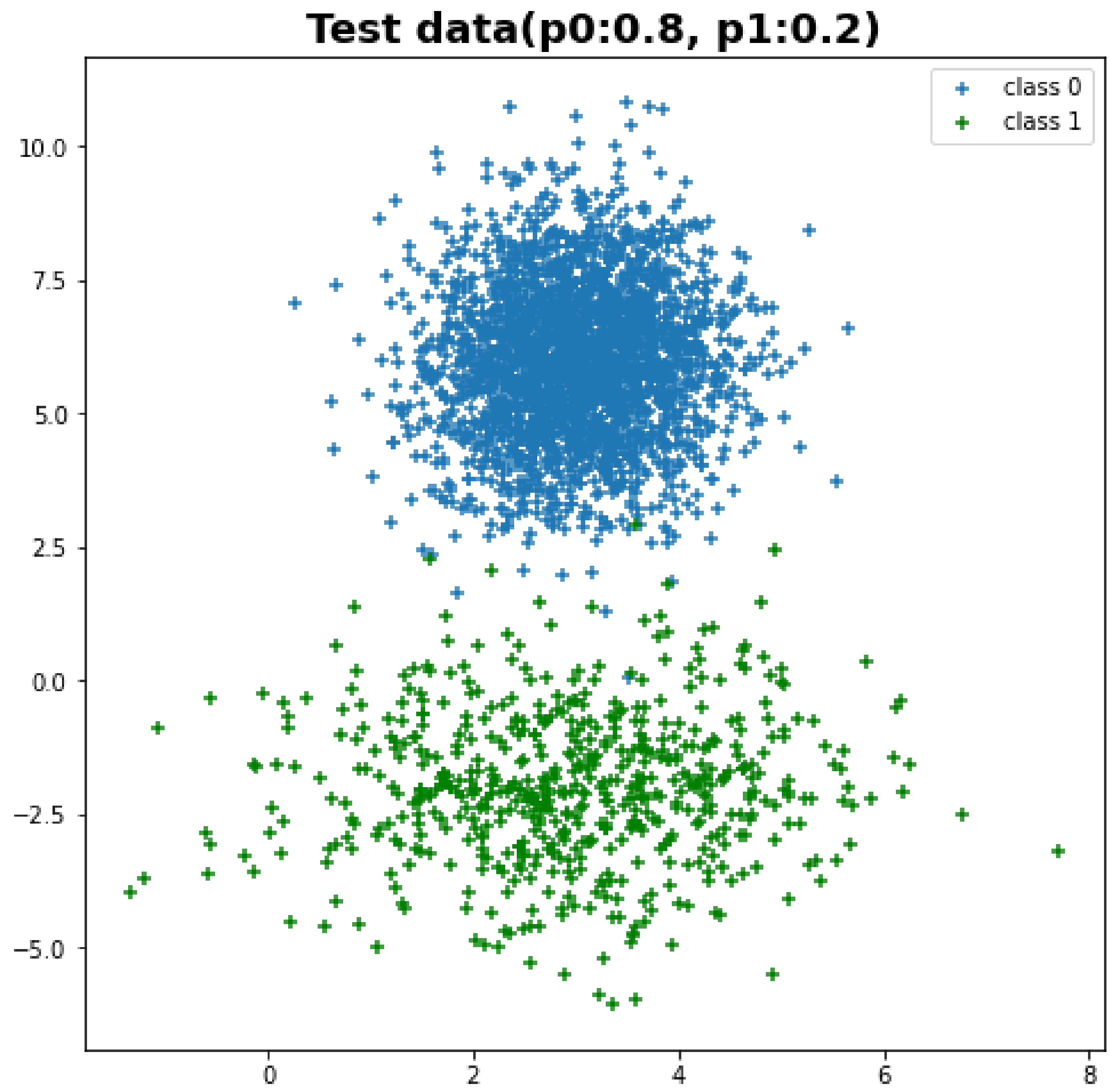
```
Train accuracy= 99.11 %
Test accuracy = 99.17 %
```

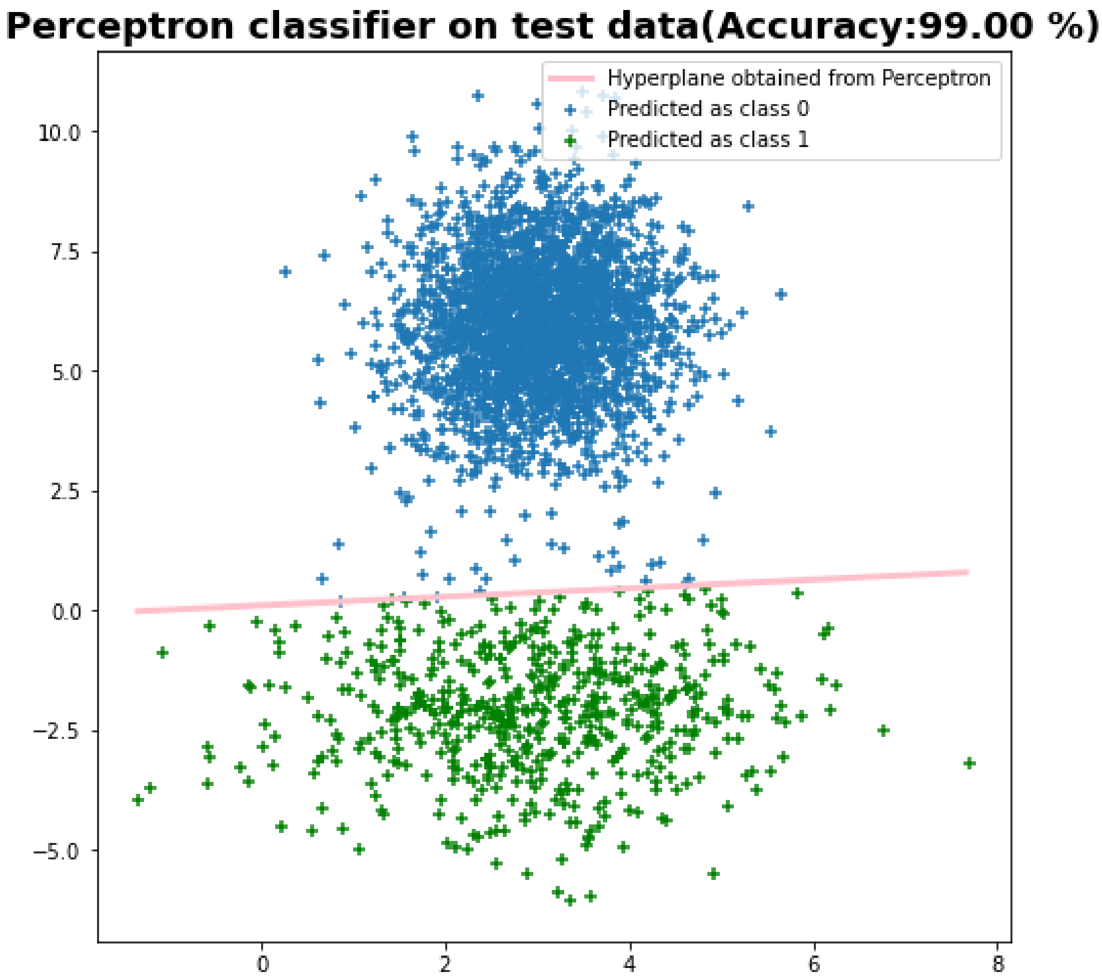


5.4.6 Plotting results:

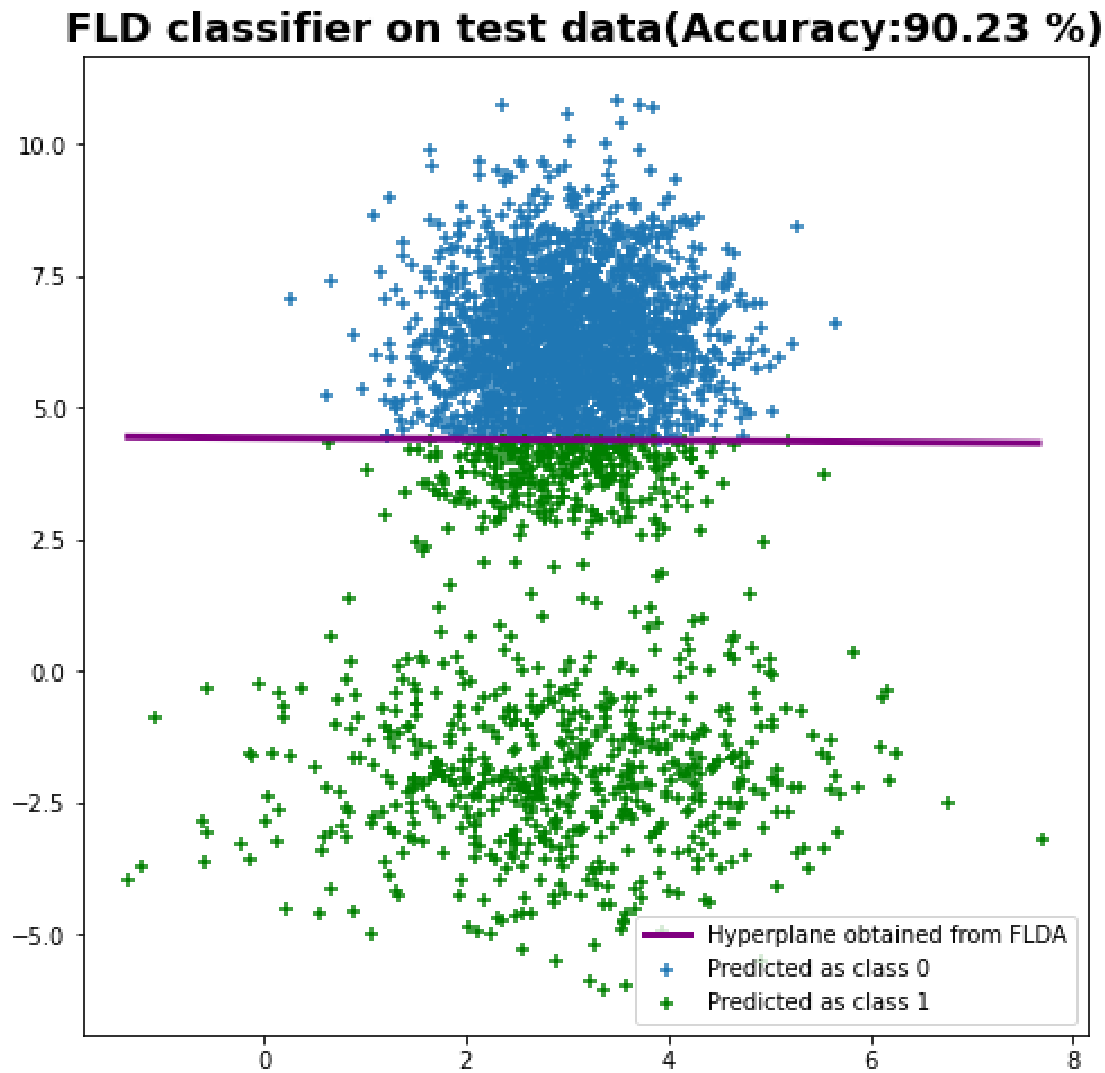
[ ]:
```
plot_res(per_cls,fld_cls,linreg_cls,logreg_cls,per_cls_in,linreg_cls_in,logreg_cls_in,train_samp
```

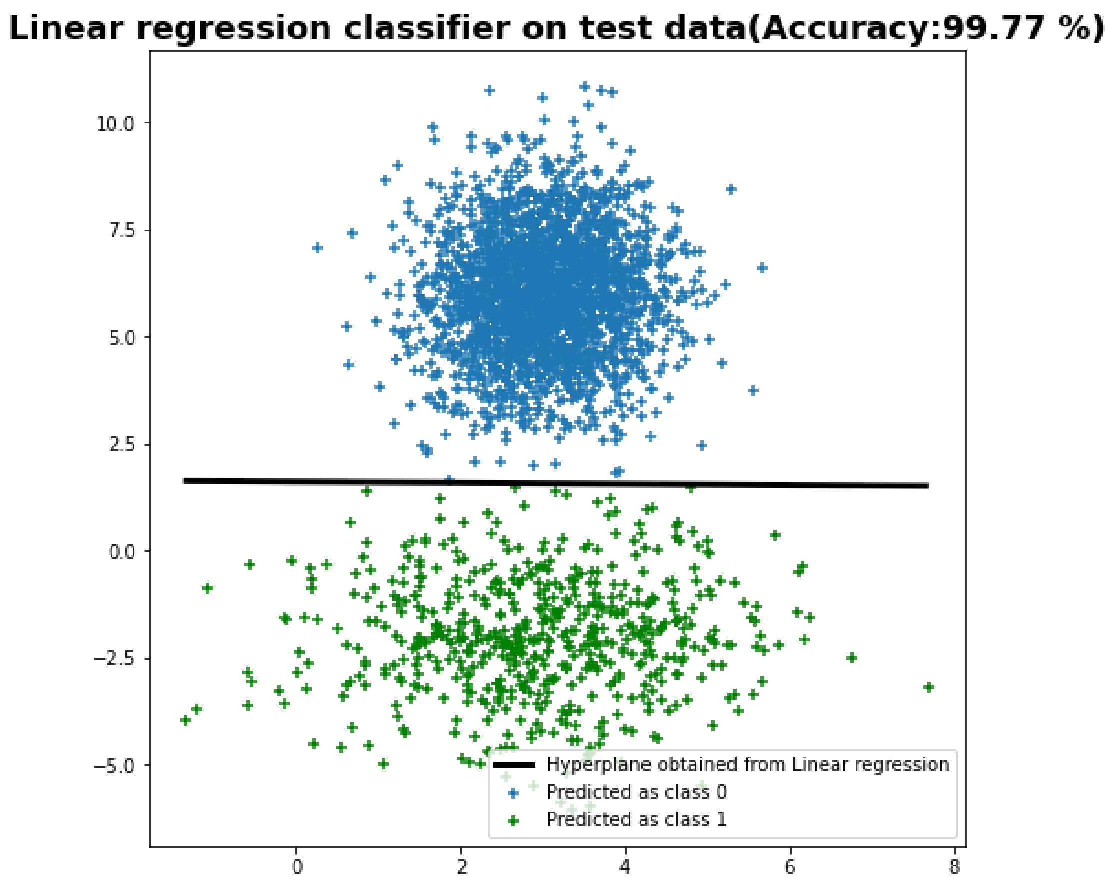

Training data(p0:0.8, p1:0.2)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
0
2
4
6
8
Hyperplane obtained from Perceptron
Hyperplane obtained from FLDA
Hyperplane obtained from Linear regression
Hyperplane obtained from Logistic regression
class 0
class 1

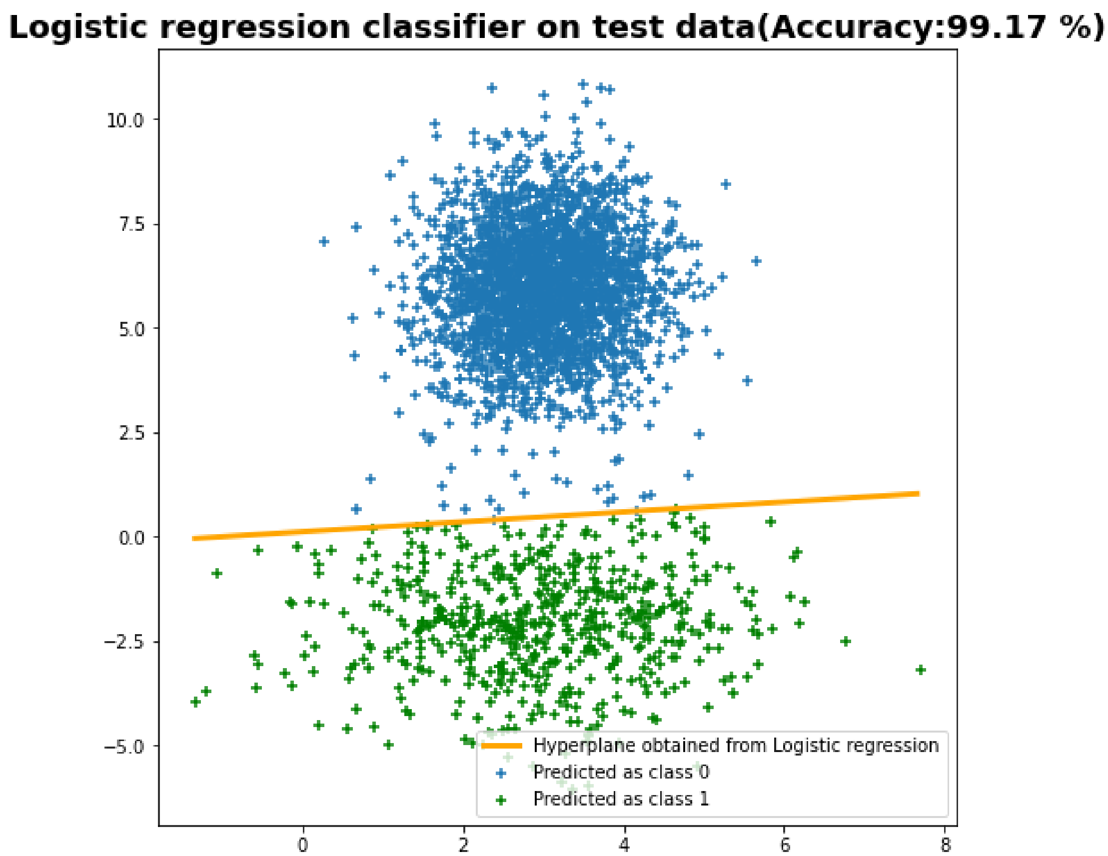

Test data(p0:0.8, p1:0.2)
class 0
class 1
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
0
2
4
6
8



Perceptron classifier on test data(Accuracy:99.00 %)
Hyperplane obtained from Perceptron
Predicted as class 0
Predicted as class 1
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
0
2
4
6
8


FLD classifier on test data(Accuracy:90.23 %)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
0
2
4
6
8
Hyperplane obtained from FLDA
Predicted as class 0
Predicted as class 1


Linear regression classifier on test data(Accuracy:99.77 %)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
0
2
4
6
8
Hyperplane obtained from Linear regression
Predicted as class 0
Predicted as class 1



6 Problem 2: German credit card data

6.1 Data generation:

[155]:
```
testdata_per = 0.2
train_samples,train_labels,test_samples,test_labels =␣
↪German_data_numeric_preprocessing(testdata_per)
testdata_per_1 = 0.3
train_samples_1,train_labels_1,test_samples_1,test_labels_1 =␣
↪German_data_numeric_preprocessing(testdata_per_1)
```

```
class probabilities: p0 = 0.3,p1 = 0.7
For train-test split : 80:20

Number of training data points : 800
Number of test data points   : 200
Dimension of feature vectors : 20

class probabilities: p0 = 0.3,p1 = 0.7
```

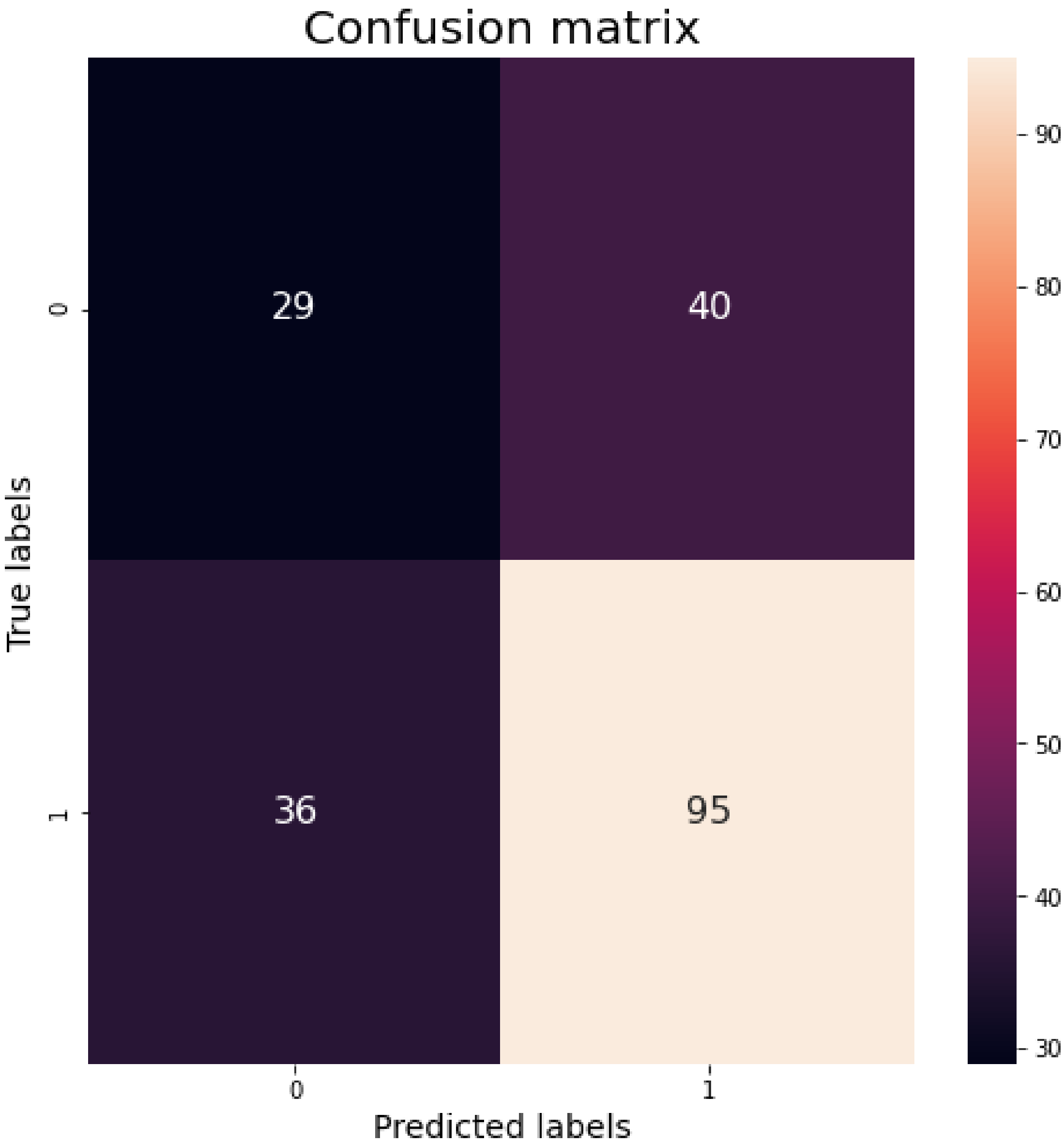
```
For train-test split : 70:30

Number of training data points : 700
Number of test data points  : 300
Dimension of feature vectors : 20
```

6.2 Perceptron:

```
[ ]: eta = 0.9
N_epochs = 30
avoid_scroll_output_window()
_,per_cls =␣
 ↪Perceptron_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs)

y_pred       = per_cls.predict(test_samples)
y_pred[y_pred == -1] = 0
f1_score_per = f1_score(test_labels,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_per}\n")

conf_mat = confusion_matrix(test_labels,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()

_,per_cls =␣
 ↪Perceptron_testing(train_samples_1,train_labels_1,test_samples_1,test_labels_1,eta,N_epochs)

y_pred       = per_cls.predict(test_samples_1)
y_pred[y_pred == -1] = 0
f1_score_per = f1_score(test_labels_1,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_per}\n")

conf_mat = confusion_matrix(test_labels_1,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()
```

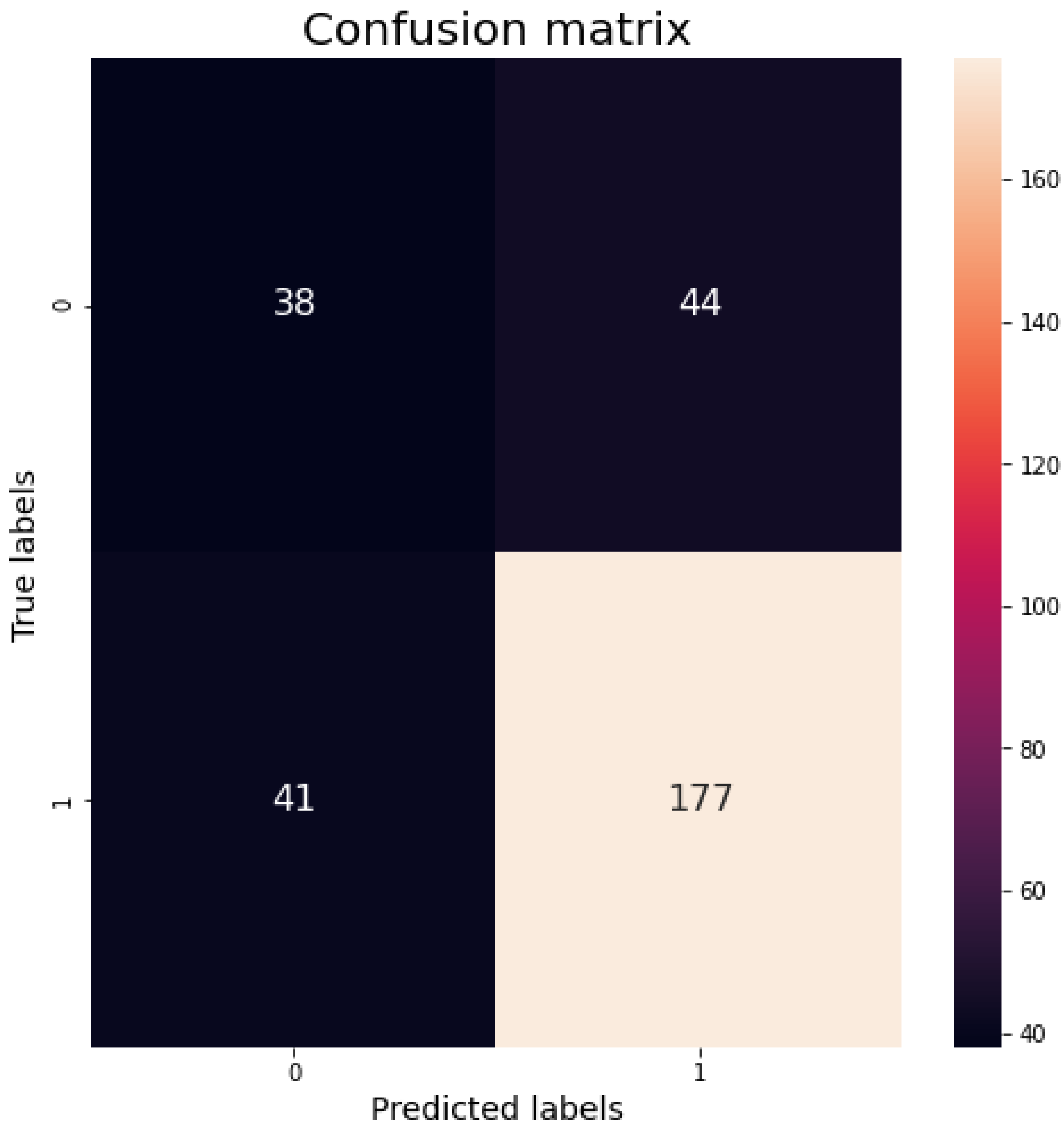
```
*********** PERCEPTRON *************

train-test split : 80:20, Learning rate : 0.9, Number of epochs : 30
```

```
Using classifier built from scratch
Train accuracy= 68.50 %
Test accuracy = 62.00%
f1_score : 0.6171855010660979
```




```
*********** PERCEPTRON *************

train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30
```

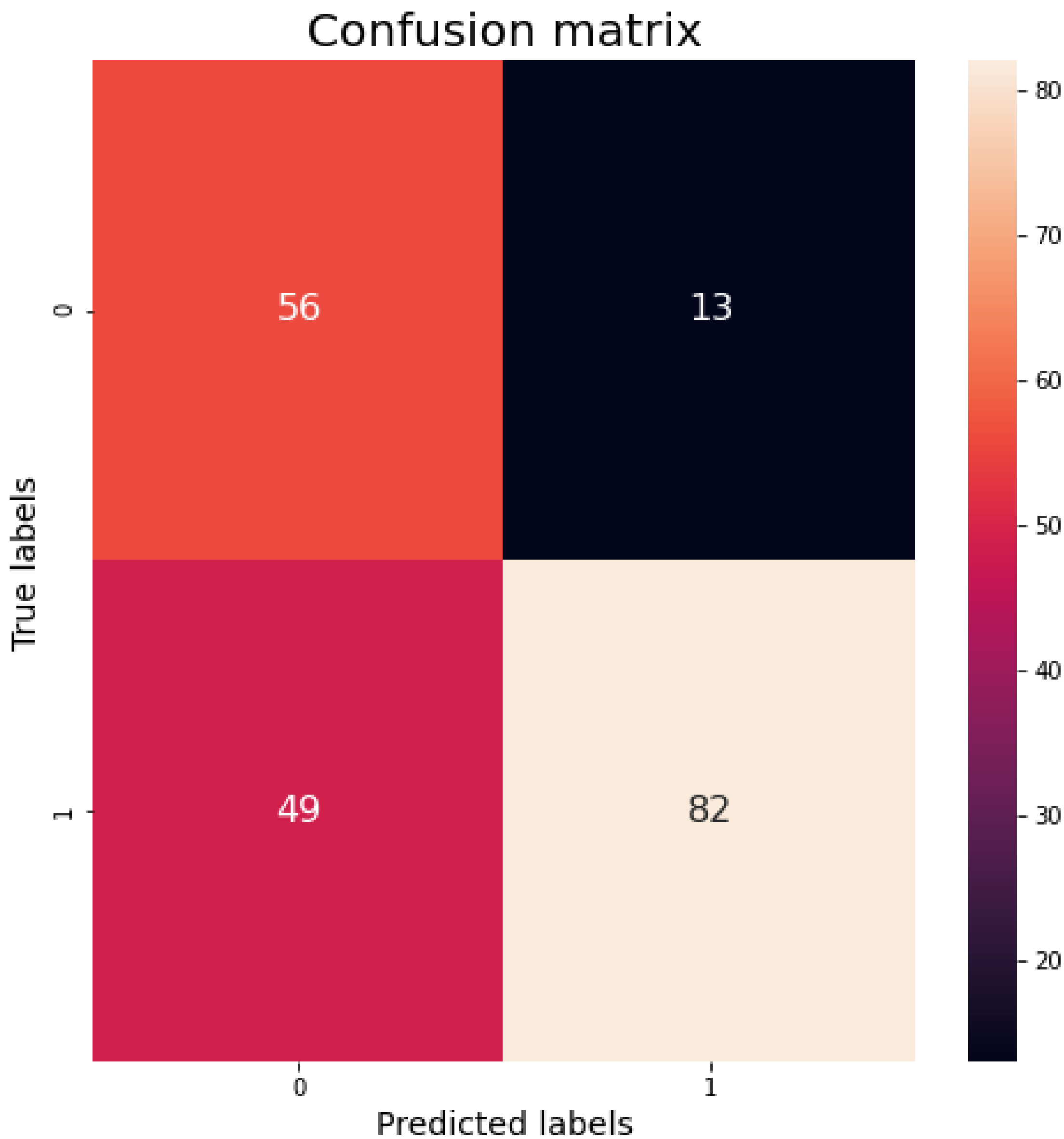
```
Using classifier built from scratch
Train accuracy= 67.71 %
Test accuracy = 71.67%
f1_score : 0.7149950244532793
```



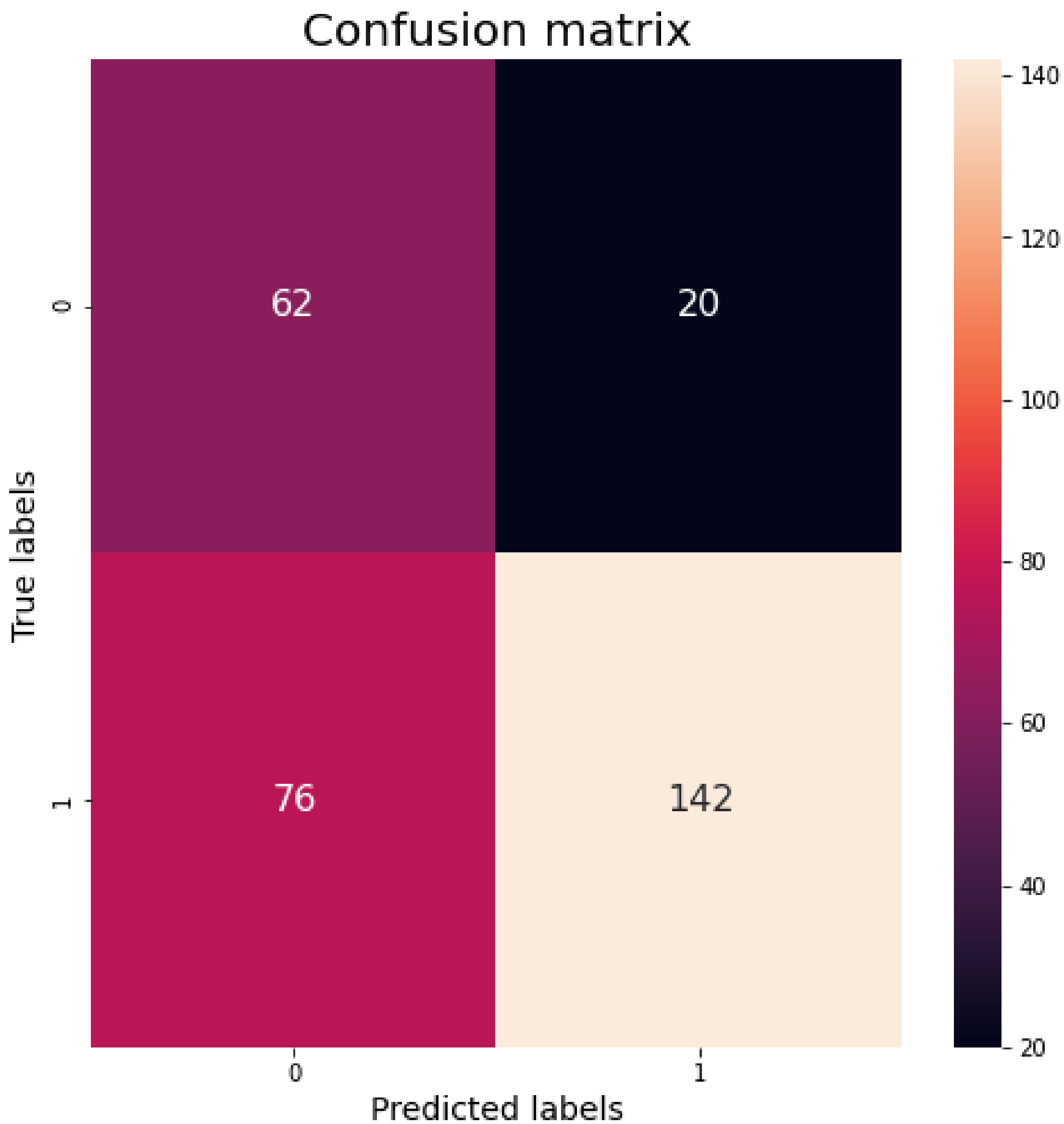
6.2.1 FLDA:

```
[ ]: avoid_scroll_output_window()
fld_cls = FLD_testing(train_samples,train_labels,test_samples,test_labels)

y_pred       = fld_cls.predict(test_samples)
f1_score_fld = f1_score(test_labels,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_fld}\n")

conf_mat = confusion_matrix(test_labels,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()

fld_cls =␣
 ↪FLD_testing(train_samples_1,train_labels_1,test_samples_1,test_labels_1)

y_pred       = fld_cls.predict(test_samples_1)
f1_score_fld = f1_score(test_labels_1,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_fld}\n")

conf_mat = confusion_matrix(test_labels_1,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()
```

```
*********** FLD *************

train-test split : 80:20

Using classifier built from scratch
Train accuracy= 69.88 %
Test accuracy = 69.00%
f1_score : 0.6973787000305158
```




```
*********** FLD *************

train-test split : 70:30

Using classifier built from scratch
Train accuracy= 69.43 %
Test accuracy = 68.00%
f1_score : 0.6971483253588516
```

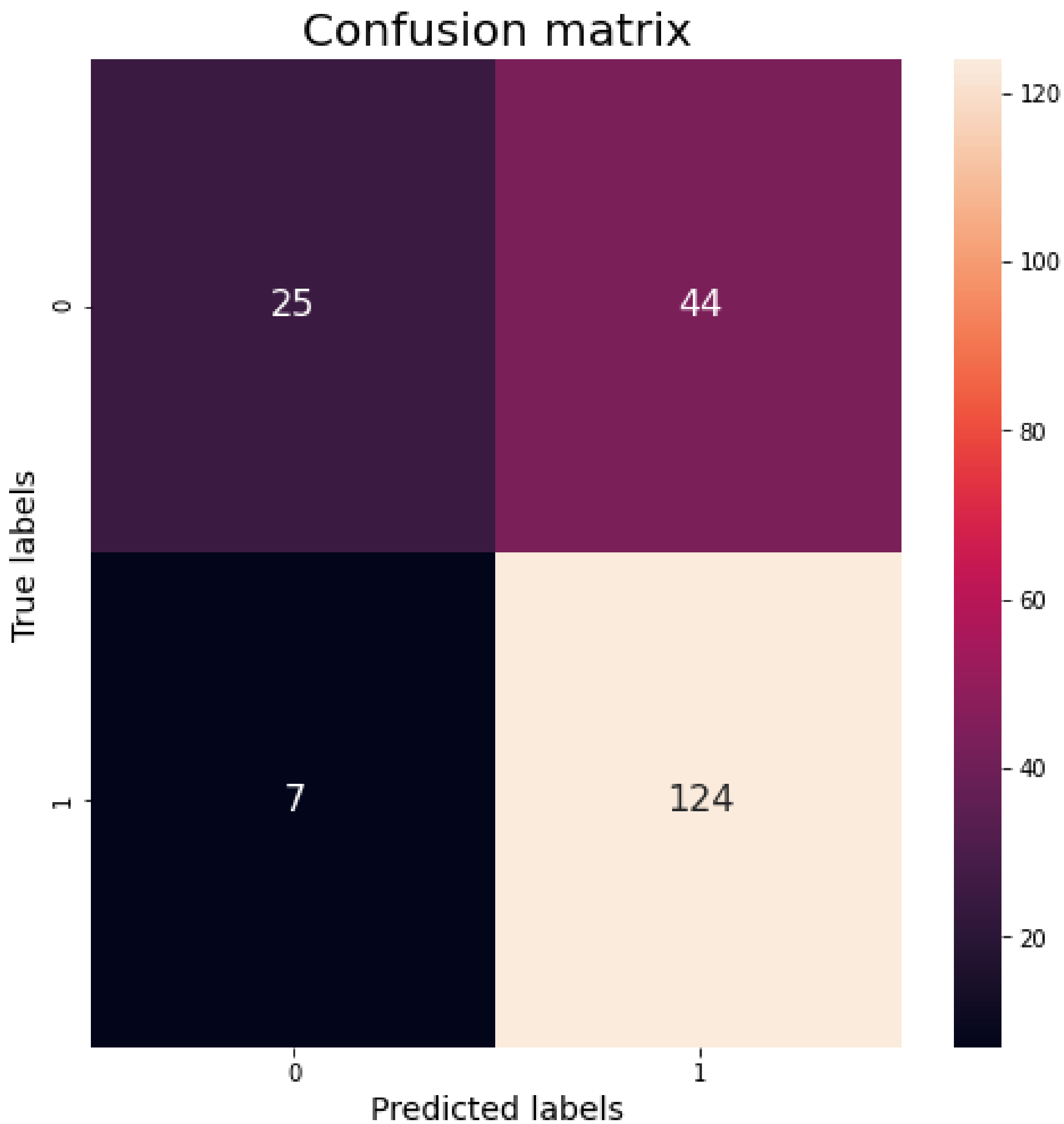


6.3 Linear regression:

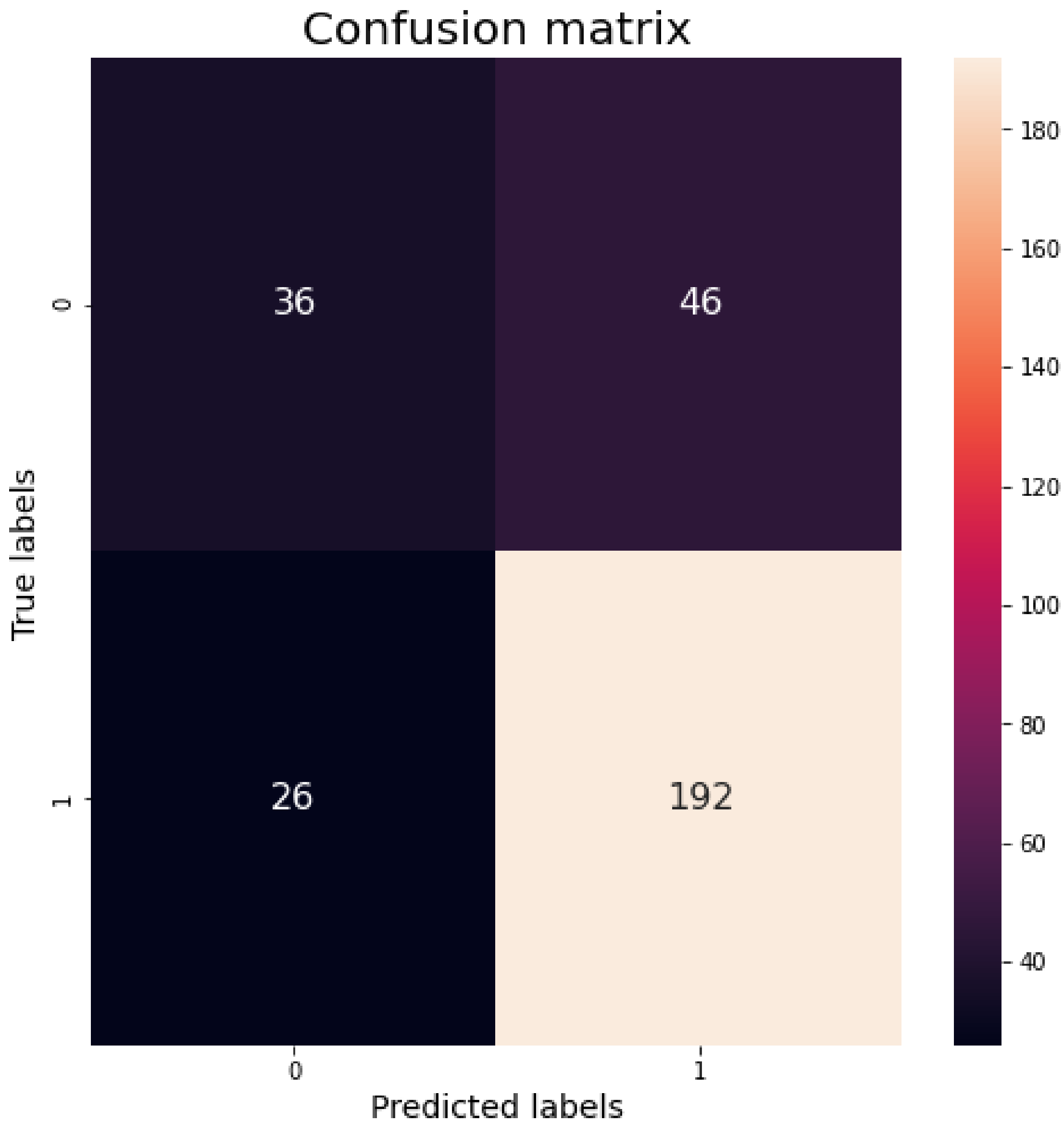
[ ]:
```
avoid_scroll_output_window()
_,linreg_cls =␣
↪Linreg_testing(train_samples,train_labels,test_samples,test_labels)

y_pred       = linreg_cls.predict(test_samples)
y_pred[y_pred == -1] = 0
f1_score_lin = f1_score(test_labels,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_lin}\n")
```

```
conf_mat = confusion_matrix(test_labels,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()

_,linreg_cls =␣
 ↪Linreg_testing(train_samples_1,train_labels_1,test_samples_1,test_labels_1)

y_pred       = linreg_cls.predict(test_samples_1)
y_pred[y_pred == -1] = 0
f1_score_lin = f1_score(test_labels_1,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_lin}\n")

conf_mat = confusion_matrix(test_labels_1,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()
```

```
*********** LINEAR REGRESSION *************

train-test split : 80:20

Using IN-BUILT classifier
Train accuracy= 77.12 %
Test accuracy = 74.50 %
Using classifier built from scratch
Train accuracy= 77.12 %
Test accuracy = 74.50 %
f1_score : 0.7140696711811652
```




```
*********** LINEAR REGRESSION *************

train-test split : 70:30

Using IN-BUILT classifier
Train accuracy= 75.86 %
Test accuracy = 76.00 %
Using classifier built from scratch
Train accuracy= 75.86 %
Test accuracy = 76.00 %
f1_score : 0.7485964912280701
```

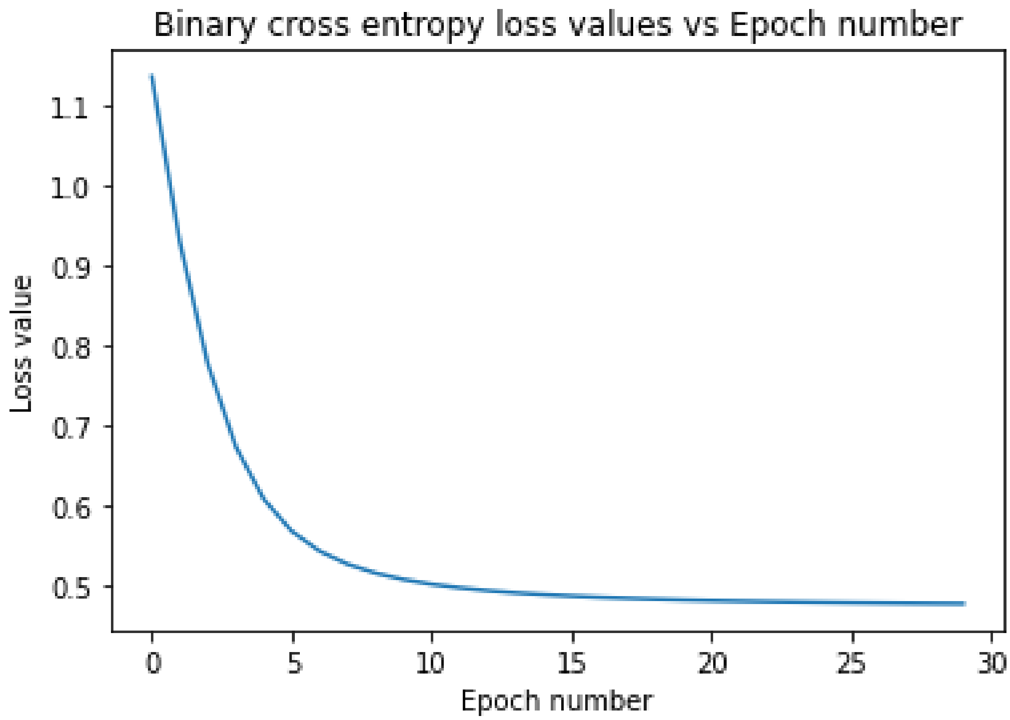


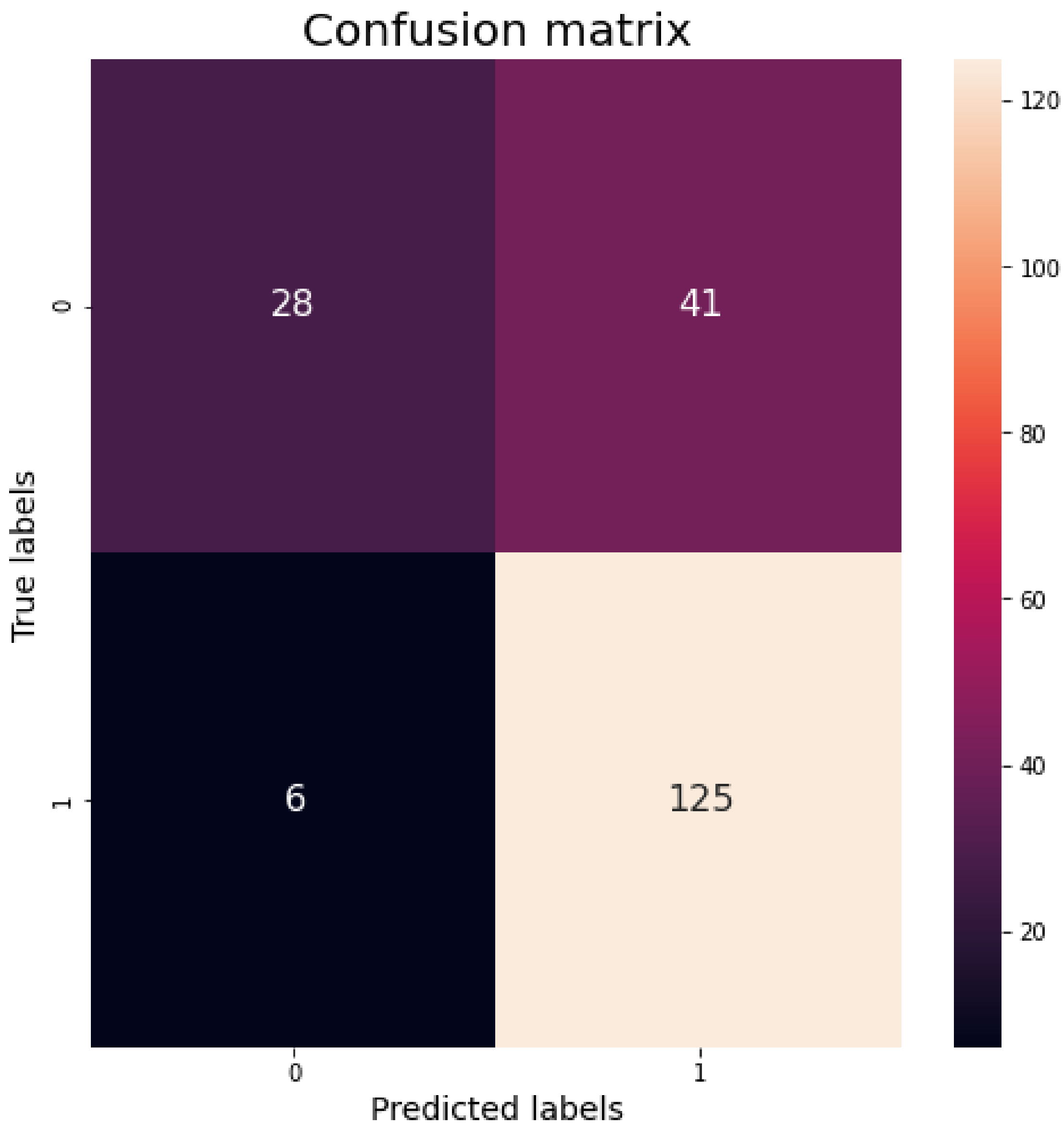
6.4 Logistic regression:

```
[ ]: eta      = 0.9
     N_epochs = 30
     avoid_scroll_output_window()
     _,logreg_cls =␣
      ↪Logreg_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs)

     y_pred       = logreg_cls.predict(test_samples)
```

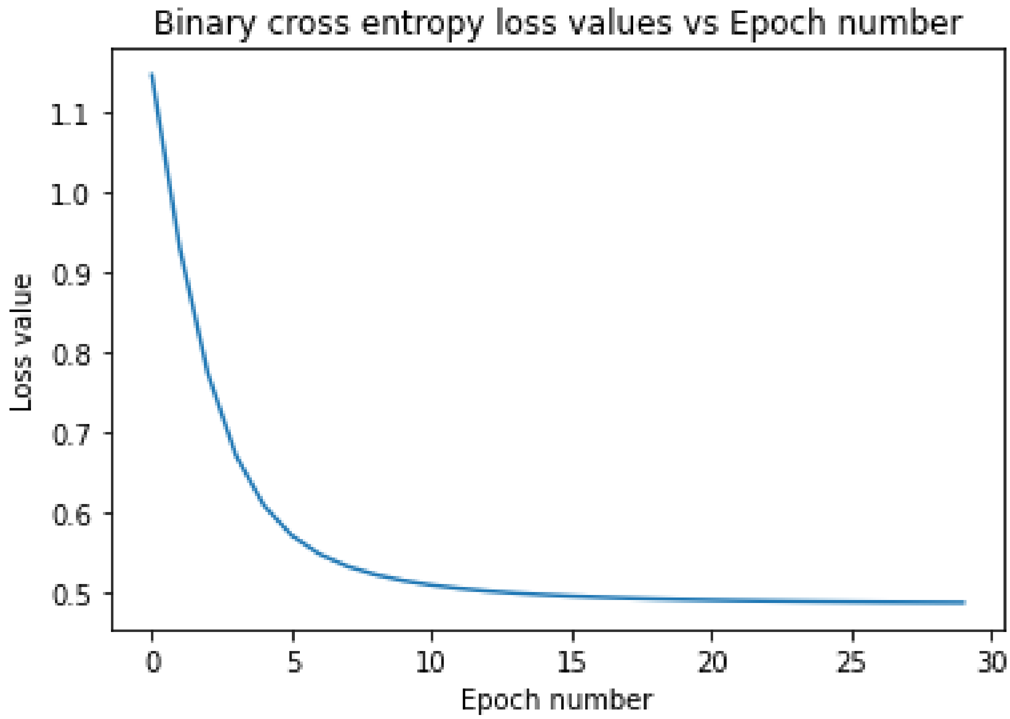
```
f1_score_log = f1_score(test_labels,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_log}\n")

conf_mat = confusion_matrix(test_labels,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()

_,logreg_cls =␣
 ↪Logreg_testing(train_samples_1,train_labels_1,test_samples_1,test_labels_1,eta,N_epochs)

y_pred       = logreg_cls.predict(test_samples_1)
f1_score_log = f1_score(test_labels_1,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_log}\n")

conf_mat = confusion_matrix(test_labels_1,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()
```

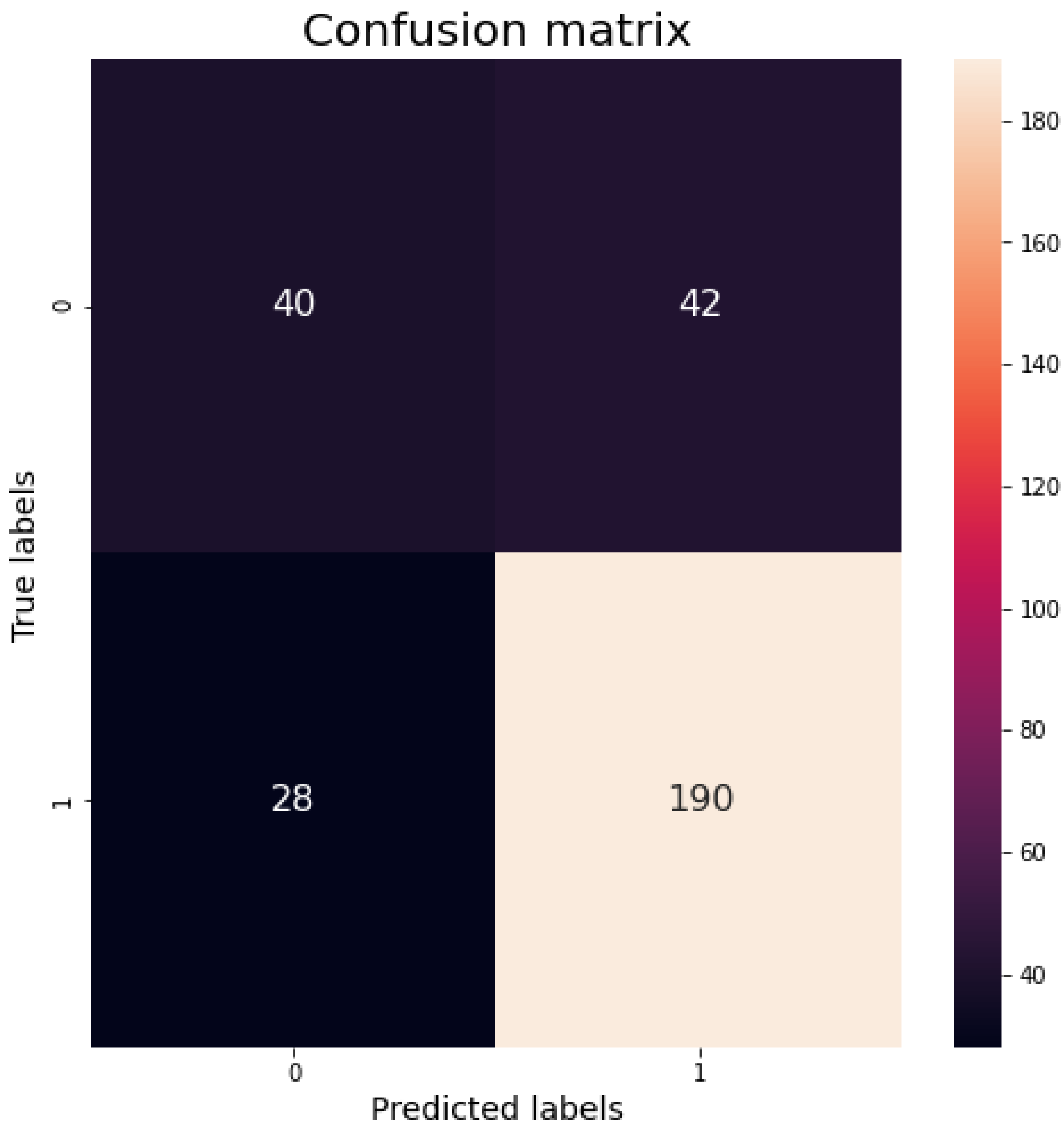
```
*********** LOGISTIC REGRESSION *************

train-test split : 80:20, Learning rate : 0.9, Number of epochs : 30

Using IN-BUILT classifier
Train accuracy= 76.62 %
Test accuracy = 74.50 %
Using classifier built from scratch
Train accuracy= 76.88 %
Test accuracy = 76.50 %
f1_score : 0.738919616880782
```


Binary cross entropy loss values vs Epoch number
Loss value
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0
5
10
15
20
25
30
Epoch number




```
*********** LOGISTIC REGRESSION *************

train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30

Using IN-BUILT classifier
Train accuracy= 76.43 %
Test accuracy = 76.33 %
Using classifier built from scratch
Train accuracy= 77.29 %
Test accuracy = 76.67 %
f1_score : 0.7594074074074073
```


Binary cross entropy loss values vs Epoch number
Loss value
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0
5
10
15
20
25
30
Epoch number

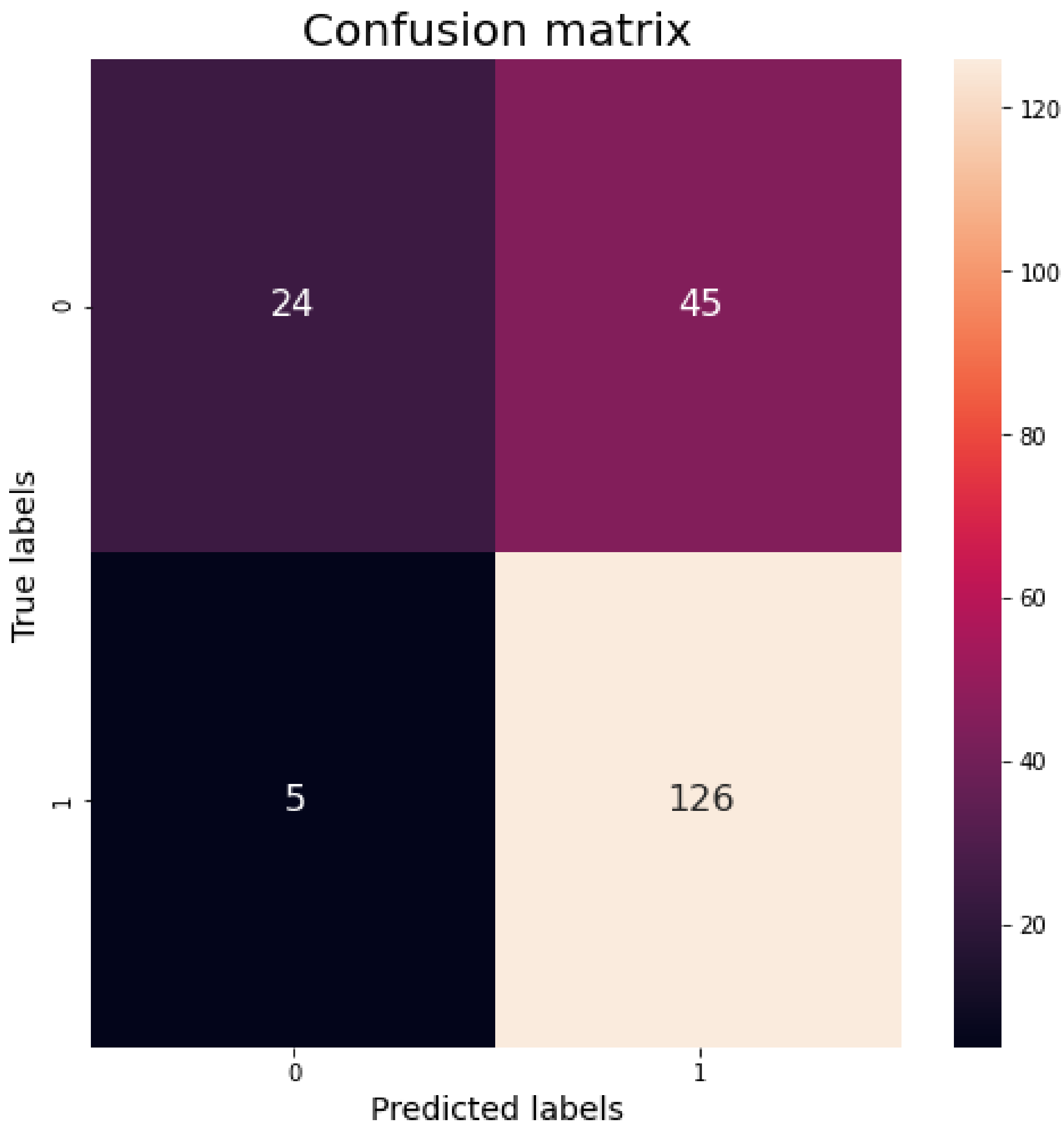


6.5 SVM:

```
[ ]: avoid_scroll_output_window()
print("************ SVM *************\n")
# Create a svm Classifier
SVM_cls = svm.SVC()

# Train the model using the training data
SVM_cls.fit(train_samples, train_labels.ravel())

# Predict the class labels for test data
```

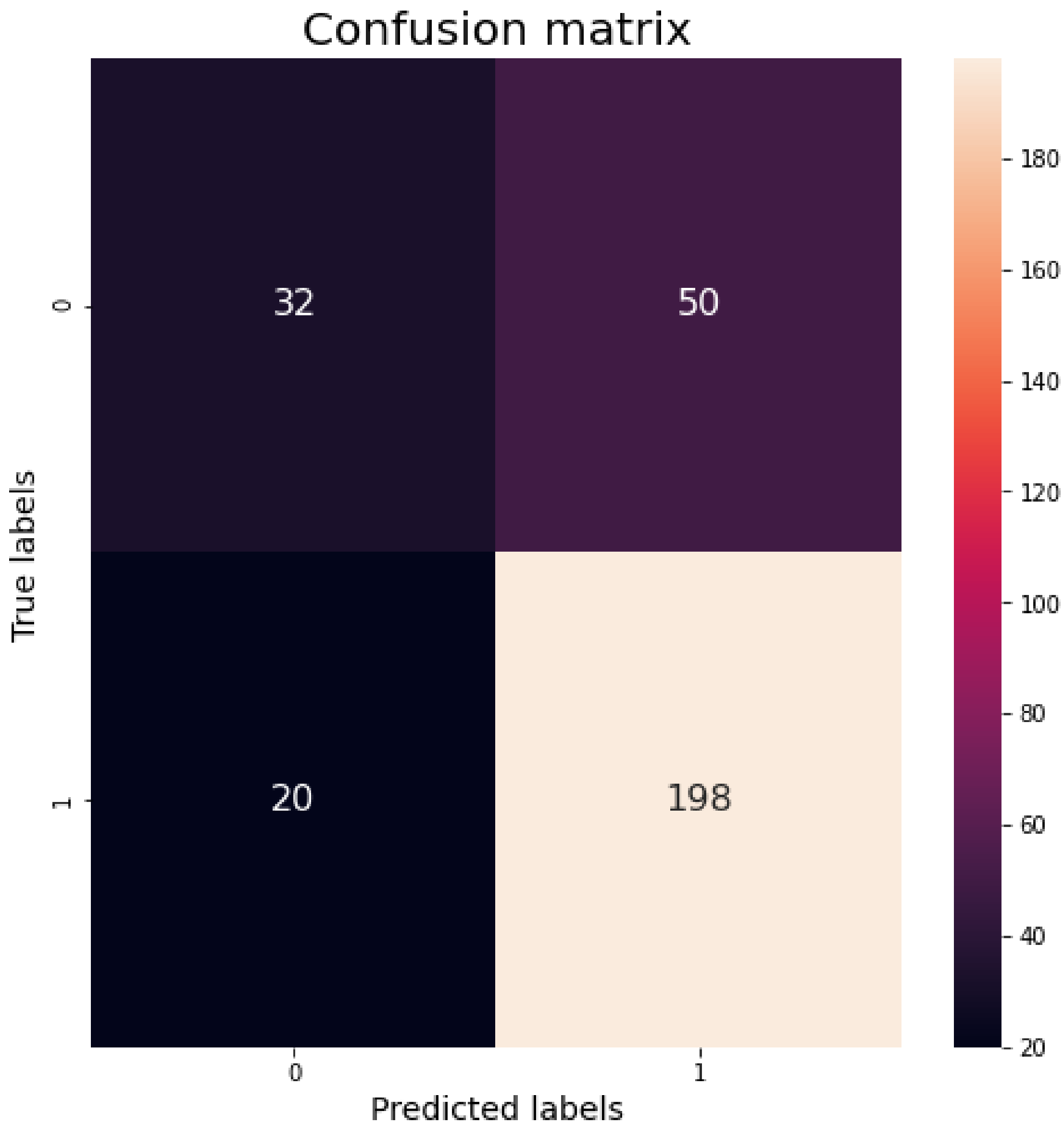
```python
y_pred_train = SVM_cls.predict(train_samples)
y_pred_test  = SVM_cls.predict(test_samples)

train_accuracy_svm = accuracy_score(train_labels,y_pred_train)*100
test_accuracy_svm  = accuracy_score(test_labels,y_pred_test)*100

f1_score_svm = f1_score(test_labels,y_pred_test,average = 'weighted')

print(f"For train-test split : {int((1-testdata_per)*100)}:
 ↪{int(testdata_per*100)}\n")
print(f"Train accuracy= {train_accuracy_svm:.2f} %")
print(f"Test accuracy = {test_accuracy_svm:.2f} %\n")
print(f"f1_score : {f1_score_svm}\n")

conf_mat = confusion_matrix(test_labels,y_pred_test)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()

# Train the model using the training data
SVM_cls.fit(train_samples_1, train_labels_1.ravel())

# Predict the class labels for test data
y_pred_train_1 = SVM_cls.predict(train_samples_1)
y_pred_test_1  = SVM_cls.predict(test_samples_1)

train_accuracy_svm = accuracy_score(train_labels_1,y_pred_train_1)*100
test_accuracy_svm  = accuracy_score(test_labels_1,y_pred_test_1)*100

f1_score_svm = f1_score(test_labels_1,y_pred_test_1,average = 'weighted')

print(f"For train-test split : {int((1-testdata_per_1)*100)}:
 ↪{int(testdata_per_1*100)}\n")
print(f"Train accuracy= {train_accuracy_svm:.2f} %")
print(f"Test accuracy = {test_accuracy_svm:.2f} %\n")
print(f"f1_score : {f1_score_svm}\n")

conf_mat = confusion_matrix(test_labels_1,y_pred_test_1)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()
```

```
************ SVM *************

For train-test split : 80:20

Train accuracy= 85.62 %
Test accuracy = 75.00 %

f1_score : 0.7155358832274631
```




```
For train-test split : 70:30
```

```
Train accuracy= 84.71 %
Test accuracy = 76.67 %

f1_score : 0.748057993295326
```



7 Problem 3: Porto-Seguro safe driver prediction data

7.1 Data generation:

[158]:
```
testdata_per = 0.2
train_samples,train_labels,test_samples,test_labels =␣
↪Safe_driver_prediction_preprocessing(testdata_per)

# Commented because it was taking more time to transform data to higher dimension

# #Transform generated d dimensional data to higher dimension where it can be␣
↪linearly separable
# #Transforming the data to higher dimension using polynomial transformation
# poly = PolynomialFeatures(54, include_bias= 1)

# train_samples_tr = poly.fit_transform(train_samples)
# test_samples_tr  = poly.fit_transform(test_samples)
# print(f"Dimension of transformed feature vectors : {np.
↪shape(train_samples_tr)[1]}")

testdata_per_1 = 0.3
train_samples_1,train_labels_1,test_samples_1,test_labels_1 =␣
↪Safe_driver_prediction_preprocessing(testdata_per_1)
```

```
class probabilities : p0 = 0.963552482140817,p1 = 0.036447517859182946
class probabilities after under sampling: p0 = 0.5,p1 = 0.5
For train-test split : 80:20

Number of training data points : 34710
Number of test data points  : 8678
Dimension of feature vectors : 54

class probabilities : p0 = 0.963552482140817,p1 = 0.036447517859182946
class probabilities after under sampling: p0 = 0.5,p1 = 0.5
For train-test split : 70:30

Number of training data points : 30371
Number of test data points  : 13017
Dimension of feature vectors : 54
```

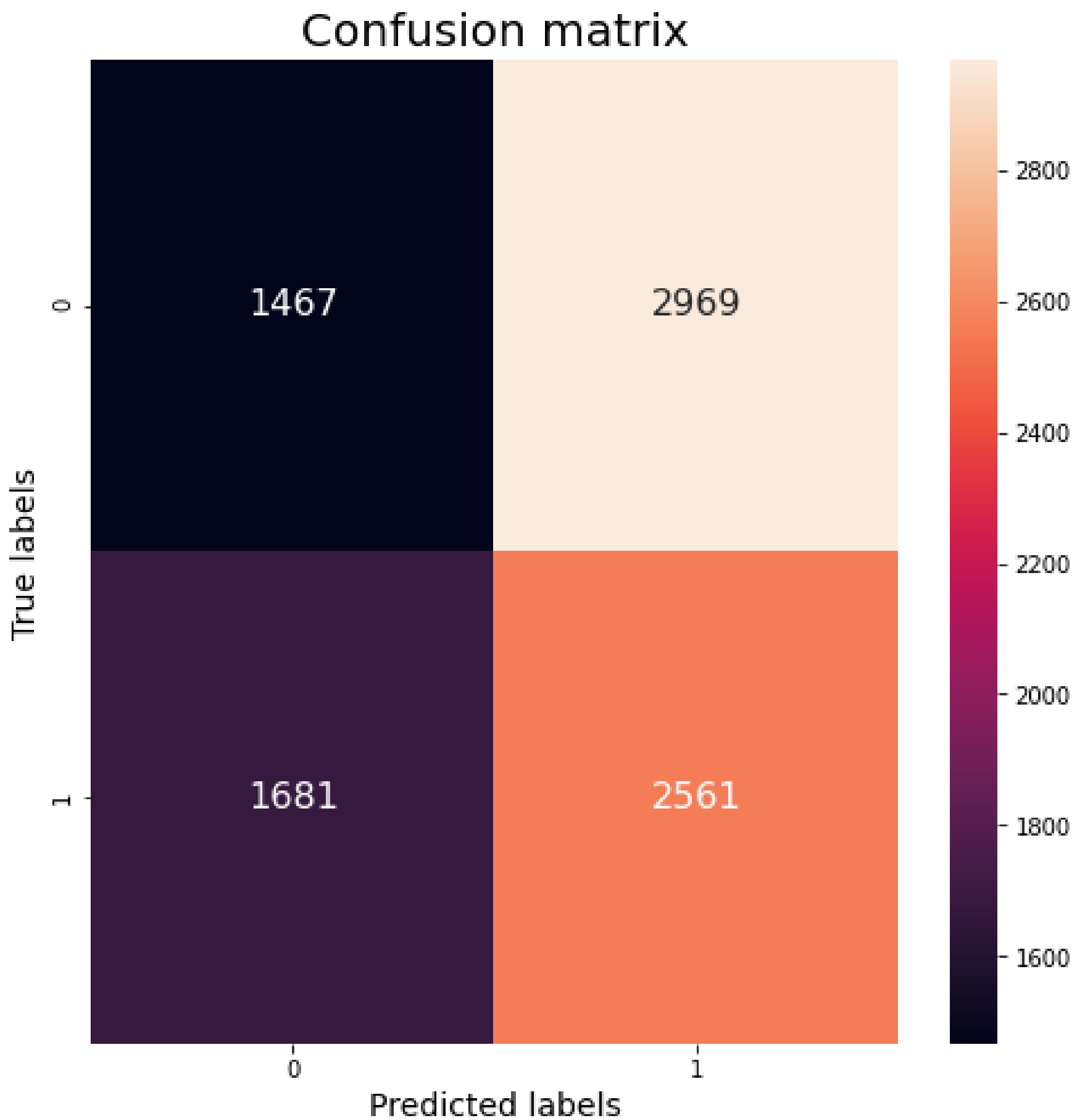
7.2 Perceptron:

[159]:
```
eta = 0.9
N_epochs = 30
avoid_scroll_output_window()
```

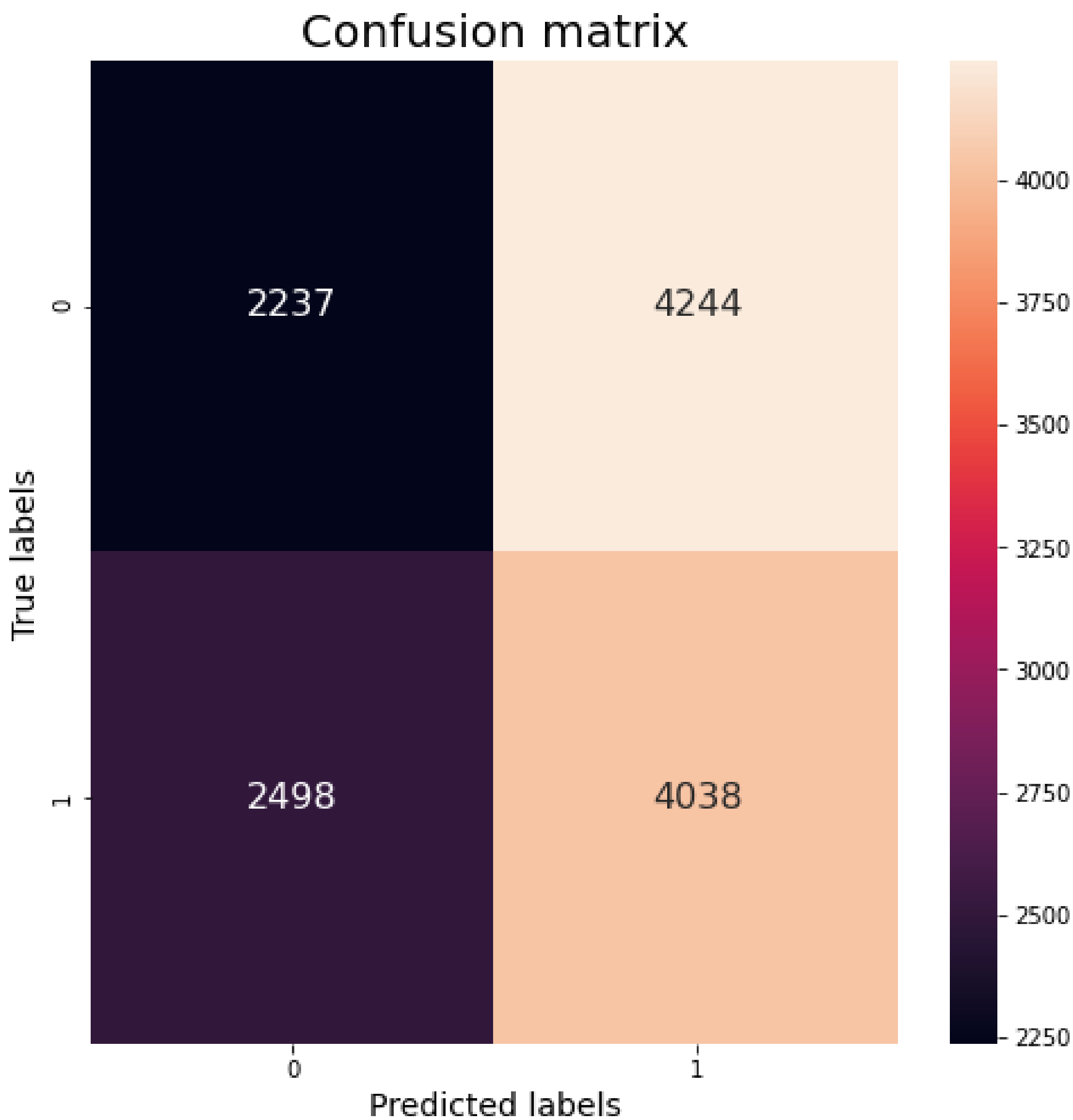
```
_,per_cls =␣
 ↪Perceptron_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs)

y_pred       = per_cls.predict(test_samples)
y_pred[y_pred == -1] = 0
f1_score_per = f1_score(test_labels,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_per}\n")

conf_mat = confusion_matrix(test_labels,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()

# # Perceptron on transformed data
# print("\nPerceptron on transformed data\n")
# _,per_cls_tr =␣
 ↪Perceptron_testing(train_samples_tr,train_labels,test_samples_tr,test_labels,eta,N_epochs)

# y_pred       = per_cls_tr.predict(test_samples_tr)
# y_pred[y_pred == -1] = 0
# f1_score_per = f1_score(test_labels,y_pred,average = 'weighted')
# print(f"f1_score : {f1_score_per}\n")

# conf_mat = confusion_matrix(test_labels,y_pred)
# fig= plt.figure(figsize=(8,8))
# sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
# plt.xlabel("Predicted labels",fontsize=14)
# plt.ylabel("True labels",fontsize=14)
# plt.title("Confusion matrix",fontsize=20)
# plt.show()

_,per_cls =␣
 ↪Perceptron_testing(train_samples_1,train_labels_1,test_samples_1,test_labels_1,eta,N_epochs)

y_pred       = per_cls.predict(test_samples_1)
y_pred[y_pred == -1] = 0
f1_score_per = f1_score(test_labels_1,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_per}\n")

conf_mat = confusion_matrix(test_labels_1,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
```

```
plt.title("Confusion matrix",fontsize=20)
plt.show()
```

```
*********** PERCEPTRON *************

train-test split : 80:20, Learning rate : 0.9, Number of epochs : 30

Using classifier built from scratch
Train accuracy= 47.61 %
Test accuracy = 46.42%
f1_score : 0.45397434852123786
```



```
*********** PERCEPTRON *************

train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30

Using classifier built from scratch
Train accuracy= 47.99 %
Test accuracy = 48.21%
f1_score : 0.4722623222910797
```



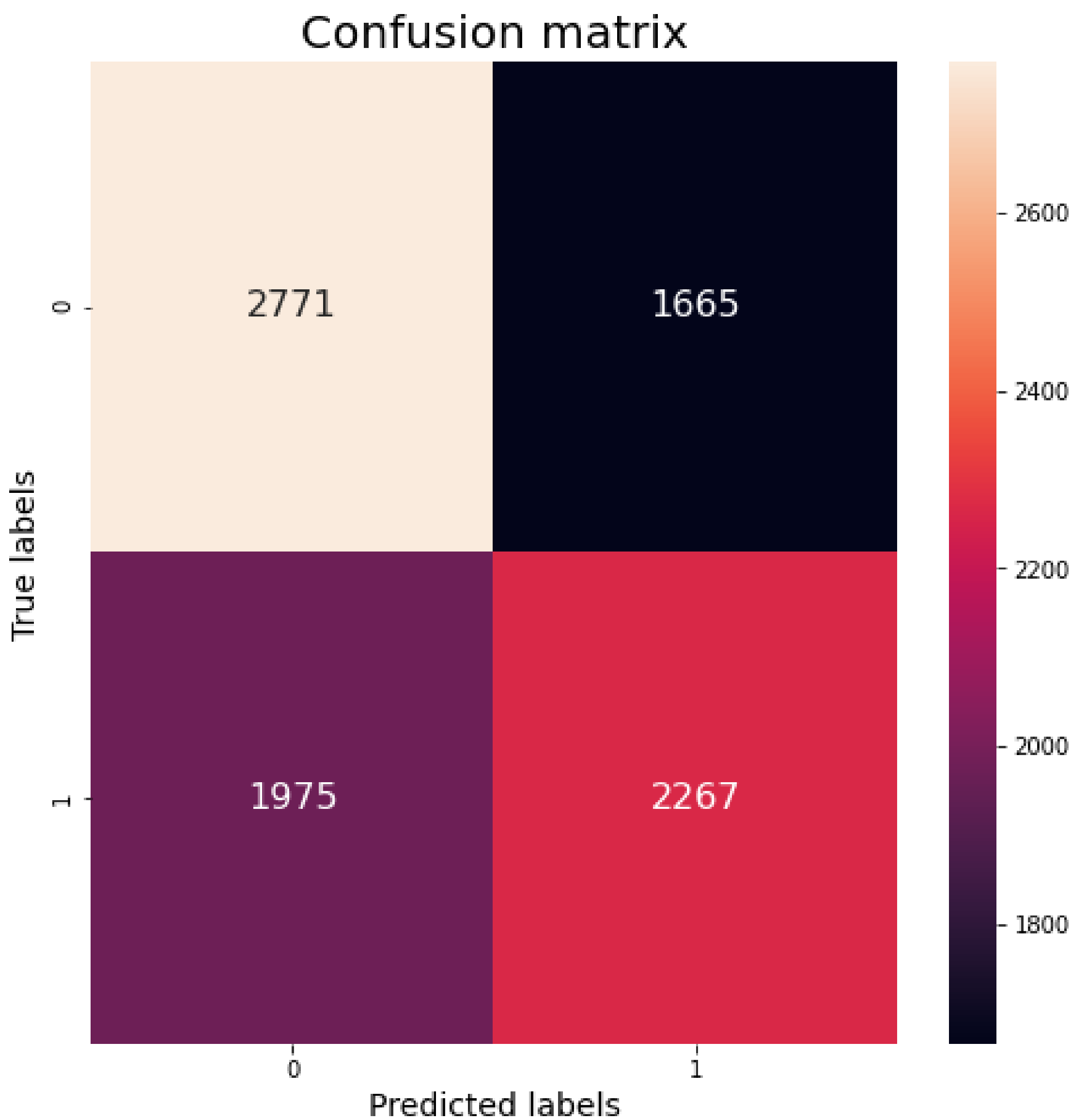
7.2.1 FLDA:

[160]:

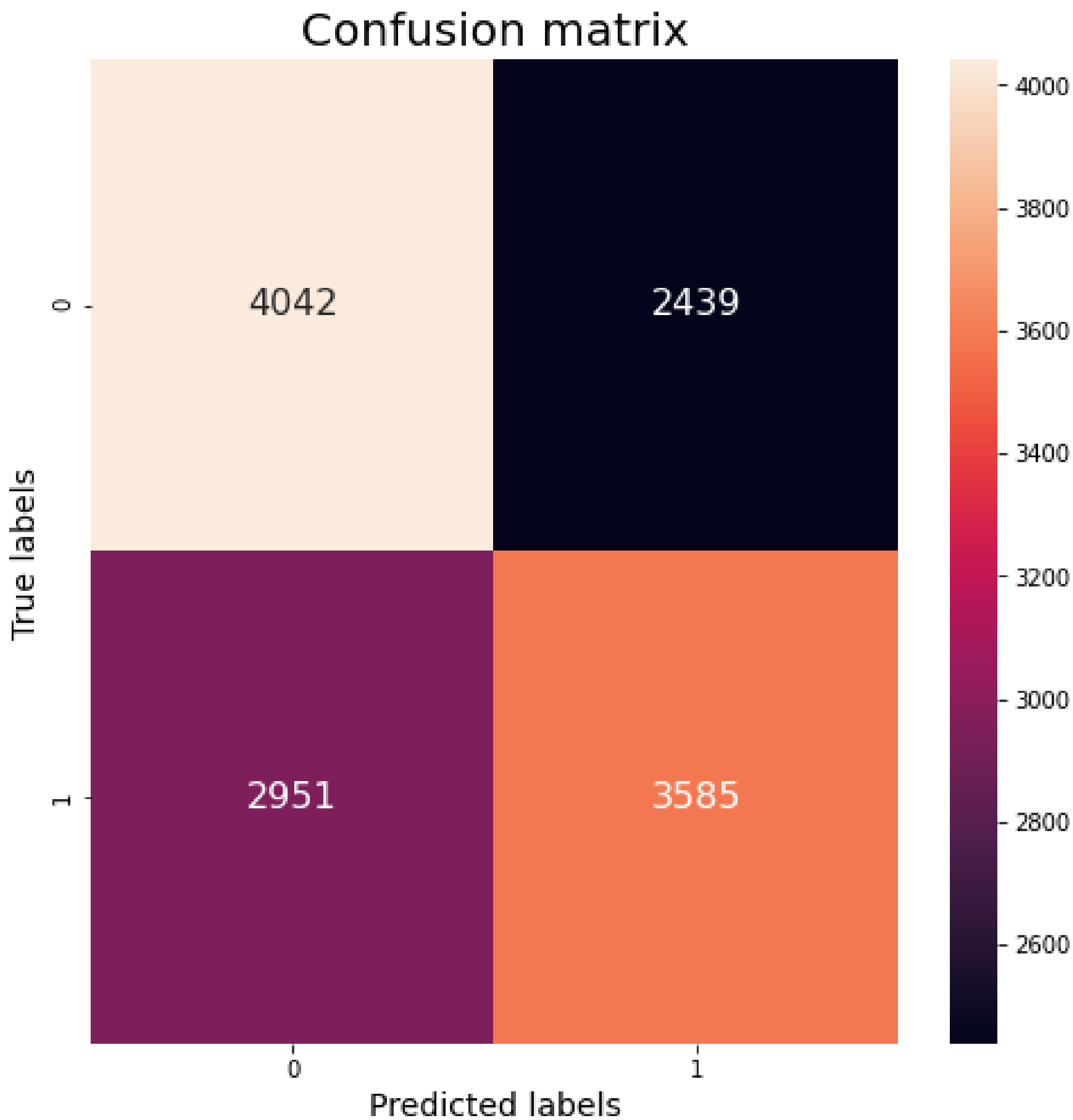
```
avoid_scroll_output_window()
fld_cls = FLD_testing(train_samples,train_labels,test_samples,test_labels)

y_pred       = fld_cls.predict(test_samples)
f1_score_fld = f1_score(test_labels,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_fld}\n")

conf_mat = confusion_matrix(test_labels,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()

# # FLDA on transformed data
# print("\nFLDA on transformed data\n")
# fld_cls_tr =␣
 ↪FLD_testing(train_samples_tr,train_labels,test_samples_tr,test_labels)

# y_pred       = fld_cls_tr.predict(test_samples_tr)
# f1_score_fld = f1_score(test_labels,y_pred,average = 'weighted')
# print(f"f1_score : {f1_score_fld}\n")

# conf_mat = confusion_matrix(test_labels,y_pred)
# fig= plt.figure(figsize=(8,8))
# sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
# plt.xlabel("Predicted labels",fontsize=14)
# plt.ylabel("True labels",fontsize=14)
# plt.title("Confusion matrix",fontsize=20)
# plt.show()

fld_cls =␣
 ↪FLD_testing(train_samples_1,train_labels_1,test_samples_1,test_labels_1)

y_pred       = fld_cls.predict(test_samples_1)
f1_score_fld = f1_score(test_labels_1,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_fld}\n")

conf_mat = confusion_matrix(test_labels_1,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
```

```
plt.show()
```

```
*********** FLD *************

train-test split : 80:20

Using classifier built from scratch
Train accuracy= 58.52 %
Test accuracy = 58.05%
f1_score : 0.579675336967485
```



```
*********** FLD *************

train-test split : 70:30

Using classifier built from scratch
Train accuracy= 58.56 %
Test accuracy = 58.59%
f1_score : 0.5853535935477001
```

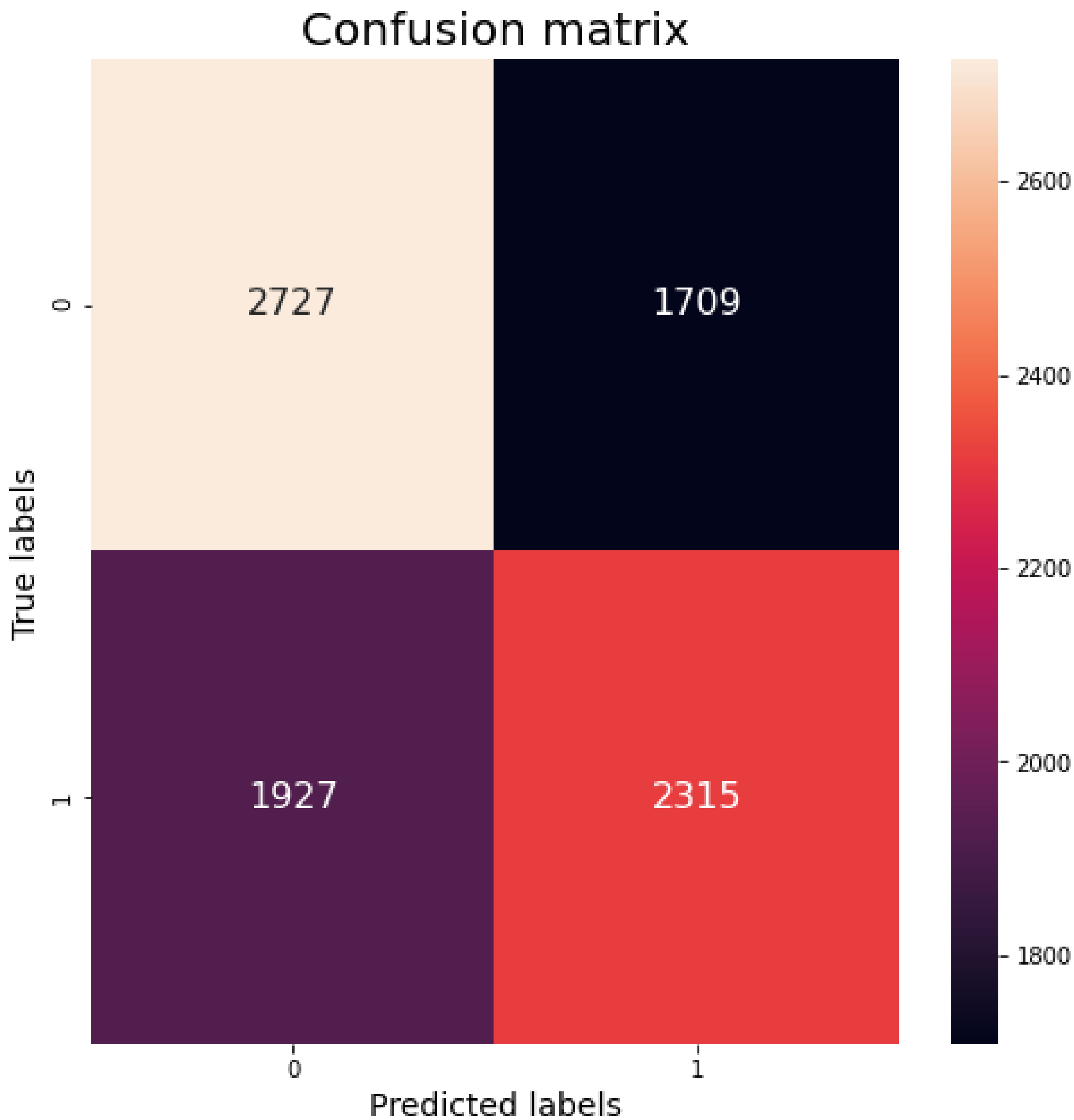


7.3 Linear regression:

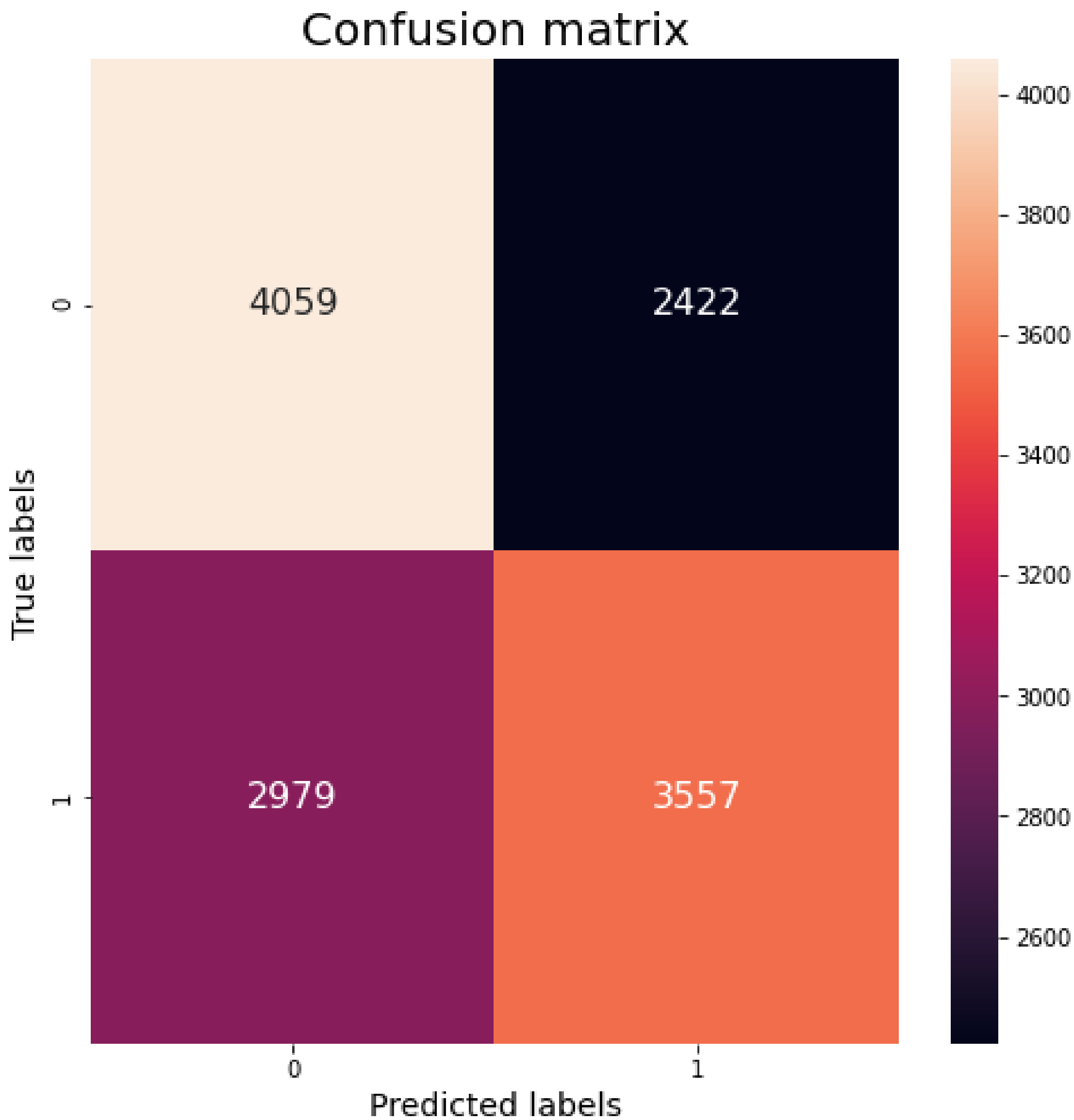
[161]:
```
avoid_scroll_output_window()
_,linreg_cls =␣
 ↪Linreg_testing(train_samples,train_labels,test_samples,test_labels)

y_pred       = linreg_cls.predict(test_samples)
y_pred[y_pred == -1] = 0
f1_score_lin = f1_score(test_labels,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_lin}\n")

conf_mat = confusion_matrix(test_labels,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()

# # Linear regression on transformed data
# print("\nLinear regression on transformed data\n")
# _,linreg_cls_tr =␣
 ↪Linreg_testing(train_samples_tr,train_labels,test_samples_tr,test_labels)

# y_pred       = linreg_cls_tr.predict(test_samples_tr)
# y_pred[y_pred == -1] = 0
# f1_score_lin = f1_score(test_labels,y_pred,average = 'weighted')
# print(f"f1_score : {f1_score_lin}\n")

# conf_mat = confusion_matrix(test_labels,y_pred)
# fig= plt.figure(figsize=(8,8))
# sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
# plt.xlabel("Predicted labels",fontsize=14)
# plt.ylabel("True labels",fontsize=14)
# plt.title("Confusion matrix",fontsize=20)
# plt.show()

_,linreg_cls =␣
 ↪Linreg_testing(train_samples_1,train_labels_1,test_samples_1,test_labels_1)

y_pred       = linreg_cls.predict(test_samples_1)
y_pred[y_pred == -1] = 0
f1_score_lin = f1_score(test_labels_1,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_lin}\n")

conf_mat = confusion_matrix(test_labels_1,y_pred)
fig= plt.figure(figsize=(8,8))
```

```python
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()
```

```
*********** LINEAR REGRESSION *************

train-test split : 80:20

Using IN-BUILT classifier
Train accuracy= 58.59 %
Test accuracy = 58.14 %
Using classifier built from scratch
Train accuracy= 58.61 %
Test accuracy = 58.10 %
f1_score : 0.5805086095885221
```




```
*********** LINEAR REGRESSION *************

train-test split : 70:30

Using IN-BUILT classifier
Train accuracy= 58.54 %
Test accuracy = 58.50 %
Using classifier built from scratch
Train accuracy= 58.55 %
Test accuracy = 58.51 %
f1_score : 0.5843953279452863
```



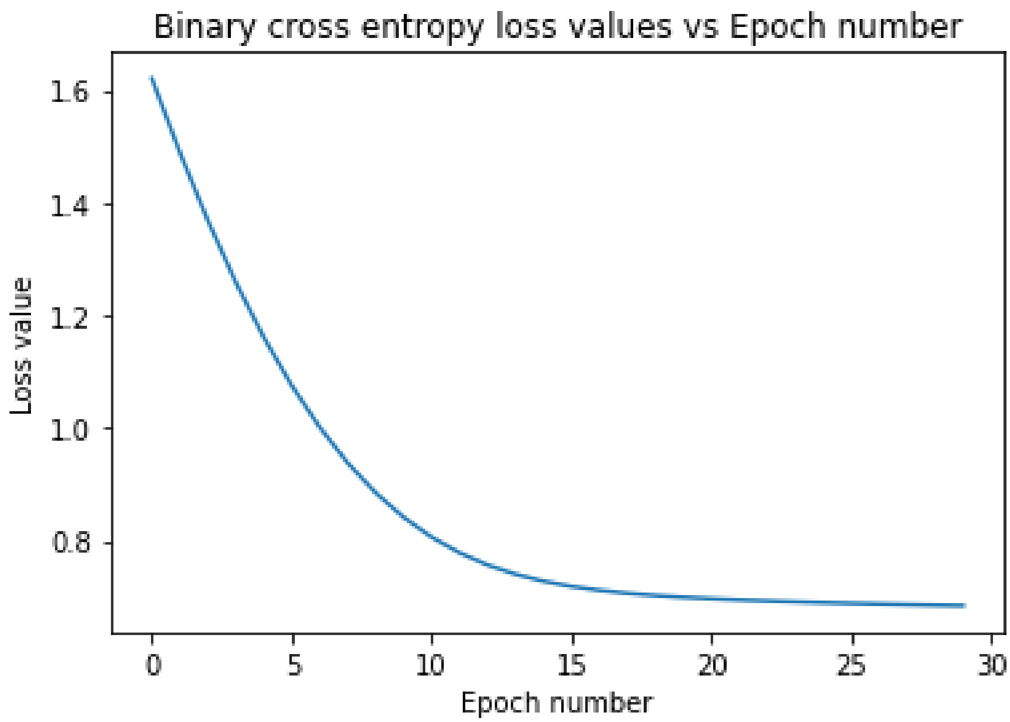
7.4 Logistic regression:

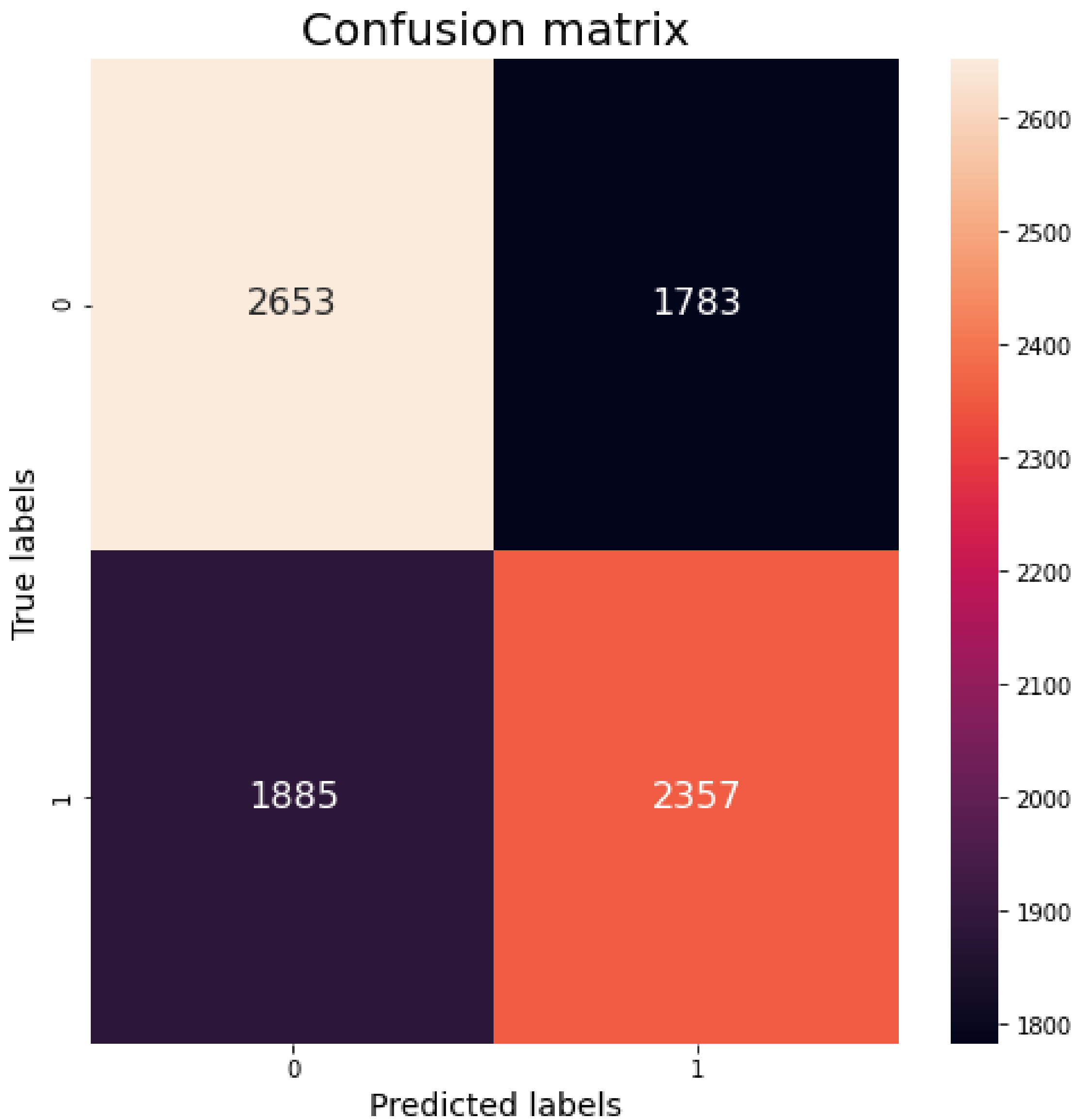
[163]:
```
eta       = 0.9
N_epochs = 30
avoid_scroll_output_window()
_,logreg_cls =␣
↪Logreg_testing(train_samples,train_labels,test_samples,test_labels,eta,N_epochs)

y_pred       = logreg_cls.predict(test_samples)
f1_score_log = f1_score(test_labels,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_log}\n")
```

```
conf_mat = confusion_matrix(test_labels,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()

# # Logistic regression on transformed data
# print("\nLogistic regression on transformed data\n")
# _,logreg_cls_tr =␣
 ↪Logreg_testing(train_samples_tr,train_labels,test_samples_tr,test_labels,eta,N_epochs,False)

# y_pred       = logreg_cls_tr.predict(test_samples_tr)
# f1_score_log = f1_score(test_labels,y_pred,average = 'weighted')
# print(f"f1_score : {f1_score_log}\n")

# conf_mat = confusion_matrix(test_labels,y_pred)
# fig= plt.figure(figsize=(8,8))
# sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
# plt.xlabel("Predicted labels",fontsize=14)
# plt.ylabel("True labels",fontsize=14)
# plt.title("Confusion matrix",fontsize=20)
# plt.show()

_,logreg_cls =␣
 ↪Logreg_testing(train_samples_1,train_labels_1,test_samples_1,test_labels_1,eta,N_epochs)

y_pred       = logreg_cls.predict(test_samples_1)
f1_score_log = f1_score(test_labels_1,y_pred,average = 'weighted')
print(f"f1_score : {f1_score_log}\n")

conf_mat = confusion_matrix(test_labels_1,y_pred)
fig= plt.figure(figsize=(8,8))
sns.heatmap(conf_mat,annot=True,annot_kws={"size":16},fmt="d")
plt.xlabel("Predicted labels",fontsize=14)
plt.ylabel("True labels",fontsize=14)
plt.title("Confusion matrix",fontsize=20)
plt.show()
```

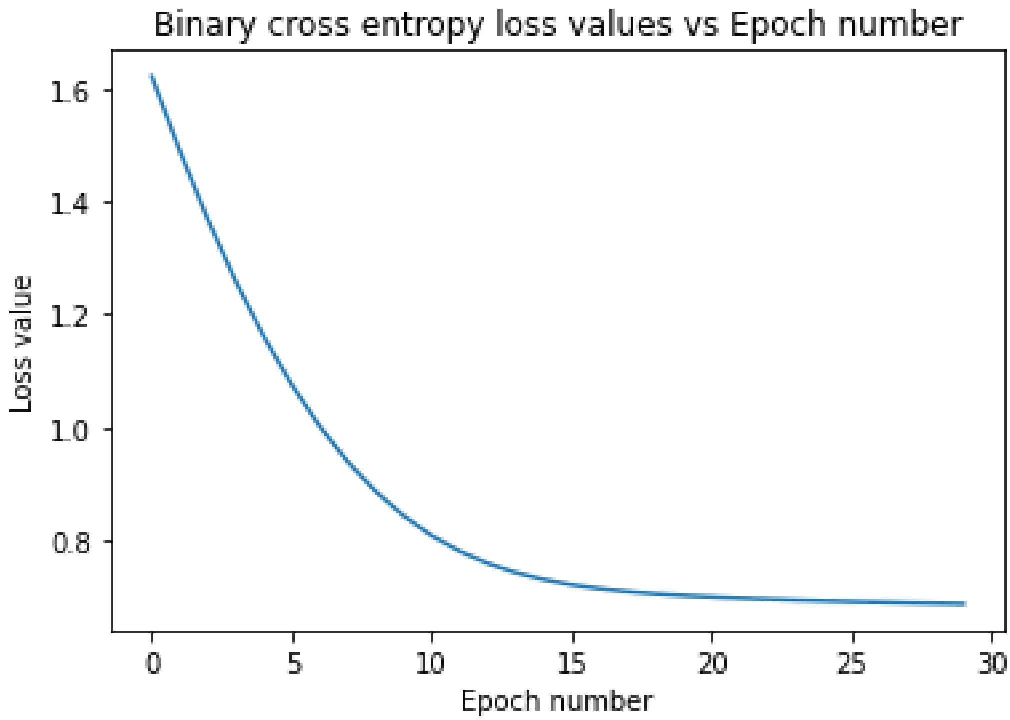
```
*********** LOGISTIC REGRESSION *************

train-test split : 80:20, Learning rate : 0.9, Number of epochs : 30

Using IN-BUILT classifier
Train accuracy= 58.60 %
```

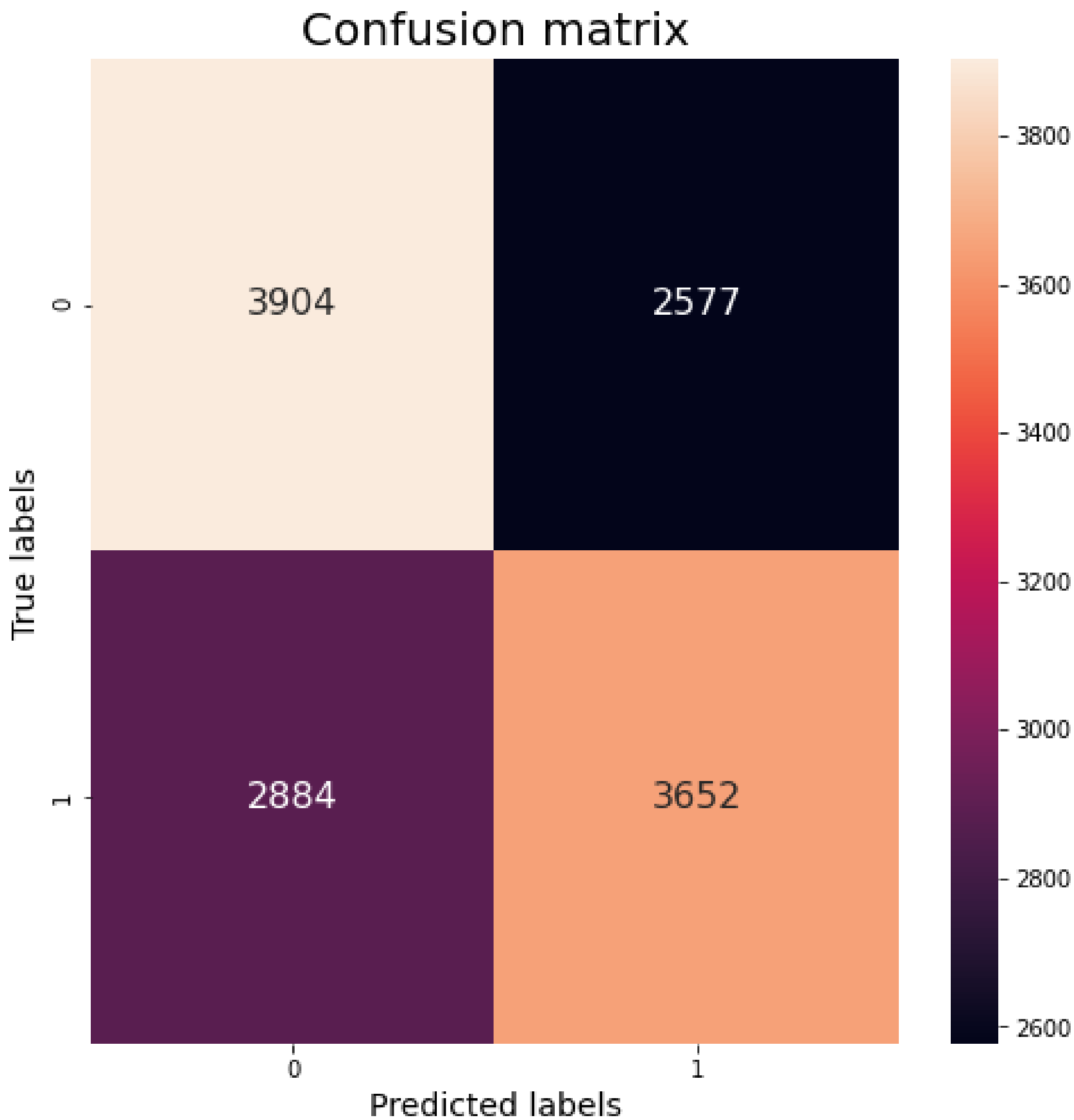
```
Test accuracy = 58.15 %
Using classifier built from scratch
Train accuracy= 57.97 %
Test accuracy = 57.73 %
f1_score : 0.5771523081255794
```






```
*********** LOGISTIC REGRESSION *************

train-test split : 70:30, Learning rate : 0.9, Number of epochs : 30

Using IN-BUILT classifier
Train accuracy= 58.55 %
Test accuracy = 58.59 %
Using classifier built from scratch
Train accuracy= 57.80 %
Test accuracy = 58.05 %
f1_score : 0.5802800707162671
```


Binary cross entropy loss values vs Epoch number
Loss value
1.6
1.4
1.2
1.0
0.8
0
5
10
15
20
25
30
Epoch number


Confusion matrix
3904
2577
2884
3652
True labels
Predicted labels
0
1
0
1
3800
3600
3400
3200
3000
2800
2600