Trabajo Practico N° 7

Repaso: Cardinal $\longrightarrow$ Cantidad de elementos de un conjunto

$X = \{1, 2, 7\} \longrightarrow |X| = 3$

$X = \mathbb{N} \longrightarrow |X| = \aleph_0$ — numerable

$\mathbb{N} = \{0, 1, 2, \ldots\}$

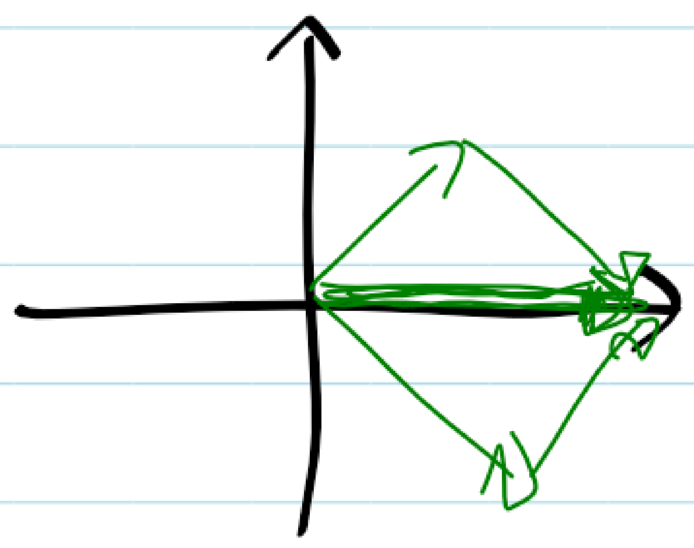
Infinitos

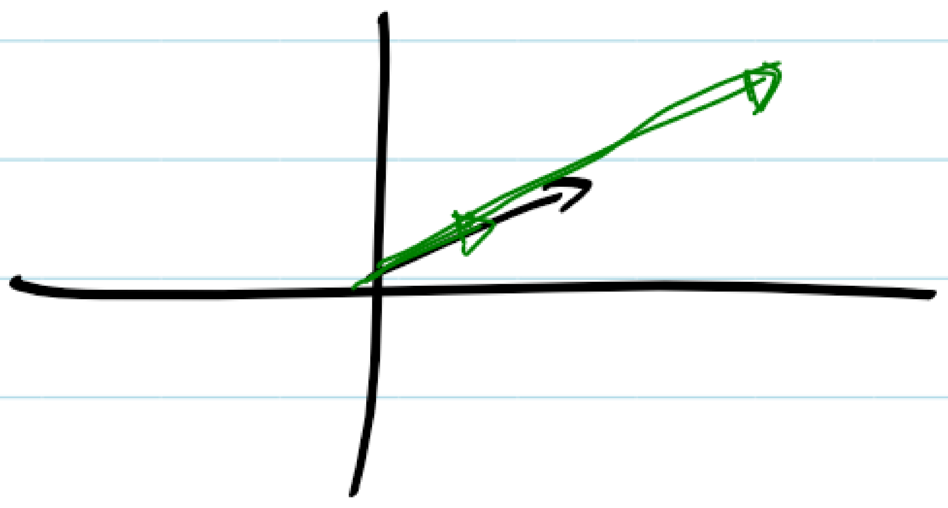
$X = \mathbb{R} \longrightarrow |X| = \aleph_1$ — no numerable

$V = \mathbb{R}^2$ Como $\mathbb{R}$-espacio vectorial

$(a, b) + (c, d) = (a+c, b+d)$

$k(a, b) = (ka, kb)$





Base de $\mathbb{R}^2 \longrightarrow$ Conjunto de Vectores
- ② Linealm Indep
- ① Genera a $V = \mathbb{R}^2$

$B_V = \{(1,0), (0,1)\}$ es una base de $V = \mathbb{R}^2$

① $(a, b) = a(1,0) + b(0,1) \longleftarrow$ Algebra de Vectores

$(a,b) = a(1,0) + b(0,1) \qquad B_V = \{(1,0),(0,1)\}$

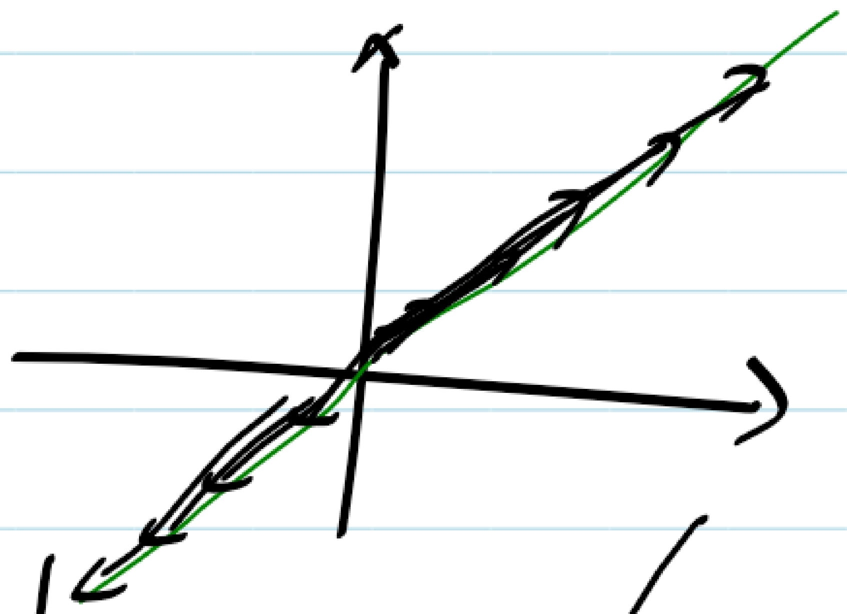
② $\underbrace{\alpha(1,0) + \beta(0,1)}_{(\alpha,\beta)} = (0,0)$

$$(\alpha,\beta) = (0,0) \iff \begin{cases} \alpha = 0 \\ \beta = 0 \end{cases}$$

Luego $\{(1,0),(0,1)\}$ es LI.

Es decir B_V es una base de $V = \mathbb{R}^2$

$\dim V = 2$



$|\mathbb{R}^2| = \aleph_1$

$|\mathbb{R}^2| = \infty$ (infinito)

$\| \quad \mathbb{R} \times \mathbb{R}$

$(a \ , \ b)$

$V = O = \{(0,0)\}$ Esp vect. trivial

$* \ |V| = 1$

$* \ \dim V = 0$

$\boxed{k=0}$

$\nexists \ (0,0) = k(0,0) \quad \forall k \in \mathbb{R}$ (no obtengo como única sol. $k=0$)

$(0,0)$ no es linealm. independiente

$X = \{(1,2)\}$ es LI | $k(1,2) = (0,0)$

$(k, 2k) = (0,0)$

$\begin{cases} k = 0 \\ 2k = 0 \end{cases}$

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$\boxed{6b}$ $\mathcal{N} = \{(1,2,0,0),(1,0,1,0)\} \subseteq \mathbb{R}^4$

* $\dim \mathbb{R}^4 = 4$ $\qquad B = \{(1,0,0,0),(0,1,0,0),(0,0,0,1)$
$(0,0,1,0)\}$ es base de $\mathbb{R}^4$

① $W = \{ \alpha(1,2,0,0) + \beta(1,0,1,0) : \alpha,\beta \in \mathbb{R} \}$

$$\alpha(1,2,0,0) + \beta(1,0,1,0) = (\alpha+\beta, 2\alpha, \beta, 0)$$

Ejemplo:

$\alpha = 1,\ \beta = 0 \longrightarrow (1,2,0,0)$

$\alpha = -2,\ \beta = 5 \longrightarrow (3,-4,5,0)$

$(b_1, b_2, b_3, b_4) \in W$ si $\begin{cases} x + y = b_1 \\ 2x = b_2 \\ y = b_3 \\ \boxed{0 = b_4} \end{cases}$ Sist.

$(0,0,0,1) \in W$ si $\begin{cases} x + y = 0 \\ 2x = 0 \\ y = 0 \\ \boxed{1 = 0} \end{cases}$

Lo cual es imposible $(1 \neq 0)$ y por lo tanto $(0,0,0,1) \notin W$

$$\mathcal{N}_2 = \{ \underbrace{(1,2,0,0),(1,0,1,0)}, \underline{(0,0,0,1)} \}$$

$$\alpha(1,2,0,0) + \beta(1,0,1,0) + \gamma(0,0,0,1) = (0,0,0,0)$$

$$(\alpha+\beta, 2\alpha, \beta, \gamma) = (0,0,0,0)$$

$$\begin{cases} \boxed{\alpha+\beta=0} \\ \boxed{2\alpha=0} \\ \boxed{\beta=0} \\ \boxed{\gamma=0} \end{cases} \quad \alpha=\beta=\gamma=0$$

Luego $\mathcal{N}_2$ es LI

Si yo agrego un vector que no está en el subesp. generado por $\mathcal{N}$ entonces el nuevo conj es LI

Veamos el subesp. generado por $\mathcal{N}_2$

$$\alpha(1,2,0,0) + \beta(1,0,1,0) + \gamma(0,0,0,1) = \underline{(\alpha+\beta, 2\alpha, \beta, \gamma)}$$

$(b_1, b_2, b_3, b_4) \in W_2$ si $\begin{cases} \alpha+\beta = b_1 \\ 2\alpha = b_2 \\ \beta = b_3 \\ \gamma = b_4 \end{cases}$

Tomamos $(0,1,0,0) \longrightarrow \begin{cases} \alpha+\beta=0 \leftarrow \\ 2\alpha=1 \\ \beta=0 \\ \gamma=0 \end{cases}$

Luego $(0,1,0,0) \notin W_2$

$$\mathcal{N}_3 = \{(1,2,0,0),(1,0,1,0),(0,0,0,1),(0,1,0,0)\}$$

$\mathcal{N}_3$ es base de $\mathbb{R}^4$ $\longrightarrow$ Verif

* $\mathcal{N}_3$ es LI : $\alpha(1,2,0,0)+\beta(1,0,1,0)+\gamma(0,0,0,1)$
$+\delta(0,1,0,0)=(0,0,0,0)$

$$(\alpha+\beta, 2\alpha+\delta, \beta, \gamma) = (0,0,0,0)$$

$$\begin{cases} \beta = 0 \\ \gamma = 0 \\ 2\alpha+\delta = 0 \\ \alpha+\beta = 0 \end{cases} \quad \Bigg| \quad \begin{array}{l} \alpha = 0 \\ 2\alpha+\delta = \delta = 0 \end{array} \quad \Bigg| \quad \begin{array}{l} \alpha = 0 \\ \beta = 0 \\ \gamma = 0 \\ \delta = 0 \end{array}$$

* $\mathcal{N}_3$ es LI
* $\mathcal{N}_3$ genera a $\mathbb{R}^4$

$\Big\}$ Luego $\mathcal{N}_3$ es base de $\mathbb{R}^4$

* $(a,b,c,d) = x(1,2,0,0)+y(1,0,1,0)+z(0,0,0,1)$
$+t(0,1,0,0)$

$$\begin{cases} x+y = a \\ 2x+t = b \\ y = c \\ z = d \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & a \\ 2 & 0 & 0 & 1 & b \\ 0 & 0 & 1 & 0 & c \\ 0 & 0 & 0 & 1 & d \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & a \\ 0 & -2 & 0 & 1 & b-2a \\ 0 & 0 & 1 & 0 & c \\ 0 & 0 & 0 & 1 & d \end{array}\right)$$

$\boxed{z = d}$ $\boxed{y = c}$

$-2y+t = b-2a \Rightarrow -2c+t = b-2a \Rightarrow \boxed{t = b-2a+2c}$

$x+y = a \Rightarrow x+c = a \Rightarrow \boxed{x = a-c}$

$V = \mathbb{R}^2 \qquad (\dim \mathbb{R}^2 = 2)$

$\mathcal{N} = \{(1,7)\}$ $\qquad W =$ Subesp gen. por $\mathcal{N}$

$* \; k(1,7) = (k, 7k)$

$(1,0) \notin W \qquad (k, 7k) = (1,0) \iff \begin{cases} k = 1 \\ 7k = 0 \end{cases}$

$\mathcal{N}_2 = \{(1,7),(1,0)\}$

$*$ $\mathcal{N}_2$ es LI $\longrightarrow \alpha(1,7) + \beta(1,0) = (0,0)$

$= (\alpha + \beta, 7\alpha)$

$*$ $\mathcal{N}_2$ genera a $\mathbb{R}^2$ (Tarea)

Si yo adiciono cualq. vector a $\mathcal{N}_2$. Nos da un conj. que es LD

Partimos de N $\longrightarrow$ Veamos si N genera a V

Paso 0 $\qquad$ Paso 1

$\boxed{\text{Si}}$ $\qquad$ $\boxed{\text{No}}$

Veamos si es LI $\qquad$ Veamos si es LI

base $\qquad$ No es base $\qquad$ Agregamos un vector que no está en el generado $\qquad$ No es base

$$W = \{(x,y,z,w,u) : \underbrace{y = x-z,\ w = x+z,\ u = 2x-3z}\}$$

$$(x,y,z,w,u) \in W \iff \underline{(x,y,z,w,u)} = (x, x-z, z, x+z, 2x-3z)$$

$$= x\underline{(1,1,0,1,2)} + z\underline{(0,-1,1,1,-3)}$$

$$\text{Base}_W = \{(1,1,0,1,2),(0,-1,1,1,-3)\}$$

$$V = \{(x,y) : \underline{y = 2x}\}$$

$$(x,y) = (x,2x) = \boxed{x(1,2)}$$

Por lo tanto todo vector de V es un escalar de $(1,2)$

$B = \{(1,2)\}$ es LI y genera a V $\therefore$ es base

<u>Cuestión</u> $V = \mathbb{R}^2$ $B = \{(1,2)\ (0,1),(-1,0)\}$

B genera a V

$\{\alpha, \beta, \gamma$ son las incognitas

$$\underline{\alpha}(1,2) + \underline{\beta}(0,1) + \underline{\gamma}(-1,0) = (p,q)$$

$$(\alpha-\gamma,\ 2\alpha+\beta) = (p,q) \longrightarrow \begin{cases} \alpha-\gamma = p \\ 2\alpha+\beta = q \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & p \\ 2 & 1 & 0 & q \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & p \\ 0 & 1 & 2 & q-2p \end{array}\right) -$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & p \\ \hline 0 & -1 & 2 & q-2p \end{array}\right) \qquad \{(p-\gamma, 2\gamma-q+2p, \gamma) : \gamma\in\mathbb{R}\}$$

$$AX = \begin{pmatrix} p \\ q \end{pmatrix}$$

$$-\beta + 2\gamma = q - 2p \longrightarrow \beta = 2\gamma - q + 2p$$

$$\alpha + \gamma = p \longrightarrow \alpha = p - \gamma$$

Tenemos que B genera a $\mathbb{R}^2$ pero existen infinitas combinaciones lineales para expresar un mismo vector

$$\boxed{|B| = 3 \qquad \dim \mathbb{R}^2 = 2}$$

* B genera a $\mathbb{R}^2 \longrightarrow$ Todo vector de $\mathbb{R}^2$ admite infinitas comb. lineales con los vectores

$$\alpha(1,2) + \beta(0,1) + \gamma(-1,0) = (0,0) \quad (*)$$

$$(\alpha - \gamma, 2\beta) = (0,0) \longrightarrow \begin{cases} \alpha - \gamma = 0 \to \alpha = \gamma \\ 2\beta = 0 \to \beta = 0 \end{cases}$$

$\alpha = 1, \beta = 0, \gamma = 1$ $\qquad \alpha = 0 \quad \beta = 0 \quad \gamma = 0$ /

$\alpha = -1, \beta = 0, \gamma = -1$

Luego B es LD => B no es base

Clase extra – Alg 2

Sea $X = \{v_1, \ldots, v_k\}$ un conjunto LI en V y sea $\tilde{X} = \{v_1, \ldots, v_s\} \subseteq X$ $\quad \{1,2\} = \{2,1\}$

Supongamos $\tilde{X}$ es LD

$\exists\, \alpha_1, \ldots \alpha_s \in \mathbb{R}$ no todos nulos tales que

$$\boxed{\alpha_1} v_1 + \underline{\boxed{\alpha_4} v_5} + \boxed{\alpha_s} v_s = 0$$

Sin perdida de generalidad, podemos suponer que $\alpha_1 \neq 0$ y de este modo

$$\Rightarrow v_1 = -\frac{\alpha_2}{\alpha_1} v_2 - \frac{\alpha_3}{\alpha_1} v_3 - \ldots - \frac{\alpha_s}{\alpha_1} v_s$$

Luego

$$1 v_1 = -\frac{\alpha_2}{\alpha_1} v_2 - \ldots - \frac{\alpha_s}{\alpha_1} v_s + 0 v_{s+1} + \ldots + 0 v_k$$

$$\underset{\neq 0}{\alpha_1} v_1 + \alpha_2 v_2 + \ldots + \alpha_s v_s + 0 v_{s+1} + \ldots + 0 v_k = 0$$

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$$A_1 = \begin{bmatrix} 1 & -2 & 0 & 3 & 7 \\ 2 & 1 & -3 & 1 & 1 \end{bmatrix} \qquad W_1 \longrightarrow A_1 X = 0$$

$$A_2 = \begin{bmatrix} 3 & 2 & 0 & 0 & 3 \\ 1 & 0 & -3 & 1 & 0 \\ -1 & 1 & -3 & 1 & -2 \end{bmatrix} \qquad W \longrightarrow A_2 X = 0$$

W_1

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & 3 & 7 & 0 \\ 2 & 1 & -3 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 3 & 7 & 0 \\ 0 & 5 & -3 & -5 & -13 & 0 \end{array}\right]$$

$$5y - 3z - 5t - 13w = 0 \Rightarrow y = \frac{3}{5}z + t + \frac{13}{5}w$$

$$x - 2y + 3t + 7w = 0 \Rightarrow x = 2y - 3t - 7w$$

$$x = \frac{6}{5}z + 2t + \frac{26}{5}w - 3t - 7w$$

$$x = \frac{6}{5}z - t - \frac{9}{5}w$$

$$Cs = \left\{ \left(\frac{6}{5}z - t - \frac{9}{5}w, \frac{3}{5}z + t + \frac{13}{5}w, z, t, w \right\} \right.$$

$$\begin{bmatrix} 3 & 2 & 0 & 0 & 3 \\ 1 & 0 & -3 & 1 & 0 \\ -1 & 1 & -3 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 3 & 2 & 0 & 0 & 3 \\ 0 & -2 & -9 & 3 & -3 \\ 0 & 5 & -9 & 3 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & 2 & 0 & 0 & 3 \\ 0 & -2 & -9 & 3 & -3 \\ 0 & 0 & 63 & -21 & 21 \end{bmatrix}$$

$$\sim\begin{bmatrix} 3 & 2 & 0 & 0 & 3 \\ 0 & -2 & -9 & 3 & -3 \\ 0 & 0 & 63 & -21 & 21 \end{bmatrix} \sim \begin{bmatrix} 3 & 2 & 0 & 0 & 3 \\ 0 & 2 & 9 & -3 & 3 \\ 0 & 0 & 3 & -1 & 1 \end{bmatrix}$$

$$3z - t + 1w = 0 \longrightarrow \boxed{z = \frac{1}{3}t - \frac{1}{3}\omega}$$

$$2y + 9z - 3t + 3w = 0 \to y = -\frac{9}{2}z + \frac{3}{2}t - \frac{3}{2}\omega$$

$$y = -\frac{3}{2}\cancel{t} + \frac{3}{2}\cancel{\omega} + \frac{3}{2}\cancel{t} - \frac{3}{2}\cancel{\omega} = 0 \qquad \boxed{y = 0}$$

$$3x + 2y + 3\omega = 0 \longrightarrow \boxed{x = -\omega}$$

$A_2X = 0$

$$CS_2 = \left\{ \left(-\omega, 0, \frac{1}{3}t - \frac{1}{3}\omega, t, \omega\right);\ t, \omega \in \mathbb{R} \right\} = W_1$$

$$CS_1 = \left\{ \underbrace{\left(\frac{6}{5}z - t - \frac{9}{5}\omega,\ \frac{3}{5}z + t + \frac{13}{5}\omega,\ z, t, \omega\right)}_{A_1X = 0} \right\} = W_2$$

$W_1 \cap W_2$

$$\left(-\omega, 0, \frac{1}{3}t - \frac{1}{3}\omega, t, \omega\right) = \left(\frac{6}{5}z - t - \frac{9}{5}\omega,\ \underbrace{\frac{3}{5}z + t + \frac{13}{5}\omega}, z, t, \omega\right)$$

$$\left.\begin{aligned} \omega &= \omega \\ t &= t \\ z &= z \\ 0 &= \frac{3}{5}z + t + \frac{13}{5}\omega \\ -\omega &= \frac{6}{5}z - t - \frac{9}{5}\omega \end{aligned}\right\}$$

$W_1 \longrightarrow A_1 X = 0$

$W_2 \longrightarrow A_2 X = 0$ $\Big\}$ Desc. de $W_1 \cap W$

$v \in W_1 \longrightarrow A_1 \cdot v = 0$

$v \in W_2 \longrightarrow A_2 \cdot v = 0$

Por lo tanto

$v \in W_1 \cap W_2 \Rightarrow \overline{A_1 \cdot v = 0}$ y $\overline{A_2 \cdot v = 0}$

Luego, la realidad es que $A \cdot v = 0$

$$A = \begin{bmatrix} 3 & 2 & 0 & 0 & 3 \\ 1 & 0 & -3 & 1 & 0 \\ -1 & 1 & -3 & 1 & -2 \\ 1 & -2 & 0 & 3 & 4 \\ 2 & 1 & -3 & 1 & 1 \end{bmatrix} = \left[\frac{A_2}{A_1}\right]$$

$\boxed{10}$ $W_1 = \left\{ (u, v, w, x, y, z) : \begin{matrix} u + v + w = 0 \\ x + y + z = 0 \end{matrix} \right\}$

$W_2 = \langle (1,-1,1,-1,1,-1), (1\ 2\ 3\ 4\ 5\ 6), (1\ 0\ -1\ -1\ 0\ 1),$
$(2,1,0,0,0,0) \rangle$

$\boxed{10}$ $W_1 = \left\{ (\underbrace{u, v, w}_{①}, \underbrace{x, y, z}_{②}) : \begin{array}{l} u + v + w = 0 \\ x + y + z = 0 \end{array} \right\}$

$W_2 = \langle (1,-1,1,-1,1,-1), (1\ 2\ 3\ 4\ 5\ 6), (1\ 0\ -1\ -1\ 0\ 1), (2,1,0,0,0,0) \rangle$

(a)

$\alpha_1 (1,-1,1,-1,1,-1) + \alpha_2 (1,2,3,4,5,6) + \alpha_3 (1,0,-1,-1,0,1) + \alpha_4 (2,1,0,0,0,0) = (\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4, -\alpha_1 + 2\alpha_2 + \alpha_4, \alpha_1 + 3\alpha_2 - \alpha_3, -\alpha_1 + 4\alpha_2 - \alpha_3, \alpha_1 + 5\alpha_2, -\alpha_1 + 6\alpha_2 + \alpha_3)$

$$\left.\begin{array}{l} (\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4) + (-\alpha_1 + 2\alpha_2 + \alpha_4) + (\alpha_1 + 3\alpha_2 - \alpha_3) = 0 \\ (-\alpha_1 + 4\alpha_2 - \alpha_3) + (\alpha_1 + 5\alpha_2) + (-\alpha_1 + 6\alpha_2 + \alpha_3) = 0 \end{array}\right\} \begin{array}{l} ① \\ ② \end{array}$$

C_S = Descripción de $W_1 \cap W_2$

$\boxed{17}$ (a) $W = \{ (x,y,z) \in \mathbb{R}^3 \ / \ z = x + y \}$

$(x,y,z) \in W \Leftrightarrow (x,y,z) = (x, y, x+y) = x(1,0,1) + y(0,1,1)$

$\Rightarrow B = \{ (1,0,1), (0,1,1) \}$ genera a W y es LI

B es base de $W \Rightarrow \dim W = 2$

(b) $U = \langle \underbrace{(1,2,0)\,(0\,1,0),(0,0,1)}_{}, \underbrace{(2\,1\,3)}_{} \rangle$

Teoría

$U < \mathbb{R}^3$

$\dim \mathbb{R}^3 = 3 \Rightarrow \dim U \leq 3$ $\left(\dim U = 3 \Rightarrow U = \mathbb{R}^3 \right)$

$$x(1,2,0) + y(0,1,0) + z(0,0,1) + t(2,1,3) = (0,0,0)$$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix} \longrightarrow A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & 3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(2,1,3) = 2(1,2,0) - 3(0,1,0) + 3(0,0,1)$$

Luego $U = \langle \underset{v_1}{(1,2,0)}, \underset{v_2}{(0,1,0)}, \underset{v_3}{(0,0,1)} \rangle$

Tarea: $v_1\ v_2\ v_3$ son L $\rightarrow \dim U = 3$