```
clc;
clear all;

% Definir parámetros de los enlaces
N = 6; % Número de enlaces
L = [0.5; 0.4; 0.35; 0.3; 0.25; 0.2]; % Longitudes de los enlaces (metros)
M = [2.0; 1.5; 1.2; 1.0; 0.8; 0.6]; % Masas de los enlaces (kg)
J = [0.1; 0.08; 0.06; 0.05; 0.04; 0.03]; % Inercias de los enlaces (kg*m^2)
f = zeros(1, N); % Sin fricción
g = 9.81; % Aceleración gravitacional (m/s²)

syms theta [N 1] real % posiciones angulares respecto a la vertical (absolutas)
syms omega [N 1] real % velocidades angulares dtheta/dt
syms a [N 1] real % aceleraciones angulares domega/dt = d^2theta/dt^2
syms tau [N 1] real % pares aplicados a cada articulación (si hay motores)
syms tau_ext [N 1] real; % Pares externos actuando en cada articulación
q = theta; % coordenadas generalizadas
dotq = omega; % velocidades generalizadas, derivadas temporales de q

syms M_f real; % Masa del efector final (kg)

%%%%%%%%%%%%%%%%%%%%%% Energia Cinetica y Potencial %%%%%%%%%%%%%%%%%%%%%%

% Inicialización de matrices y vectores
PosArticulacion = sym(zeros(2, N + 1));
PosCdG = sym(zeros(2, N));
VelCdG = sym(zeros(2, N));

% Inicialización de energías
E_Potencial = 0;
E_Cinetica = 0;
%arreglo para guardarlo por eslabon
E_Pot = [];
E_Cin = [];

RayleighV = 0.5 * dotq(1)^2 * f(1); % Amortiguamiento de Rayleigh en la primera articulación

% Calculo de energías potencial y cinética
for i = 1:N + 1
    if i > 1
        PosArticulacion(:, i) = PosArticulacion(:, i - 1) + L(i - 1) * [sin(q(i - 1)); cos(q(i
        if i <= N
            RayleighV = RayleighV + 0.5 * (dotq(i) - dotq(i - 1))^2 * f(i); % La velocidad angu
        end
    end

    if i <= N
        PosCdG(:, i) = PosArticulacion(:, i) + L(i) / 2 * [sin(q(i)); cos(q(i))];
        VelCdG(:, i) = jacobian(PosCdG(:, i), q) * dotq; % Cinemática de la velocidad
```

```
        E_Potencial = simplify(E_Potencial + M(i) * g * PosCdG(2, i));
        E_Pot=[E_Pot; M(i)*g*PosCdG(2,i)];
        E_Cinetica = simplify(E_Cinetica + 0.5 * M(i) * (VelCdG(:, i)' * VelCdG(:, i)) + 0.5 *
        E_Cin = [E_Cin; 0.5*M(i)*(VelCdG(:,i)'*VelCdG(:,i)) + 0.5*J(i)*dotq(i)^2];
    else
        VelPunta = jacobian(PosArticulacion(:, i), q) * dotq;
        E_Potencial = simplify(E_Potencial + M_f * g * PosArticulacion(2, i));
        E_Cinetica = simplify(E_Cinetica + 0.5 * M_f * (VelPunta' * VelPunta));
    end
end
```

Implementemos ahora las ecuaciones Euler-Lagrange del movimiento ( $N$ G.L. $\rightarrow$ $N$ ecuaciones):

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q} + Q$$

```
%%%%%%%%%% Ecuaciones de Lagrange %%%%%%%%%%%%

Lagrangiano = E_Cinetica - E_Potencial; % Lagrangiano
dL_dq = jacobian(Lagrangiano, q).' % Gravitatoria // Derivada del lagrangiano con respecto a la
```

```
dL_dq =
```

$$\begin{pmatrix} \frac{59841\sin(\theta_1)}{2000} + \frac{M_f\ (\omega_1\cos(\theta_1)\,\sigma_2 - \omega_1\sin(\theta_1)\,\sigma_1)}{2} + \frac{981\,M_f\sin(\theta_1)}{200} - \frac{3\,\omega_1\sin(\theta_1)\ (\sigma_{20}+\sigma_{21}+\sigma_{11})}{200} + \\ \frac{85347\sin(\theta_2)}{5000} + \frac{M_f\left(\frac{4\,\omega_2\cos(\theta_2)\,\sigma_2}{5} - \frac{4\,\omega_2\sin(\theta_2)\,\sigma_1}{5}\right)}{2} + \frac{981\,M_f\sin(\theta_2)}{250} - \frac{3\,\omega_2\sin(\theta_2)\ (\sigma_{20}+\sigma_{21}+\sigma_{11})}{250} \\ \frac{20601\sin(\theta_3)}{2000} + \frac{M_f\left(\frac{7\,\omega_3\cos(\theta_3)\,\sigma_2}{10} - \frac{7\,\omega_3\sin(\theta_3)\,\sigma_1}{10}\right)}{2} + \frac{6867\,M_f\sin(\theta_3)}{2000} - \frac{21\,\omega_3\sin(\theta_3)\ (\sigma_{20} \\ \frac{55917\sin(\theta_4)}{10000} + \frac{M_f\left(\frac{3\,\omega_4\cos(\theta_4)\,\sigma_2}{5} - \frac{3\,\omega_4\sin(\theta_4)\,\sigma_1}{5}\right)}{2} + \frac{29} \\ \frac{981\sin(\theta_5)}{400} + \frac{M_f}{} \\ \frac{2943}{5} \end{pmatrix}$$

where

$$\sigma_1 = \frac{\omega_1\cos(\theta_1)}{2} + \frac{\sigma_{24}}{5} + \frac{\sigma_{11}}{20} + \frac{\sigma_9}{10} + \frac{\omega_5\cos(\theta_5)}{4} + \frac{\omega_6\cos(\theta_6)}{5}$$

$$\sigma_2 = \frac{\omega_1\sin(\theta_1)}{2} + \frac{\sigma_{23}}{5} + \frac{\sigma_{14}}{20} + \frac{\sigma_{10}}{10} + \frac{\omega_5\sin(\theta_5)}{4} + \frac{\omega_6\sin(\theta_6)}{5}$$

$$\sigma_3 = \sigma_{12} + \sigma_{13} + \sigma_{11} + 6\,\omega_4\cos(\theta_4) + \sigma_{22} + 2\,\omega_6\cos(\theta_6)$$

$$\sigma_4 = \sigma_{15} + \sigma_{16} + \sigma_{14} + 6\,\omega_4\sin(\theta_4) + \sigma_{19} + 2\,\omega_6\sin(\theta_6)$$

$$\sigma_5 = \sigma_{17} + \sigma_{18} + 14\,\omega_3\sin(\theta_3) + 12\,\omega_4\sin(\theta_4) + \sigma_{19}$$

$$\sigma_6 = \sigma_{20} + \sigma_{21} + 14\,\omega_3\cos(\theta_3) + 12\,\omega_4\cos(\theta_4) + \sigma_{22}$$

$$\sigma_7 = 5\,\omega_1\sin(\theta_1) + \sigma_{23}$$

$$\sigma_8 = 5\,\omega_1\cos(\theta_1) + \sigma_{24}$$

$$\sigma_9 = 3\,\omega_4\cos(\theta_4)$$

$$\sigma_{10} = 3\,\omega_4\sin(\theta_4)$$

$$\sigma_{11} = 7\,\omega_3\cos(\theta_3)$$

$$\sigma_{12} = 10\,\omega_1\cos(\theta_1)$$

```
dL_ddotq = jacobian(Lagrangiano, dotq).'; % Moomentum Derivada del lagrangiano con respecto a l



% Derivadas con respecto al tiempo
d_dt_dL_ddotq = sym(zeros(N, 1));
for i = 1:N
    d_dt_dL_ddotq(i) = diff(dL_ddotq(i), q(i)) * dotq(i) + diff(dL_ddotq(i), dotq(i)) * a(i);
end
% Ecuaciones de movimiento
Ecuaciones_Lagrange = d_dt_dL_ddotq - dL_dq == tau + tau_ext;

% Fuerzas de Coriolis y centrífugas
C = sym(zeros(N, N));
for i = 1:N
    for j = 1:N
        for k = 1:N
            C(i, j) = C(i, j) + 0.5 * (diff(dL_ddotq(i), q(j)) + diff(dL_ddotq(j), q(i)) - dif
        end
    end
end
Ecuaciones_Lagrange = simplify(Ecuaciones_Lagrange);
C = simplify(C);

% y al fin, los resultados
disp('Posiciones del Centro de Masa:');
```

```
Posiciones del Centro de Masa:
```

```
PosCdG = simplify(PosCdG)
```

```
PosCdG =
```

$$\begin{pmatrix} \frac{\sin(\theta_1)}{4} & \frac{\sin(\theta_1)}{2}+\frac{\sin(\theta_2)}{5} & \frac{\sin(\theta_1)}{2}+\sigma_1+\frac{7\sin(\theta_3)}{40} & \frac{\sin(\theta_1)}{2}+\sigma_1+\sigma_3+\frac{3\sin(\theta_4)}{20} & \frac{\sin(\theta_1)}{2}+\sigma_1+\sigma_3+\sigma \\ \frac{\cos(\theta_1)}{4} & \frac{\cos(\theta_1)}{2}+\frac{\cos(\theta_2)}{5} & \frac{\cos(\theta_1)}{2}+\sigma_2+\frac{7\cos(\theta_3)}{40} & \frac{\cos(\theta_1)}{2}+\sigma_2+\sigma_4+\frac{3\cos(\theta_4)}{20} & \frac{\cos(\theta_1)}{2}+\sigma_2+\sigma_4+\sigma \end{pmatrix}$$

where

$$\sigma_1 = \frac{2\sin(\theta_2)}{5}$$

$$\sigma_2 = \frac{2\cos(\theta_2)}{5}$$

$$\sigma_3 = \frac{7\sin(\theta_3)}{20}$$

$$\sigma_4 = \frac{7\cos(\theta_3)}{20}$$

$$\sigma_5 = \frac{3\sin(\theta_4)}{10}$$

$$\sigma_6 = \frac{3\cos(\theta_4)}{10}$$

```
disp('Velocidades del Centro de Masa:');
```

```
Velocidades del Centro de Masa:
```

```
VelCdG = simplify(VelCdG)
```

```
VelCdG =
```

$$\begin{pmatrix} \frac{\omega_1 \cos(\theta_1)}{4} & \sigma_2 + \frac{\omega_2 \cos(\theta_2)}{5} & \sigma_2 + \sigma_4 + \frac{\sigma_7}{40} & \sigma_2 + \sigma_4 + \frac{\sigma_7}{20} + \frac{\sigma_8}{20} & \sigma_2 + \sigma_4 + \frac{\sigma_7}{20} + \frac{\sigma_8}{10} + \frac{\omega_5 \cos(\theta_5)}{8} & \sigma \\ -\frac{\omega_1 \sin(\theta_1)}{4} & -\sigma_1 - \frac{\omega_2 \sin(\theta_2)}{5} & -\sigma_1 - \sigma_3 - \frac{\sigma_5}{40} & -\sigma_1 - \sigma_3 - \frac{\sigma_5}{20} - \frac{\sigma_6}{20} & -\sigma_1 - \sigma_3 - \frac{\sigma_5}{20} - \frac{\sigma_6}{10} - \frac{\omega_5 \sin(\theta_5)}{8} & - \end{pmatrix}$$

where

$$\sigma_1 = \frac{\omega_1 \sin(\theta_1)}{2}$$

$$\sigma_2 = \frac{\omega_1 \cos(\theta_1)}{2}$$

$$\sigma_3 = \frac{2\,\omega_2 \sin(\theta_2)}{5}$$

$$\sigma_4 = \frac{2\,\omega_2 \cos(\theta_2)}{5}$$

$$\sigma_5 = 7\,\omega_3 \sin(\theta_3)$$

$$\sigma_6 = 3\,\omega_4 \sin(\theta_4)$$

$$\sigma_7 = 7\,\omega_3 \cos(\theta_3)$$

$$\sigma_8 = 3\,\omega_4 \cos(\theta_4)$$

```
disp('Energía Potencial Total:');
```

```
Energía Potencial Total:
```

```
simplify(E_Potencial)
```

```
ans =
```

$$\frac{59841 \cos(\theta_1)}{2000} + \frac{85347 \cos(\theta_2)}{5000} + \frac{20601 \cos(\theta_3)}{2000} + \frac{55917 \cos(\theta_4)}{10000} + \frac{981 \cos(\theta_5)}{400} + \frac{2943 \cos(\theta_6)}{5000} + \frac{981\, M_f\, \mathrm{co}}{200}$$

```
disp('Energía Potencial Por Eslabon:');
```

```
Energía Potencial Por Eslabon:
```

```
simplify(E_Pot)
```

```
ans =
```

$$\begin{pmatrix} \sigma_2 \\ \frac{2943\cos(\theta_1)}{400}+\frac{2943\cos(\theta_2)}{1000} \\ \frac{2943\cos(\theta_1)}{500}+\frac{2943\cos(\theta_2)}{625}+\sigma_1 \\ \sigma_2+\frac{981\cos(\theta_2)}{250}+\frac{6867\cos(\theta_3)}{2000}+\frac{2943\cos(\theta_4)}{2000} \\ \frac{981\cos(\theta_1)}{250}+\frac{1962\cos(\theta_2)}{625}+\frac{6867\cos(\theta_3)}{2500}+\frac{2943\cos(\theta_4)}{1250}+\frac{981\cos(\theta_5)}{1000} \\ \frac{2943\cos(\theta_1)}{1000}+\frac{2943\cos(\theta_2)}{1250}+\sigma_1+\frac{8829\cos(\theta_4)}{5000}+\frac{2943\cos(\theta_5)}{2000}+\frac{2943\cos(\theta_6)}{5000} \end{pmatrix}$$

where

$$\sigma_1 = \frac{20601\cos(\theta_3)}{10000}$$

$$\sigma_2 = \frac{981\cos(\theta_1)}{200}$$

```
disp('Energía Cinética Total:');
```

```
Energía Cinética Total:
```

```
simplify(E_Cinetica)
```

```
ans =
```

$$\frac{(\sigma_{10} + \sigma_{11} + 14\,\omega_3\cos(\theta_3) + 12\,\omega_4\cos(\theta_4) + \sigma_{12})^2}{4000} + \frac{3\,(\sigma_6 + \sigma_7 + \sigma_1 + 6\,\omega_4\sin(\theta_4) + \sigma_5 + 2\,\omega_6\sin(\theta_6))^2}{4000} + \frac{3}{}$$

where

$$\sigma_1 = 7\,\omega_3\sin(\theta_3)$$

$$\sigma_2 = 7\,\omega_3\cos(\theta_3)$$

$$\sigma_3 = 20\,\omega_1\sin(\theta_1)$$

$$\sigma_4 = 16\,\omega_2\sin(\theta_2)$$

$$\sigma_5 = 5\,\omega_5\sin(\theta_5)$$

$$\sigma_6 = 10\,\omega_1\sin(\theta_1)$$

$$\sigma_7 = 8\,\omega_2\sin(\theta_2)$$

$$\sigma_8 = 3\,\omega_4\sin(\theta_4)$$

$$\sigma_9 = 2\,\omega_2\sin(\theta_2)$$

$$\sigma_{10} = 20\,\omega_1\cos(\theta_1)$$

$$\sigma_{11} = 16\,\omega_2\cos(\theta_2)$$

$$\sigma_{12} = 5\,\omega_5\cos(\theta_5)$$

$$\sigma_{13} = 10\,\omega_1\cos(\theta_1)$$

$$\sigma_{14} = 8\,\omega_2\cos(\theta_2)$$

$$\sigma_{15} = 3\,\omega_4\cos(\theta_4)$$

$$\sigma_{16} = 2\,\omega_2\cos(\theta_2)$$

```
disp('Energía Cinética por eslabon:');
```

```
Energía Cinética por eslabon:
```

```
simplify(E_Cin)
```

ans =

$$\begin{pmatrix} \frac{9\,{\omega_1}^2}{80} \\ \frac{3\,(5\,\omega_1\cos(\theta_1)+2\,\omega_2\cos(\theta_2))^2}{400}+\frac{3\,(5\,\omega_1\sin(\theta_1)+2\,\omega_2\sin(\theta_2))^2}{400}+\frac{{\omega_2}^2}{25} \\ \frac{3\,(\sigma_8+\sigma_9+\sigma_2)^2}{8000}+\frac{3\,{\omega_3}^2}{100}+\frac{3\,(\sigma_3+\sigma_4+\sigma_1)^2}{8000} \\ \frac{(\sigma_{11}+\sigma_{12}+\sigma_2+3\,\omega_4\cos(\theta_4))^2}{800}+\frac{(\sigma_6+\sigma_7+\sigma_1+3\,\omega_4\sin(\theta_4))^2}{800}+\frac{{\omega_4}^2}{40} \\ \frac{(\sigma_8+\sigma_9+14\,\omega_3\cos(\theta_3)+12\,\omega_4\cos(\theta_4)+\sigma_{10})^2}{4000}+\frac{(\sigma_3+\sigma_4+14\,\omega_3\sin(\theta_3)+12\,\omega_4\sin(\theta_4)+\sigma_5)^2}{4000}+ \\ \frac{3\,(\sigma_6+\sigma_7+\sigma_1+6\,\omega_4\sin(\theta_4)+\sigma_5+2\,\omega_6\sin(\theta_6))^2}{4000}+\frac{3\,(\sigma_{11}+\sigma_{12}+\sigma_2+6\,\omega_4\cos(\theta_4)+\sigma_{10}+2\,\omega_6\cos(\theta_6)}{4000} \end{pmatrix}$$

where

$$\sigma_1 = 7\,\omega_3\sin(\theta_3)$$

$$\sigma_2 = 7\,\omega_3\cos(\theta_3)$$

$$\sigma_3 = 20\,\omega_1\sin(\theta_1)$$

$$\sigma_4 = 16\,\omega_2\sin(\theta_2)$$

$$\sigma_5 = 5\,\omega_5\sin(\theta_5)$$

$$\sigma_6 = 10\,\omega_1\sin(\theta_1)$$

$$\sigma_7 = 8\,\omega_2\sin(\theta_2)$$

$$\sigma_8 = 20\,\omega_1\cos(\theta_1)$$

$$\sigma_9 = 16\,\omega_2\cos(\theta_2)$$

$$\sigma_{10} = 5\,\omega_5\cos(\theta_5)$$

$$\sigma_{11} = 10\,\omega_1\cos(\theta_1)$$

$$\sigma_{12} = 8\,\omega_2\cos(\theta_2)$$

```
disp('Ecuaciones de Lagrange:');
```

```
Ecuaciones de Lagrange:
```

```
simplify(Ecuaciones_Lagrange)
```

```
ans =
```

$$\begin{pmatrix} \\ \\ \\ \\ \\ \\ \\ 5000\,\tau_6 + 5000\,\tau_{ext6} + 2943\,\sin(\theta_6) + 15\,{\omega_6}^2\,\sin(2\,\theta_6) + 9810\,M_f\,\sin(\theta_6) + 150\,\omega_1\,\omega_6\,\cos(\theta_1)\,\sin(\theta_6) + 12( \end{pmatrix}$$

```
disp('Fuerzas de Coriolis y centrífugas:');
```

```
Fuerzas de Coriolis y centrífugas:
```

```
C
```

```
C =
```

$$\left(-\left(\frac{\omega_1}{200}+\frac{\omega_2}{200}+\frac{\omega_3}{200}+\frac{\omega_4}{200}+\frac{\omega_5}{200}+\frac{\omega_6}{200}\right)\ (174\,\omega_2\sin(\theta_1-\theta_2)+105\,\omega_3\sin(\theta_1-\theta_3)+57\,\omega_4\sin(\theta_1-\theta_4)\right.$$

where

$$\sigma_1=-\frac{21\sin(\theta_3-\theta_4)\ (10\,M_f+19)\ (\omega_3-\omega_4)\,\sigma_{16}}{4000}$$

$$\sigma_2=-\frac{7\sin(\theta_3-\theta_6)\ (10\,M_f+3)\ (\omega_3-\omega_6)\,\sigma_{16}}{2000}$$

$$\sigma_3=-\frac{3\sin(\theta_4-\theta_6)\ (10\,M_f+3)\ (\omega_4-\omega_6)\,\sigma_{16}}{1000}$$

$$\sigma_4=-\frac{3\sin(\theta_2-\theta_4)\ (10\,M_f+19)\ (\omega_2-\omega_4)\,\sigma_{16}}{500}$$

$$\sigma_5=-\frac{3\sin(\theta_1-\theta_4)\ (10\,M_f+19)\ (\omega_1-\omega_4)\,\sigma_{16}}{400}$$

$$\sigma_6=-\frac{\sin(\theta_5-\theta_6)\ (10\,M_f+3)\ (\omega_5-\omega_6)\,\sigma_{16}}{400}$$

$$\sigma_7=-\frac{\sin(\theta_1-\theta_2)\ (20\,M_f+87)\ (\omega_1-\omega_2)\,\sigma_{16}}{200}$$

$$\sigma_8=-\frac{\sin(\theta_2-\theta_6)\ (10\,M_f+3)\ (\omega_2-\omega_6)\,\sigma_{16}}{250}$$

$$\sigma_9=-\frac{\sin(\theta_1-\theta_6)\ (10\,M_f+3)\ (\omega_1-\omega_6)\,\sigma_{16}}{200}$$

$$\sigma_{10}=-\frac{7\sin(\theta_3-\theta_5)\ (\omega_3-\omega_5)\ (M_f+1)\,\sigma_{16}}{160}$$

$$\sigma_{11}=-\frac{7\sin(\theta_2-\theta_3)\ (\omega_2-\omega_3)\ (M_f+3)\,\sigma_{16}}{100}$$