CSCI 443: LECTURE 7 POPULATIONS AND SAMPLES

Professor David Harrison

OFFICE HOURS

Tuesday	4:00-5:00 PM
Wednesday	12:30-2:30 PM

HOMEWORK 2

Due Today at 11:00 PM.

Today is February 15

NOTE REGARDING EXAMS

Note regarding the midterm and final: The midterm and final will be written, so students will not have access to Databricks or Jupyter or Python. The questions asked here on an exam would be computed on small datasets as are used for question 31 in Part 7 and all the problems in Parts 8 and 9. I recommend that you answer the questions in these sections without using Python or a calculator. The problems are not difficult and doing them by hand may prepare you for answering such questions on the exams.

E, X, $ $ $ $

Why dollar signs in the homework?

The LaTeX Project

E

$$E$$

$\int_{x=0}^{\infty} f(x) dx$

$$\int_0^\infty f(x)\,dx$$

OUTCOME VS OUTCOME

"In healthcare, the term trial is [...] understood to involve systematic investigations to assess medical interventions' effectiveness and safety. In statistics, the term *trial* refers to a single instance of conducting a random experiment [...] Each individual roll constitutes a trial. [...] Within statistics, the *outcome* is the result observed from a single trial, exemplified by rolling a die and obtaining a 5. In statistics, an outcome is not a statistical measure like the mean [...]. In a clinical or animal trial, an outcome might refer to a statistical value calculated across a group of pateints, such as the mortality rate."

DATES OF INTEREST

February 8	HW2 handed out
February 15	HW2 due,
February 15 /16	HW3 handed out
February 22	HW3 due
February 27	Review
February 29	Midterm (must be before progress reports)
March 4	Progress Reports
March 8	Deadline for Withdrawal
March 9-17	Spring Break

BLACKBOARD & GITHUB

Slides up through lecture 7 on blackboard.

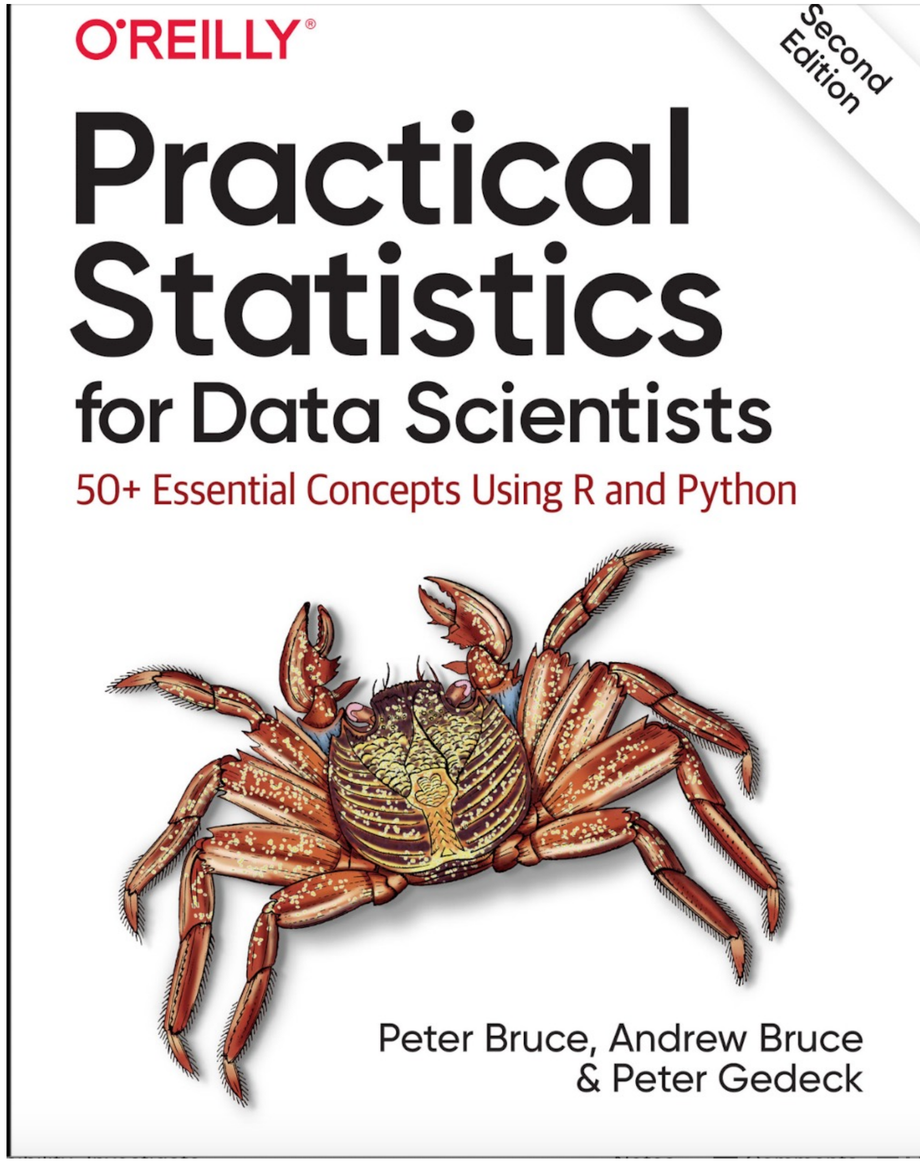
Lecture slides and examples committed to GitHub also up through lecture 7.

The project is at

https://github.com/dosirrah/CSCI443_AdvancedDataScience

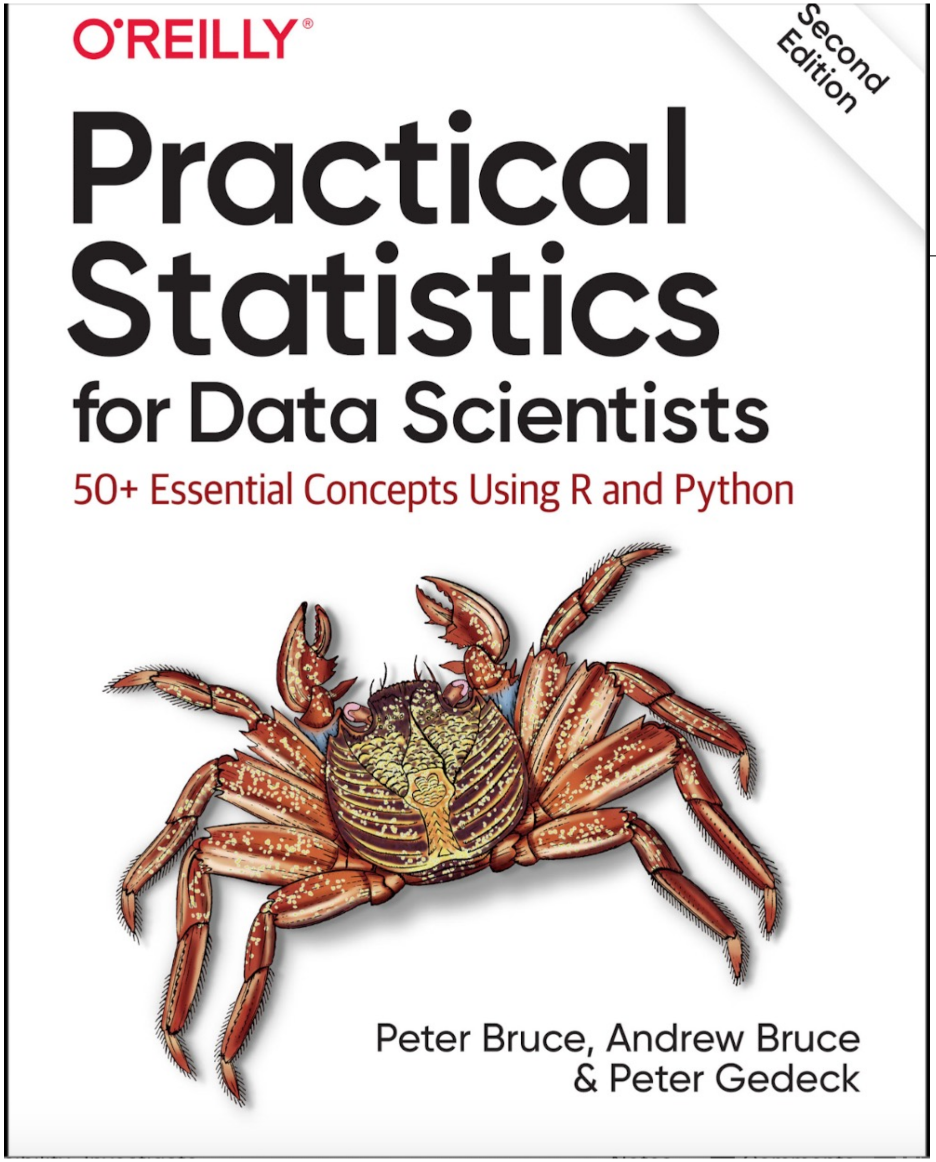
ASKED YOU TO READ

- Weighted mean
- Weighted median
- Trimmed mean
- Modes
- Bar charts
- Pie charts
- **Contour plots**



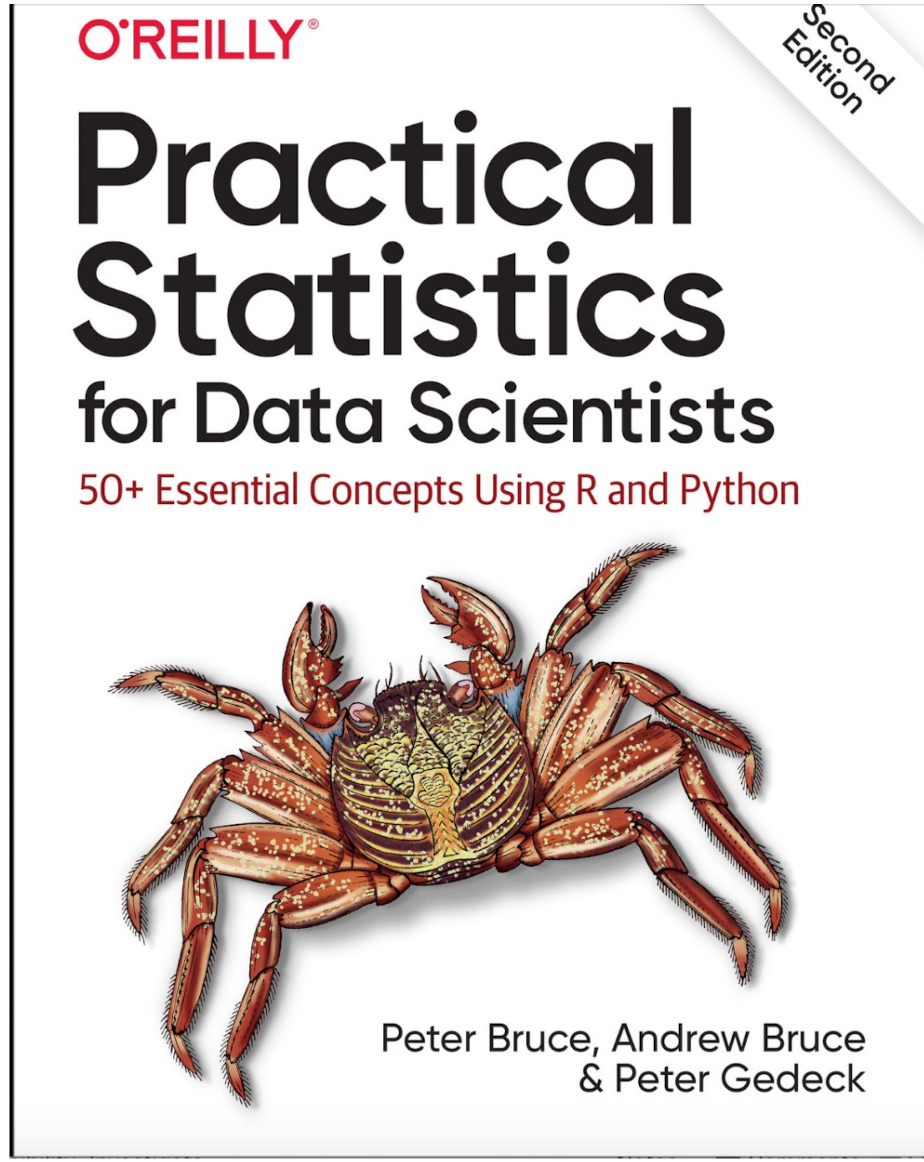
READ ABOUT

- Bias
  - Examples were already given in class, but book provides good example of selection bias.
- Random selection
  - Examples were already given in class, but book provides good example



ADD ONE MORE TO READ

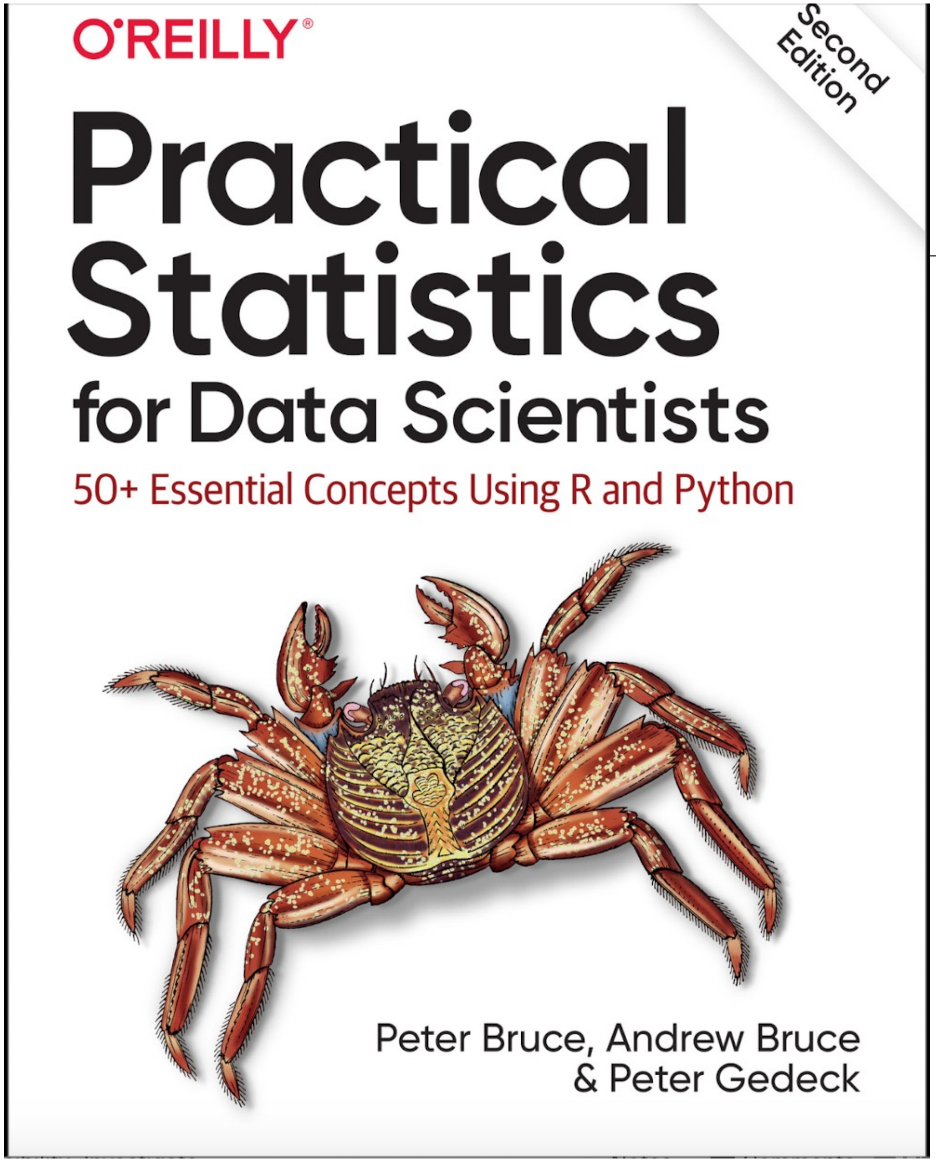
- Weighted mean
- Weighted median
- Trimmed mean
- Modes
- Bar charts
- Pie charts
- **Contour plots**



SKIP PARTS OF CHAPTER 1

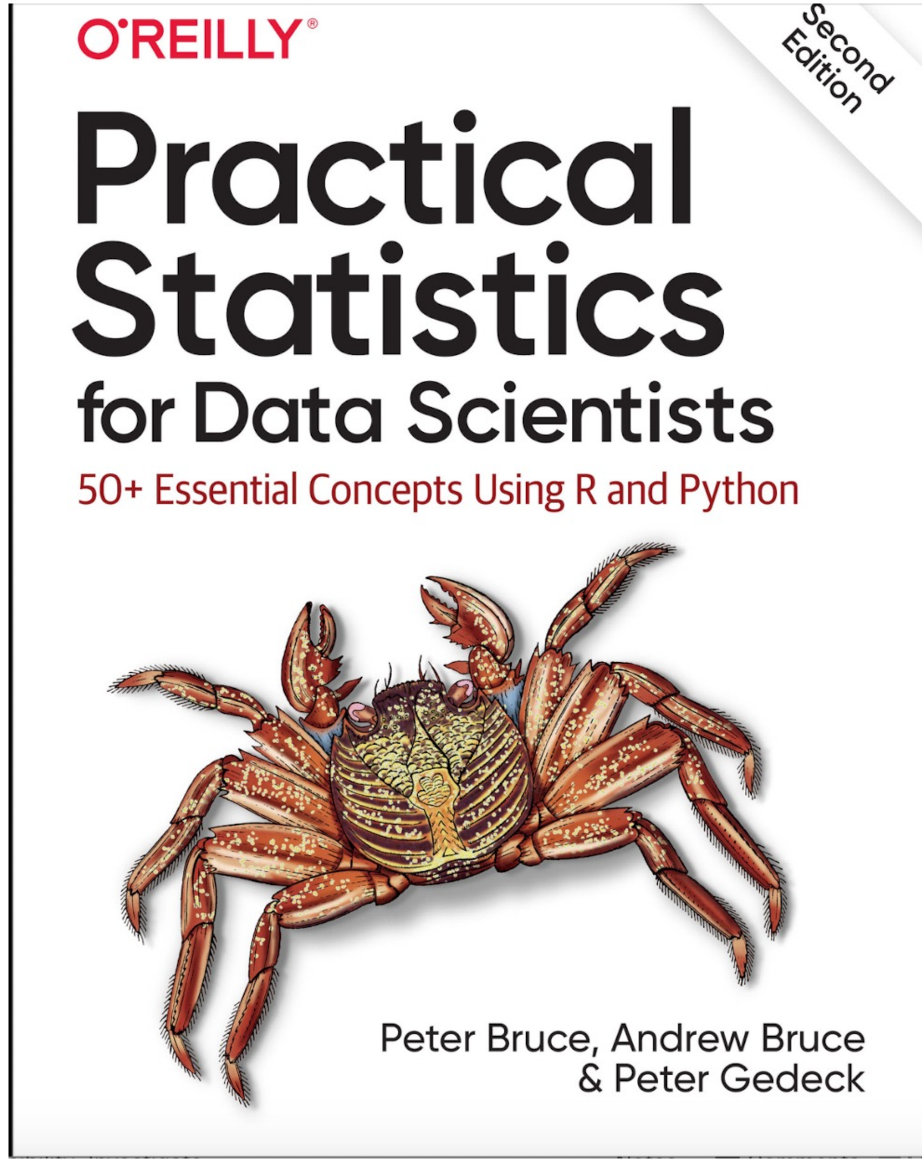
We will not cover in this class the following:

- Hexagonal binning
- Violin Plots
- Contingency Tables



THINGS I WANT TO COVER TODAY

- Chapter 2
  - Distribution vs. Sample vs. Population
  - How to evaluate Gaussian (without a computer)



PREVIOUS LECTURE: CORRELATION

KEY TERMS FOR CORRELATION

Correlation coefficient

A metric that measures the extent to which numeric variables are associated with one another (ranges from –1 to +1).

Correlation matrix

A table where the variables are shown on both rows and columns, and the cell values are the correlations between the variables.

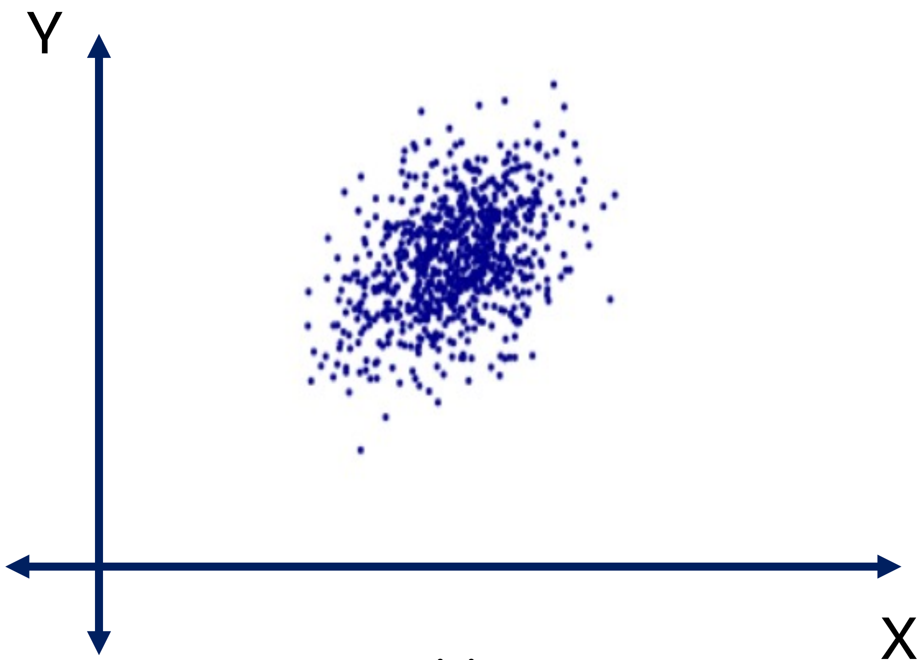
Scatterplot

A plot in which the x-axis is the value of one variable, and the y-axis the value of another.

PREVIOUS LECTURE: CORRELATION

Correlation between two random variables means they tend to move together.

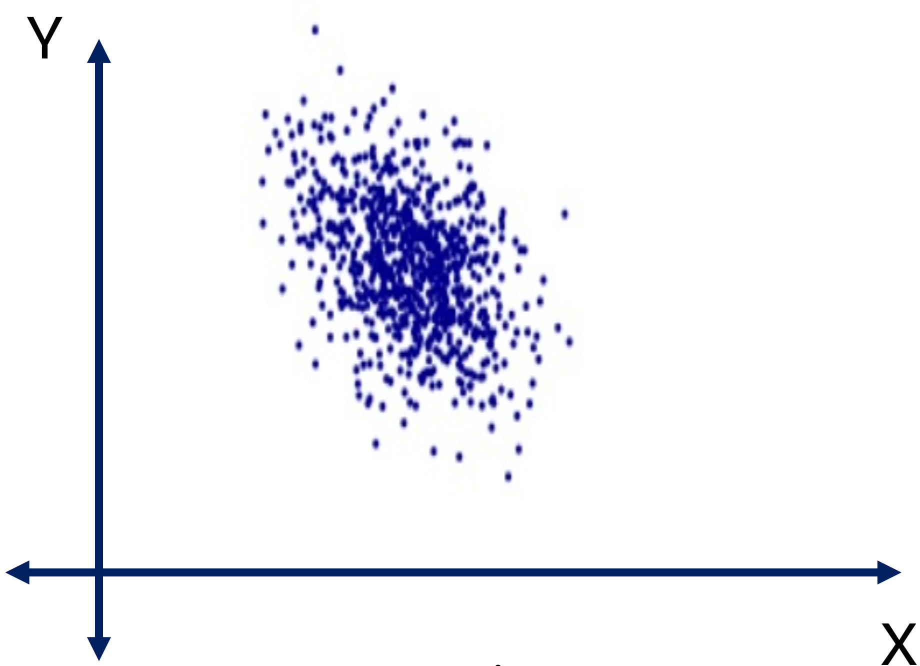
When one increases the other does, and vice versa.



Positive correlation

PREVIOUS LECTURE: CORRELATION

Negative correlation means they tend to move opposite to one another.

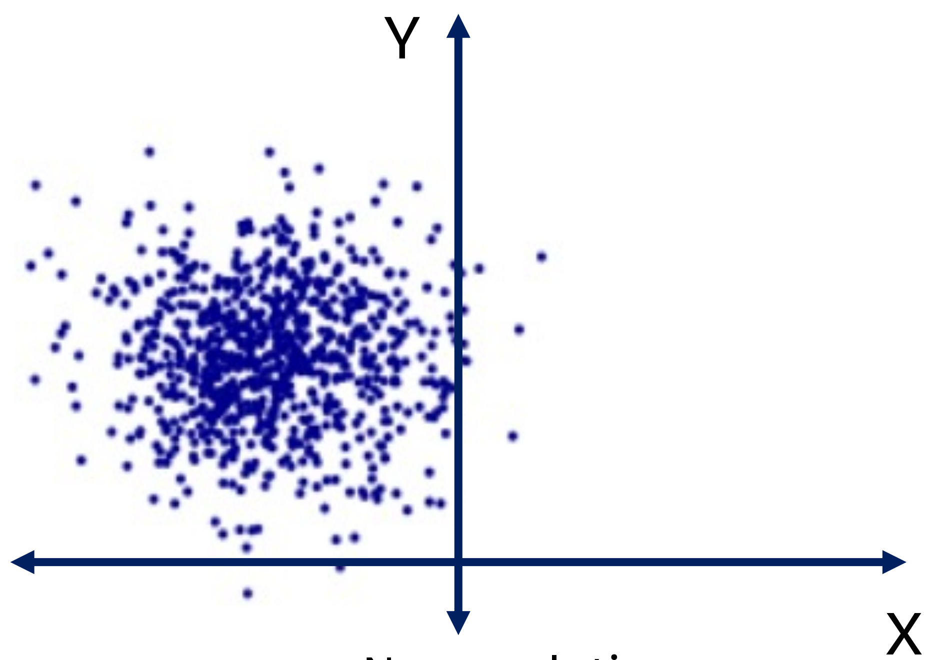


Negative correlation

PREVIOUS LECTURE: CORRELATION

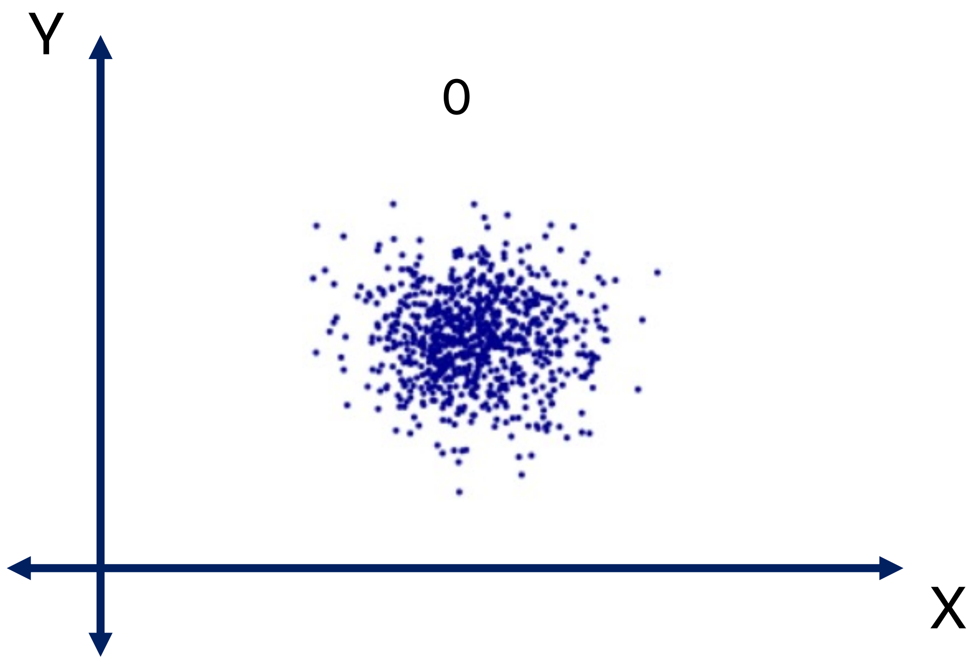
There is no correlation if they do not move together.

On a Cartesian plane this appears as NO tilt to the scatter of samples.

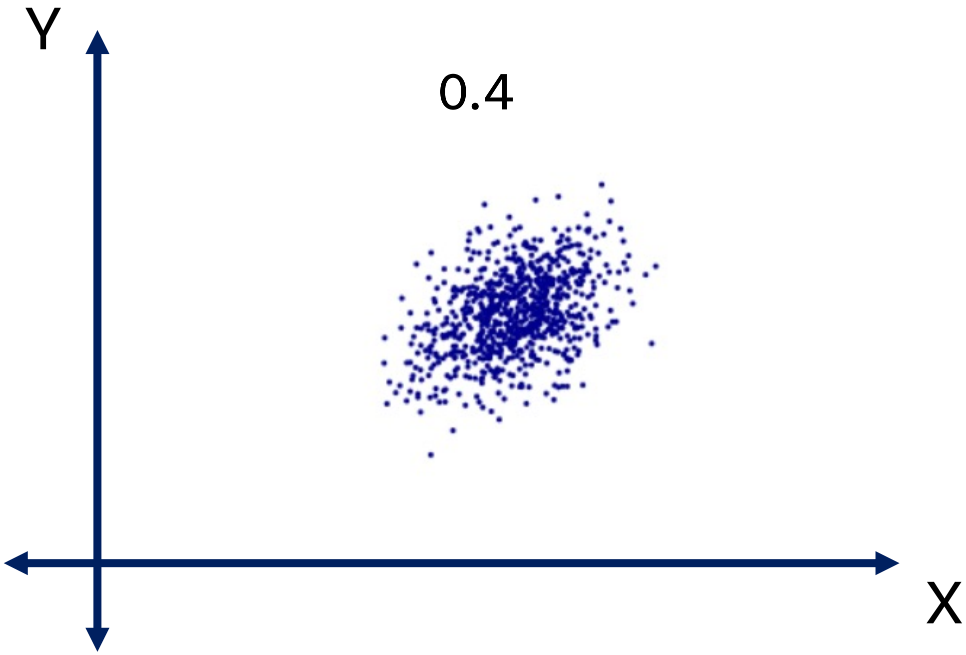


No correlation

PREVIOUS LECTURE: PEARSON CORRELATION



PREVIOUS LECTURE: PEARSON CORRELATION



COVARIANCE

Let X and Y be two random variables, covariance is

$$cov(X,Y) = E[(X-\mu_x)(Y-\mu_Y)]$$

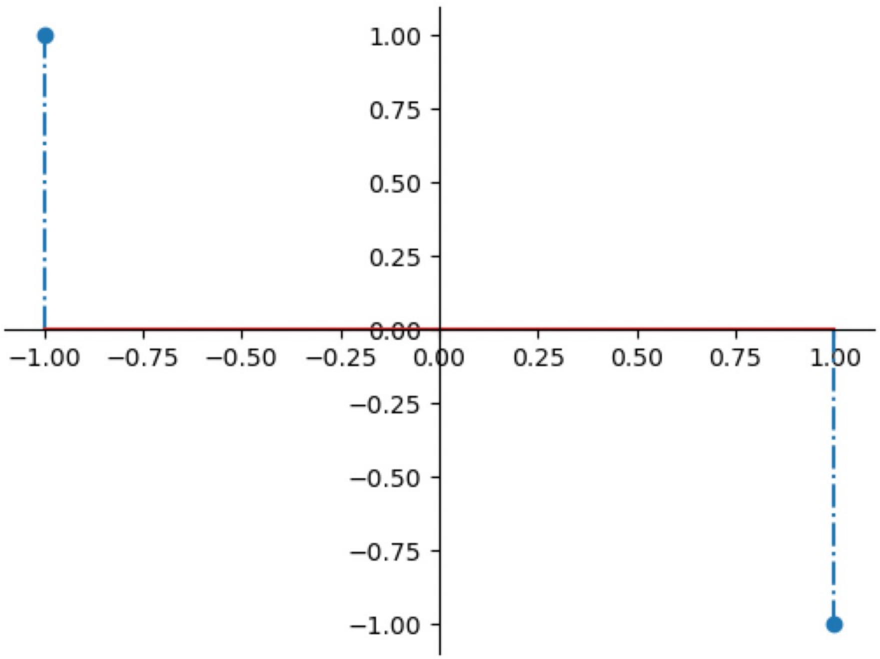
If $\mu_x = 0$ and $\mu_y = 0$ then this simplifies to

$$cov(X,Y) = E[XY]$$

$$E[XY] = \frac{1}{n}\sum_{i=1}^{n} x_i y_i$$

$$S = [(-1,1),(1,-1)]$$

$$E[XY] = \frac{1}{2}[(-1\cdot 1)+(1\cdot -1)] = -1$$



REMINDER! Specifically chose mean = 0 to simplify equation.

POPULATION COVARIANCE

Let X and Y be two random variables, covariance is

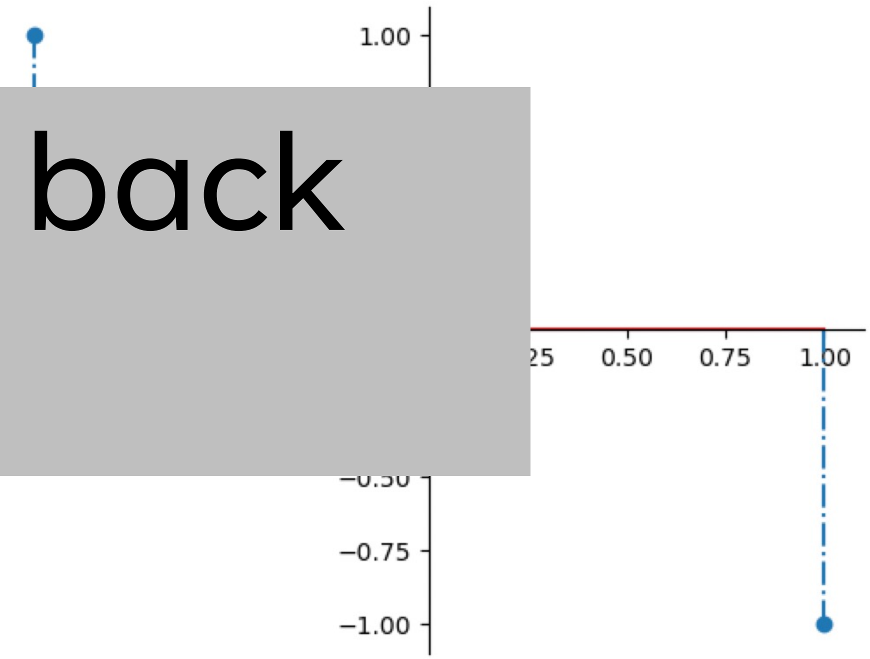
$$cov(X,Y) =$$

If $\mu_x = 0$ and $\mu_y = 0$

$$cov(X$$

$$E[XY$$

I'll come back to this.



$$S = [(-1, 1), (1, -1)]$$

$$E[XY] = \frac{1}{2}[(-1 \cdot 1) + (1 \cdot -1)] = -1$$

REMINDER! Specifically chose mean = 0 to simplify equation.

COVARIANCE

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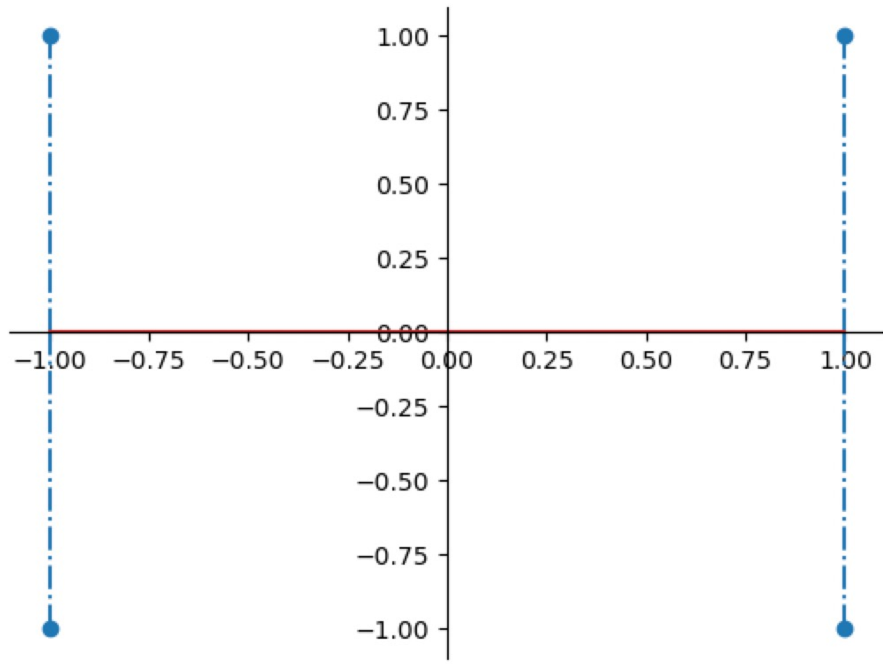

$$E[XY] = \frac{1}{4}[(-1 \cdot 1) + (-1 \cdot -1) + (1 \cdot 1) + (1 \cdot -1)] = 0$$

REMINDER! Specifically chose mean = 0 to simplify equation.

POPULATION COVARIANCE

Let X and Y be two random variables, covariance is

$$cov(X, Y) = E[(X$$

If $\mu_x = 0$ and $\mu_y = 0$

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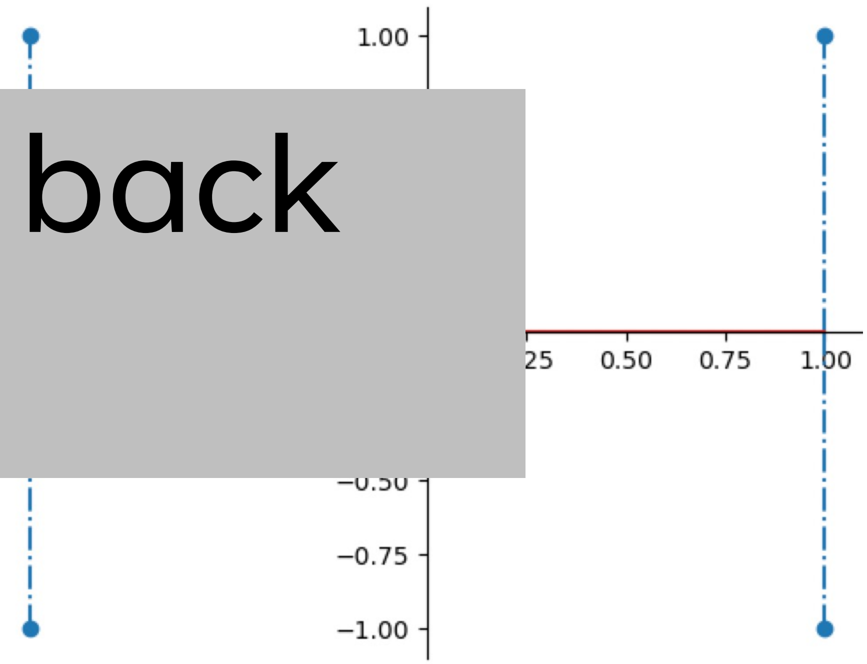

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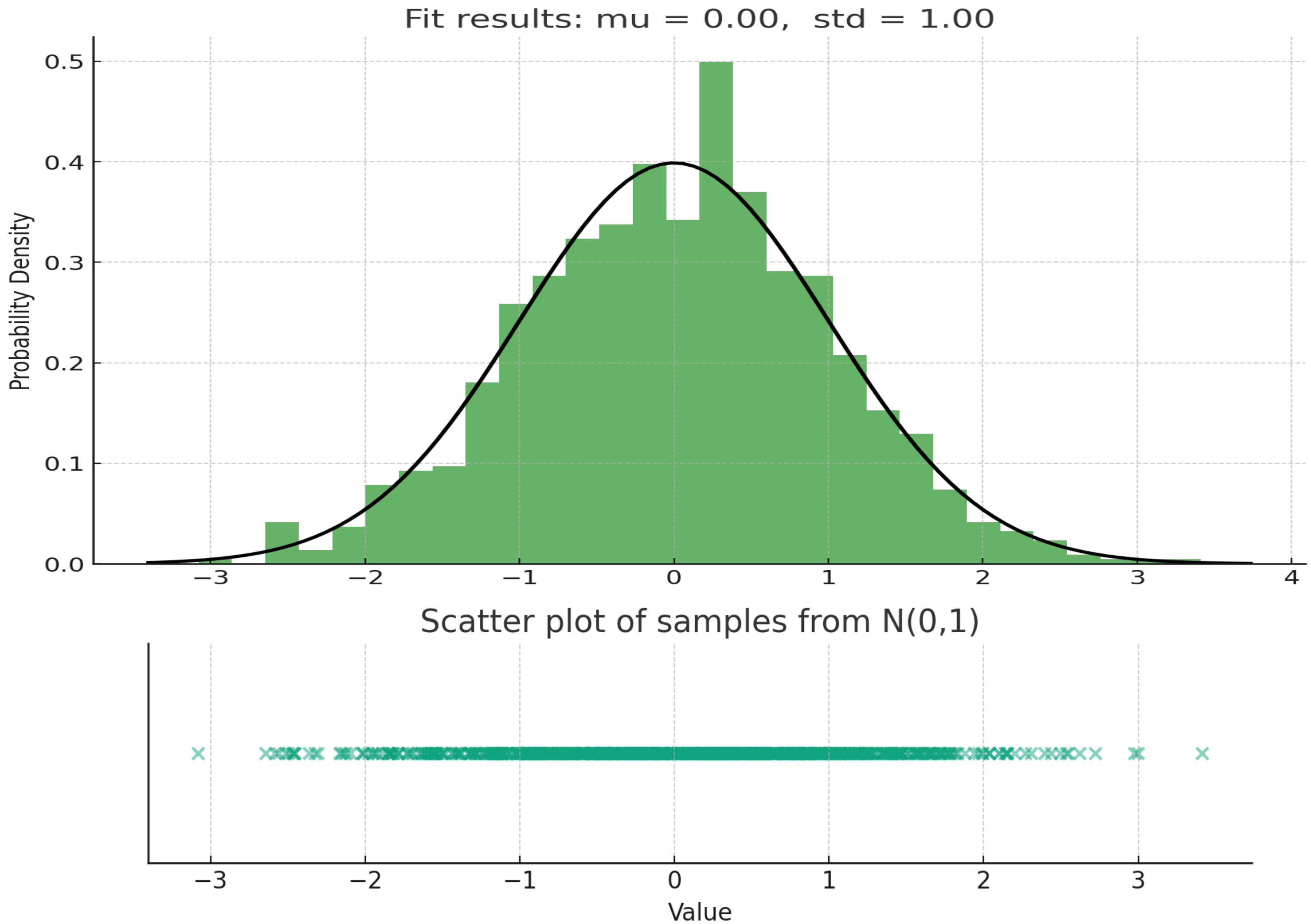
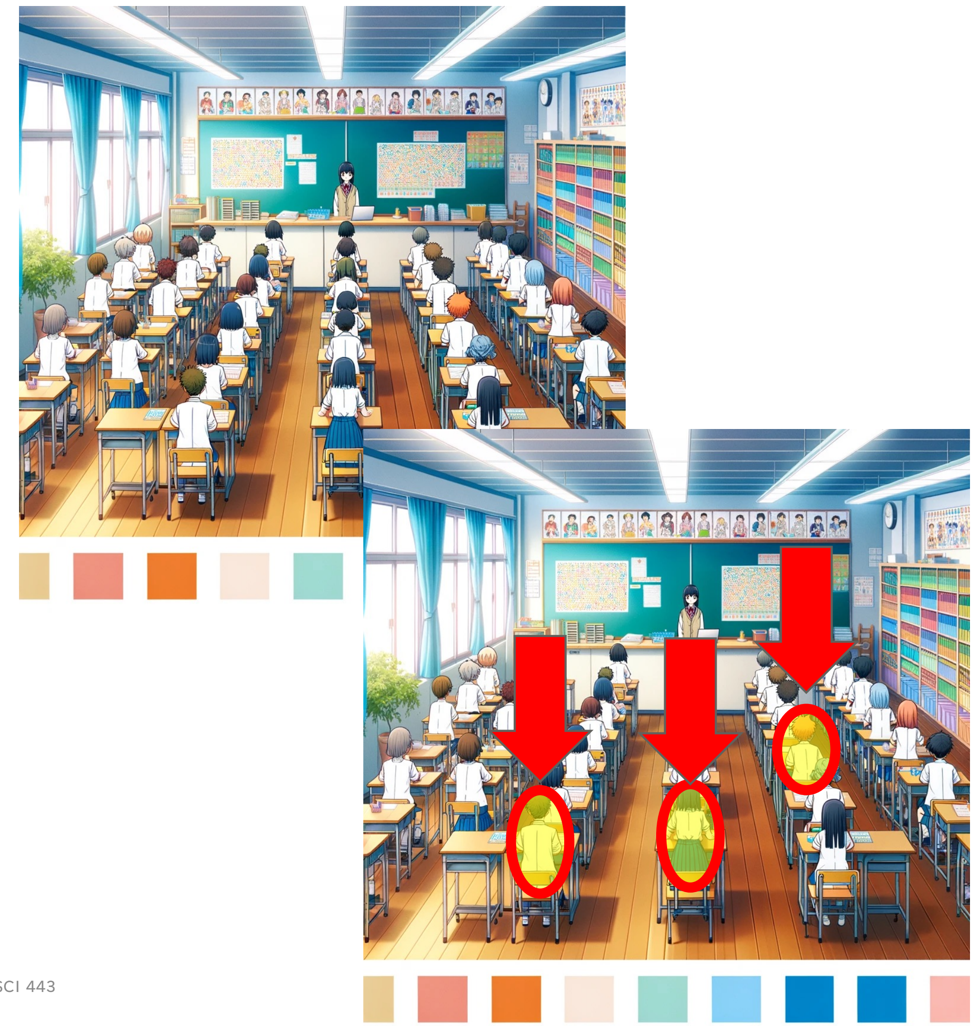
DISTRIBUTION VS. SAMPLE

- Random variable obeys a distribution.
- Draw samples of a random variable.
- We did this in HW2.



POPULATION VS. SAMPLE



- Population = my samples include all instances.
  - **All people in a class**
  - **All voters on election day**
- Sample
  - **Subset of people in the class.**
  - **Ex: poll a few voters.**

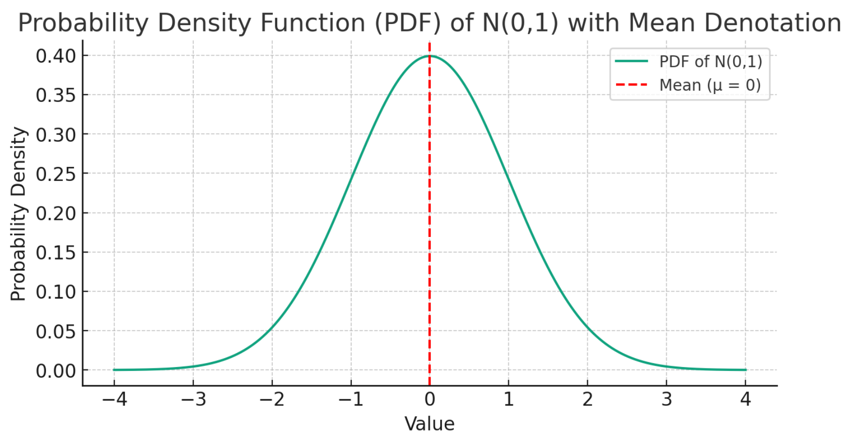
WHY SAMPLE?

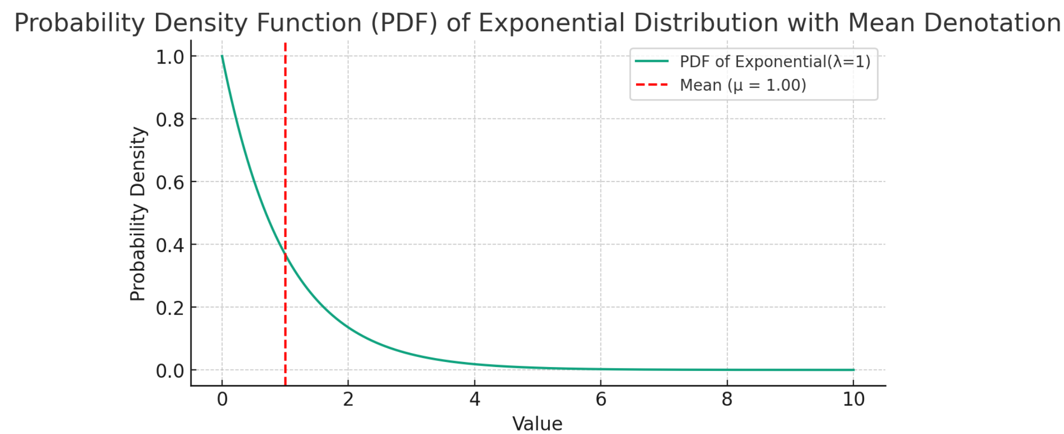
- Too expensive or time-consuming to talk to everyone in the population.
- Or when considering natural phenomena
  - **Alpha decay (U-238 -> Th-234)**
    - **Time series which goes on forever**
  - **Matter across the universe**
    - **Beyond our ability to count**
- There are cases when we can never compute a statistic over every member

DISTRIBUTION MEAN

- Distribution mean (μ) can be determined by integrating the pdf.

$$\mu = E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$





DISTRIBUTION MEAN VS. POPULATION MEAN VS. SAMPLE MEAN

- Distribution mean (μ) can be determined by integrating the pdf.

$$\mu = E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

- Doesn't work if we don't know the probability density function
- Population mean (μ) where population is size *N*

$$\mu = \frac{\sum_{i=1}^{N} X_i}{N}$$

X_i = the ith member of the population.

- Sample mean ($\bar{x}$) with sample size *n*

x_i = the ith sample in the sample set.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

LAW OF LARGE NUMBERS

- We can use a sample to estimate a population mean or distribution mean.
- Why? Law of Large Numbers.
- Wikipedia says:

In probability theory, the **law of large numbers** (**LLN**) is a mathematical theorem that states that the average of the results obtained from a large number of independent and identical random samples converges to the true value, if it exists.[1] More formally, the LLN states that given a sample of independent and identically distributed values, the sample mean converges to the true mean.

$$\lim_{n\to\infty} \tfrac{1}{n}(X_1 + X_2 + \ldots + X_n) = \mu$$

DISTRIBUTION VS. POPULATION VS. SAMPLE VARIANCE

Distribution variance

$$E[(X-\mu)^2] \qquad \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

Population variance

$$\sigma^2 = \frac{\sum_{i=1}^{N}(X_i-\mu)^2}{N}$$

Sample variance

$$s^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$$

DISTRIBUTION VS. POPULATION VS. SAMPLE VARIANCE

Distribution variance

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Population variance

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

?

WHAT HAPPENS WITH N RATHER THAN N-1 IN THE DENOMINATOR?

Distribution variance

true mean (μ) = 5, standard deviation (σ) = 2

Samples = [5.99,4.72,6.30,8.05,4.53]

Sample mean= 5.92

$$s^2_{\text{incorrect}} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{5}(x_i - 5.918)^2}{5} = 1.605$$

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \frac{\sum_{i=1}^{5}(x_i - 5.918)^2}{5-1} = 2.006$$

POPULATION VS. SAMPLE STANDARD DEVIATION

Standard deviation is the square root of the variance.

$$\sigma = \sqrt{\int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx}$$

Population standard deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}}$$

Sample standard deviation

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

POPULATION VS. SAMPLE COVARIANCE

Distribution Covariance

$$\mathrm{Cov}(X, Y) = \iint (x - \mu_X)(y - \mu_Y) f(x, y)\, dx\, dy$$

Population Covariance

$$\sigma_{XY} = \frac{1}{N} \sum_{i=1}^{N} (X_i - \mu_X)(Y_i - \mu_Y)$$

Sample Covariance

$$s_{XY} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

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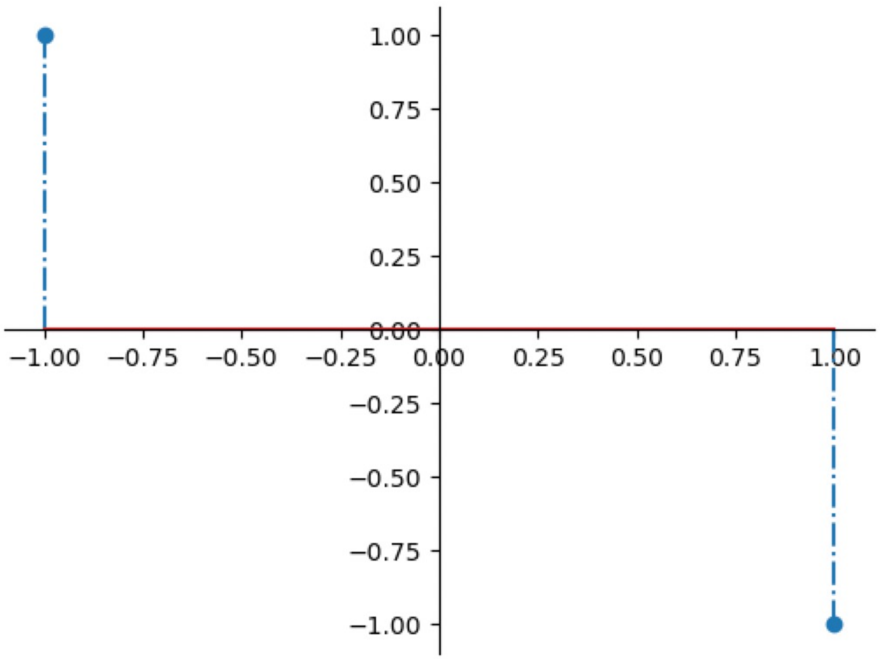

Am I assuming population covariance or sample covariance?

COVARIANCE

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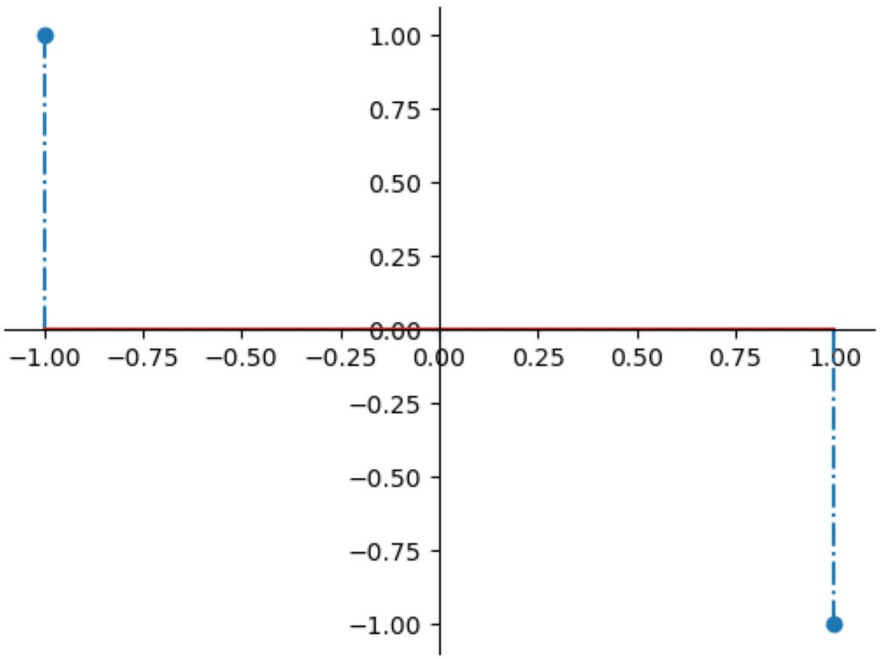

This is an example of population covariance

THANK YOU

David Harrison

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