Python 기반 기초통계 실습

2021. 9. 13

정 준 수 Ph.D

모집단과 샘플(표본)

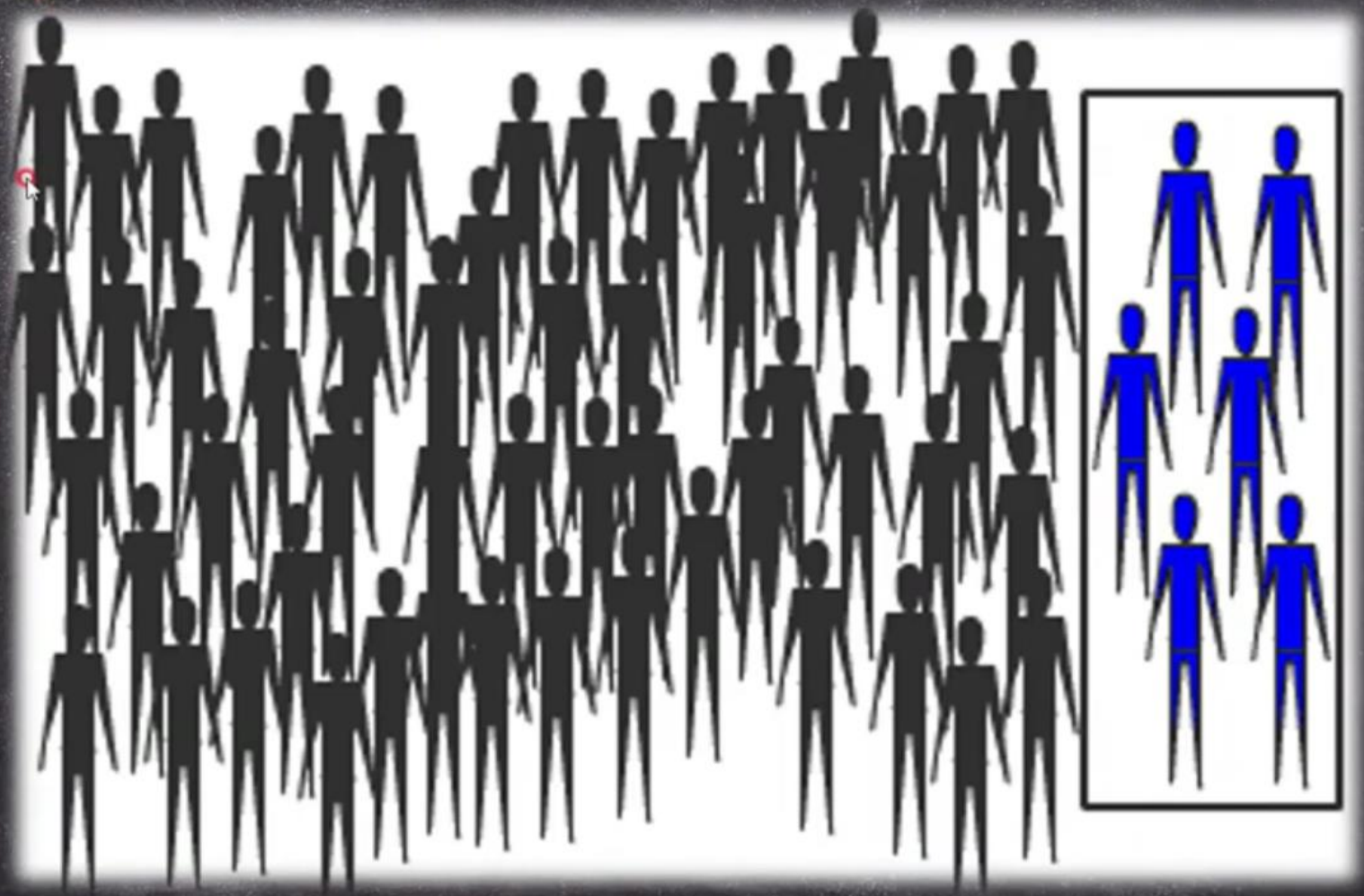
모집단

관 측 치 = N

평 균 값 = μ

분　　산 = σ^2

표준편차 = σ



표본(샘플)

관 측 치 = n

평 균 값 = $\bar{X}$

분　　산 = s^2

표준편차 = s

정규분포와 표준편차

확률변수 X가 평균이 μ, 표준편차가 σ인 정규분포를 따르면 확률밀도함수는

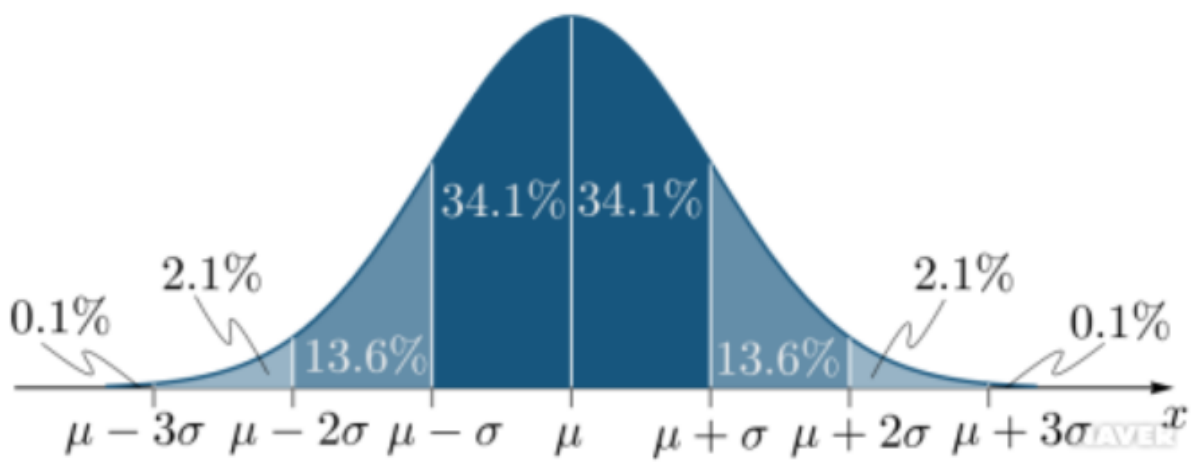
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

이다. 모집단에서 임의로 하나의 값을 취할 때 이 값이

구간 $[\mu - \sigma, \mu + \sigma]$에 속할 확률은 $68.27\,\%$

구간 $[\mu - 2\sigma, \mu + 2\sigma]$에 속할 확률은 $95.45\,\%$

구간 $[\mu - 3\sigma, \mu + 3\sigma]$에 속할 확률은 $99.73\,\%$이다.



Sampling을 통한 통계 예제

– MLB 선수연봉 데이터

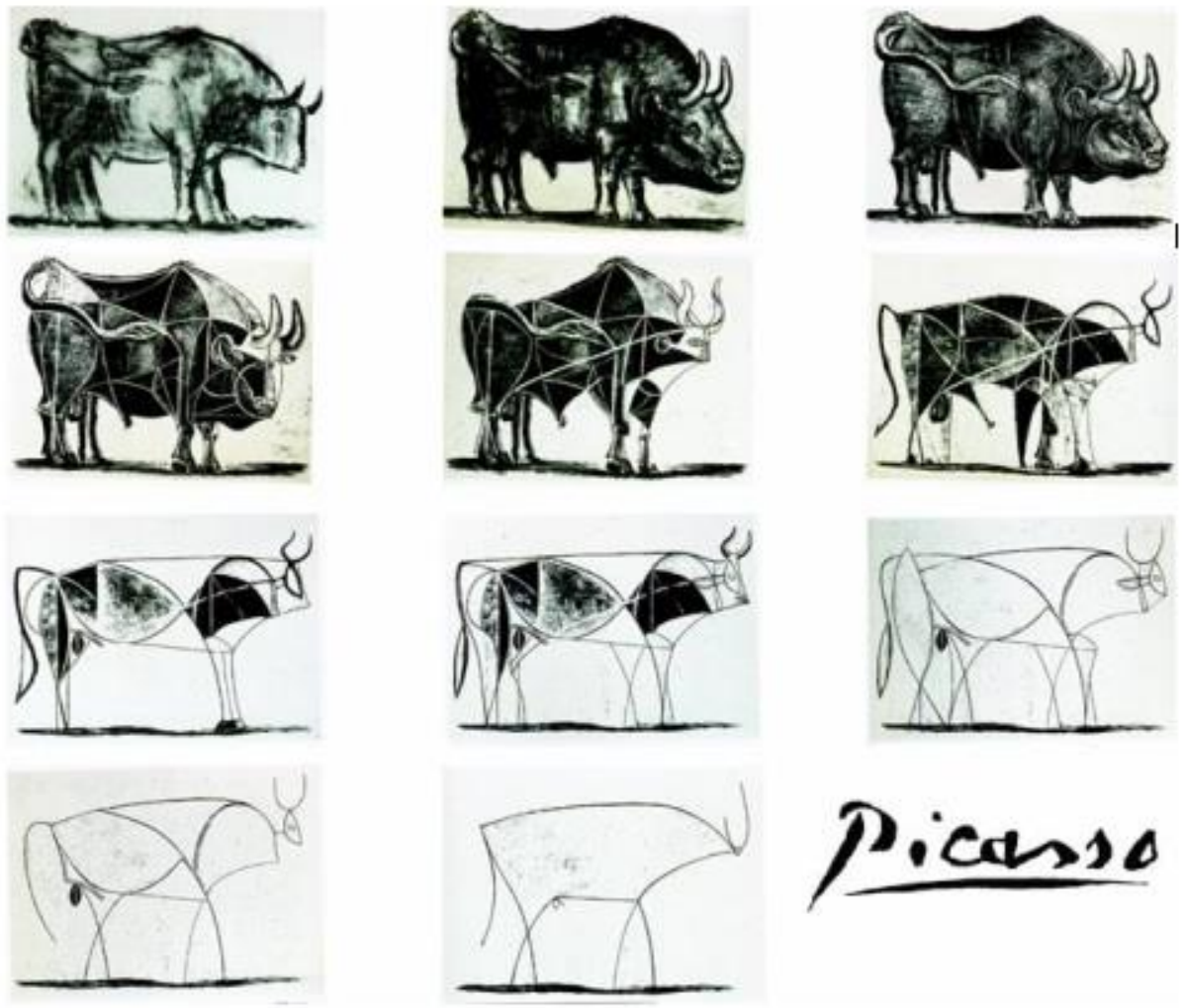
https://github.com/JSJeong-me/SEMICON-BigData/blob/main/statistics-intro.ipynb

Charles Spearman

- Statistics-factor analysis
- Creator of "Spearman's rank correlation coefficient" (-1 to +1)
- Intelligence theories
- General Intelligence or "g-factor"—positive correlations among cognitive abilities that account for most of IQ
- Much of intell. is heritable

https://en.wikipedia.org/wiki/Spearman%27s_rank_correlation_coefficient

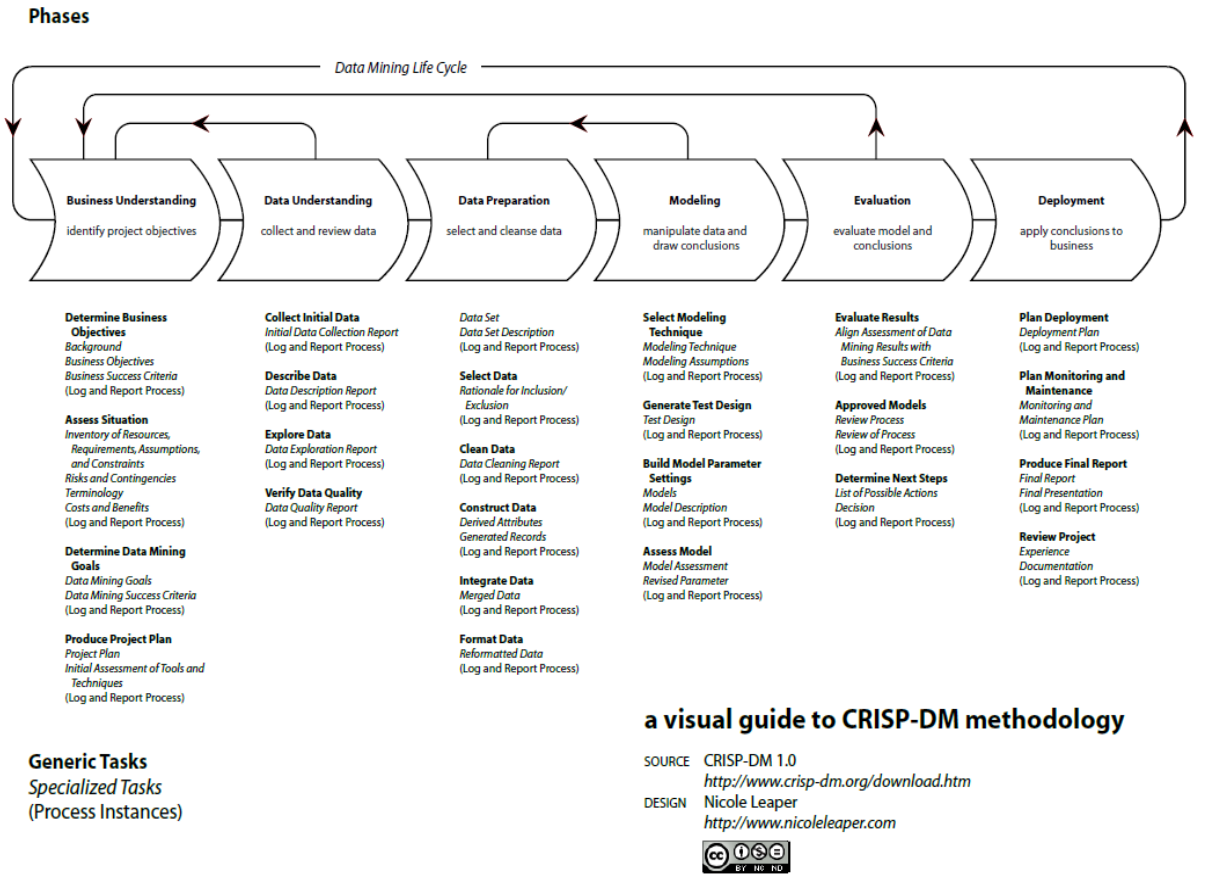
◈ 추상화 (Abstract)



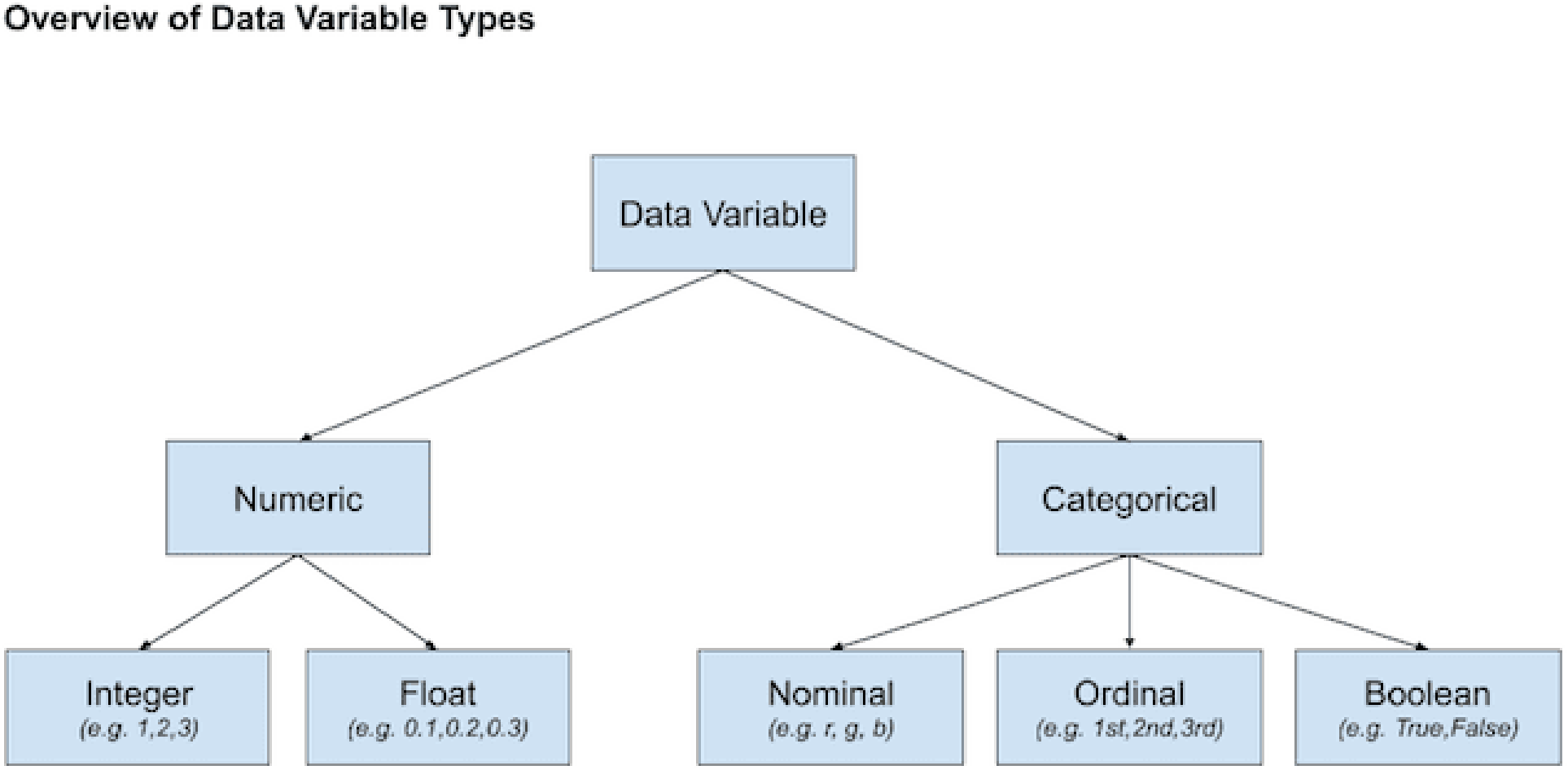
Pablo Picasso, Bull (plates I - XI) 1945

CRISP-DM (Cross Industry Standard Process for Data Mining)

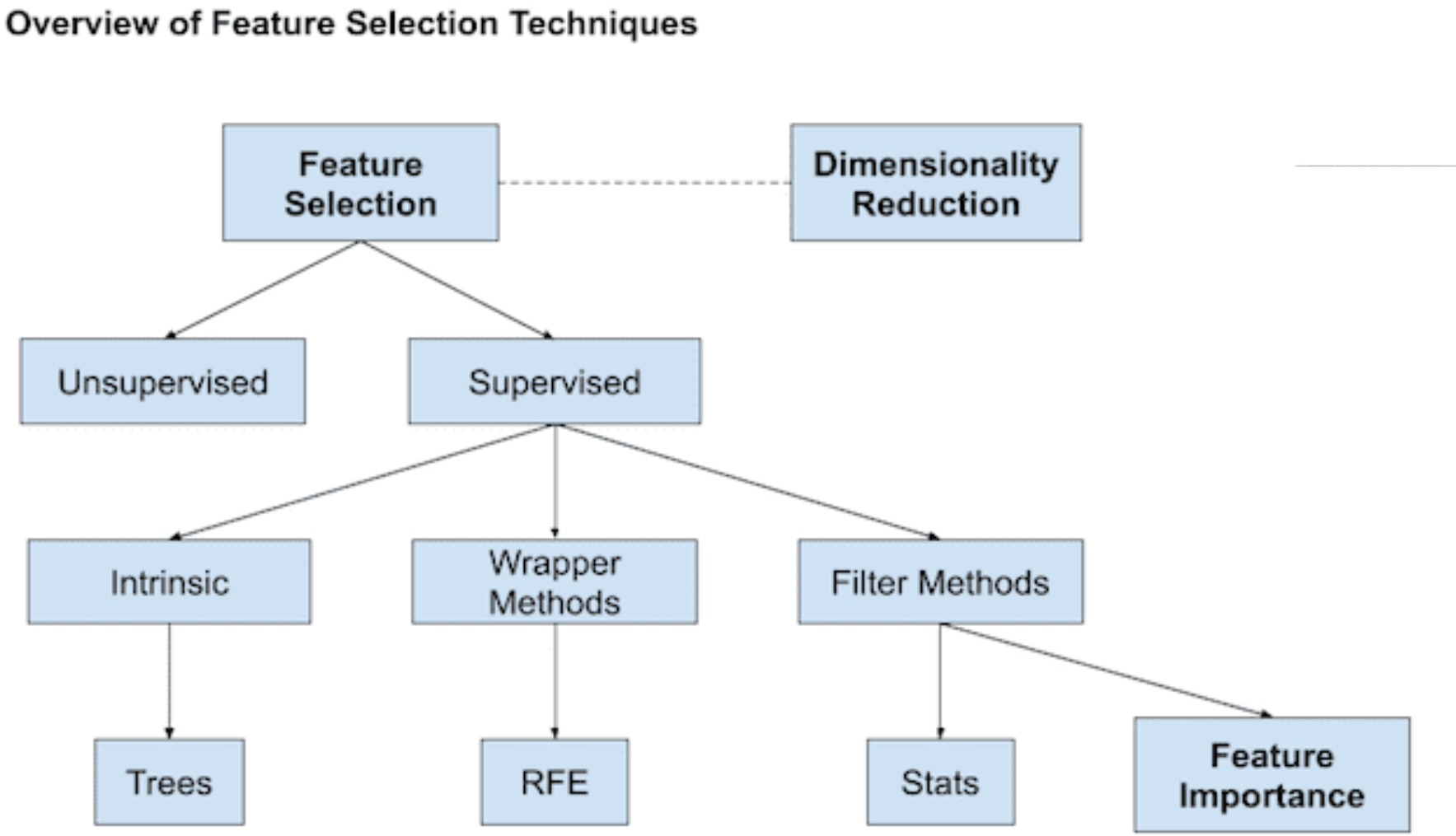
CRISP-DM(Cross Industry Standard Process for Data Mining)은 데이터 마이닝 전문가가 사용하는 일반적인 접근 방식을 설명한 가장 널리 사용되는 공개 표준 분석 모델입니다.



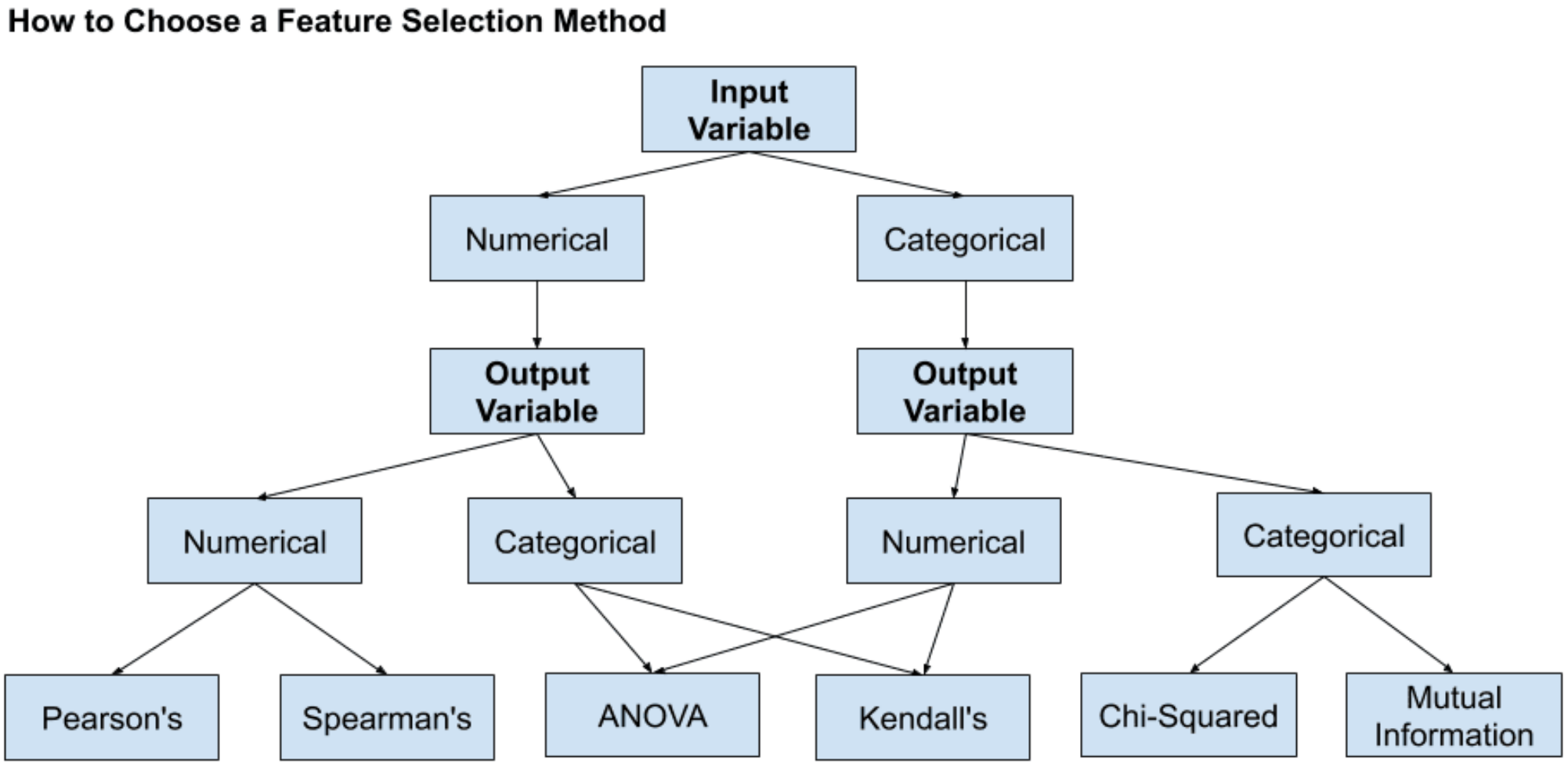
Data 변수 타입 분류



Feature Selection 분류 과정



Feature Selection 분류 방법



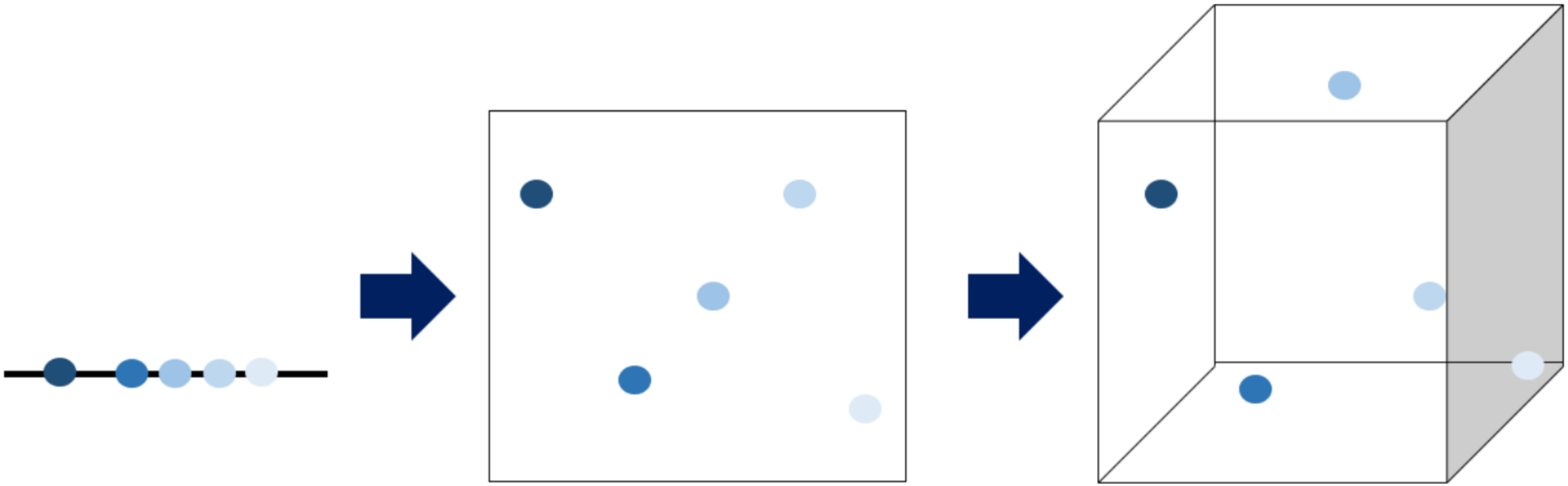
차원의 저주란,

*데이터 학습을 위해 차원이 증가하면서 학습데이터 수가 차원의 수보다 적어져 성능이 저하되는 현상.
*차원이 증가할 수록 개별 차원 내 학습할 데이터 수가 적어지는(sparse) 현상 발생

*해결책: 차원을 줄이거나(축소시키거나) 데이터를 많이 획득

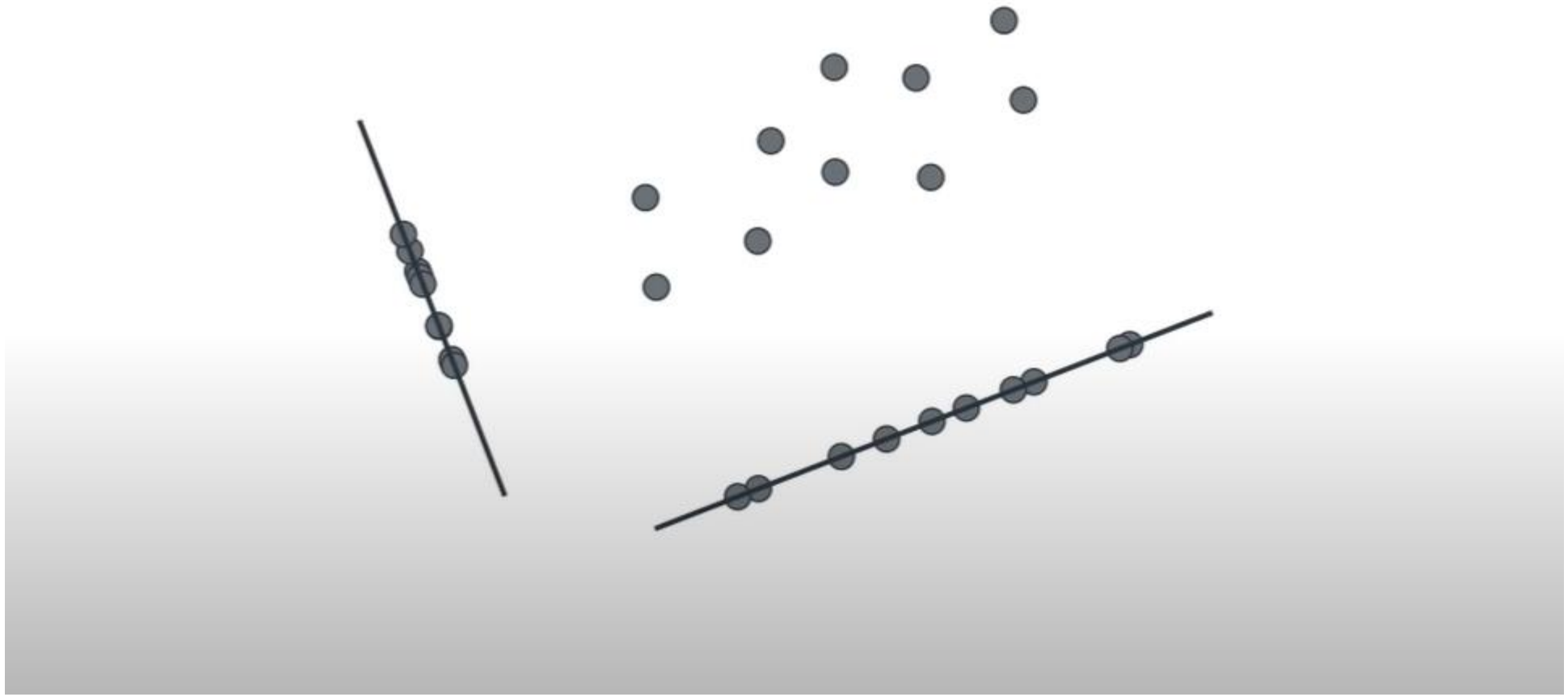
즉, 간단히 말해서
차원이 증가함에 따라(=변수의 수 증가) 모델의 성능이 안 좋아지는 현상을 의미합니다.
무조건 변수의 수가 증가한다고 해서 차원의 저주 문제가 있는 것이 아니라, 관측치 수보다 변수의 수가 많아지면 발생합니다. (예를들어, 관측치 개수는 200개인데, 변수는 7000개)

왜 이런 현상이 발생할까요?

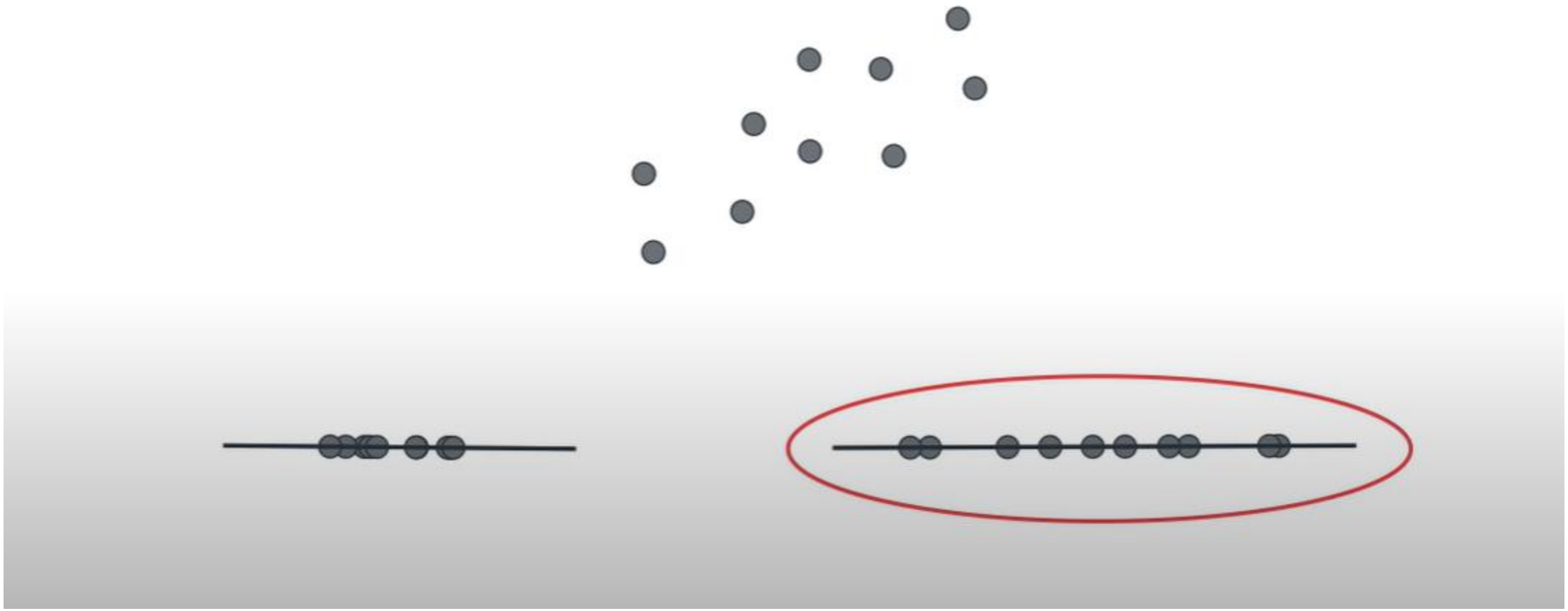


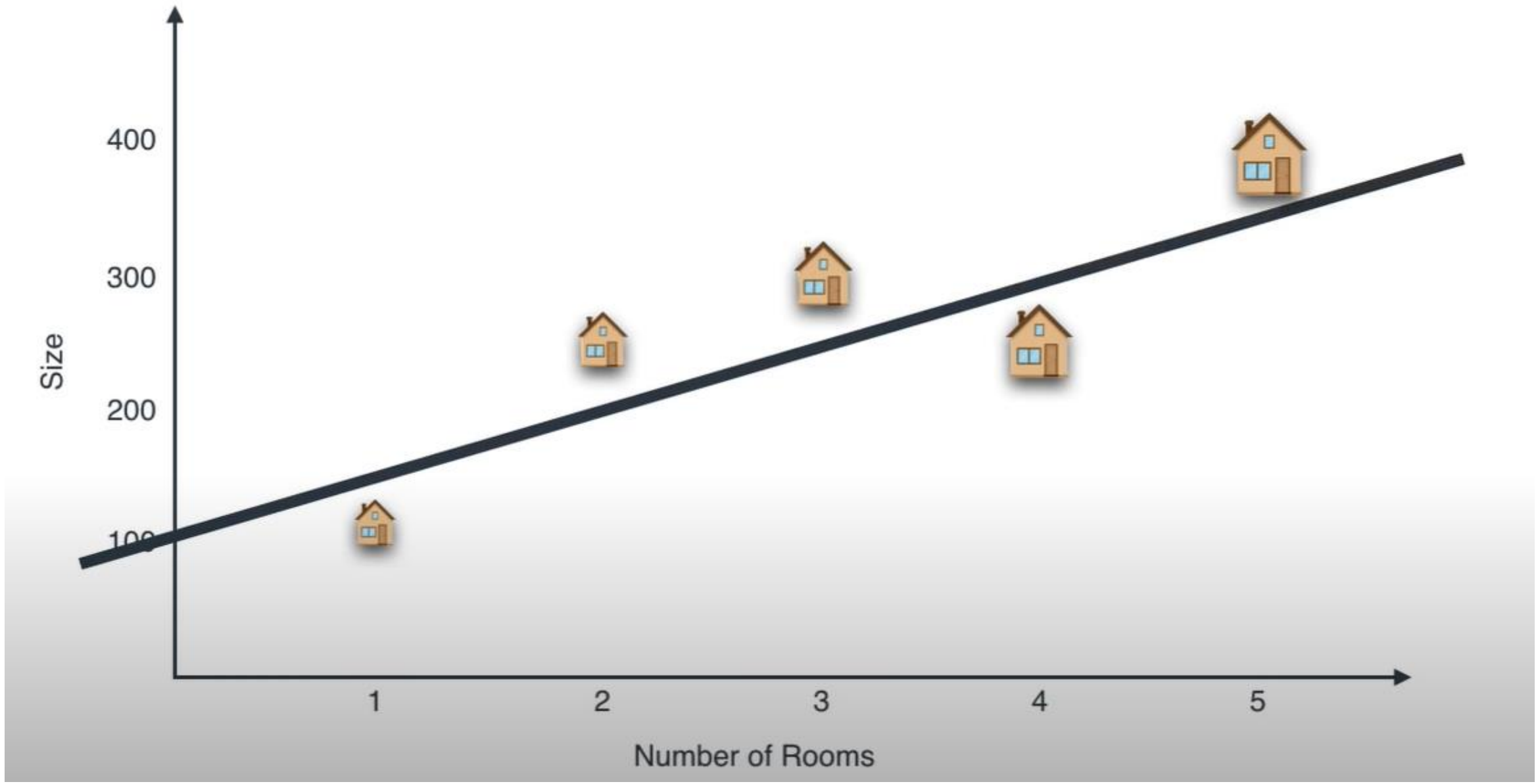
Made by: ta-daa

Dimensionality Reduction



Dimensionality Reduction




400
300
Size
200
100
1
2
3
4
5
Number of Rooms

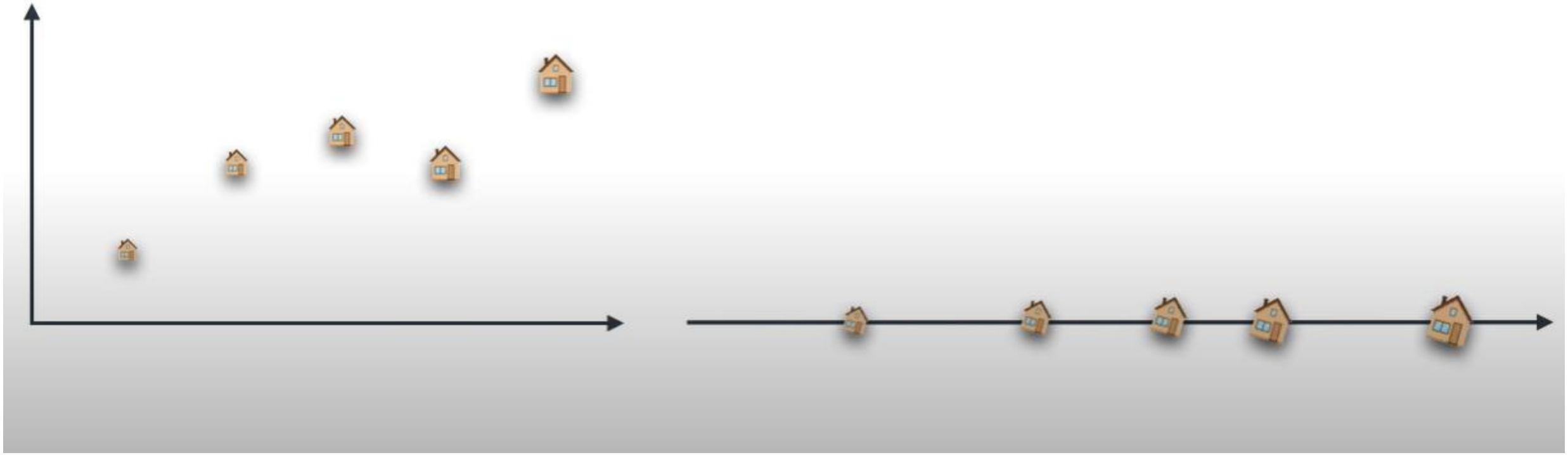
Size feature

2 dimensions

size
number of rooms

1 dimension

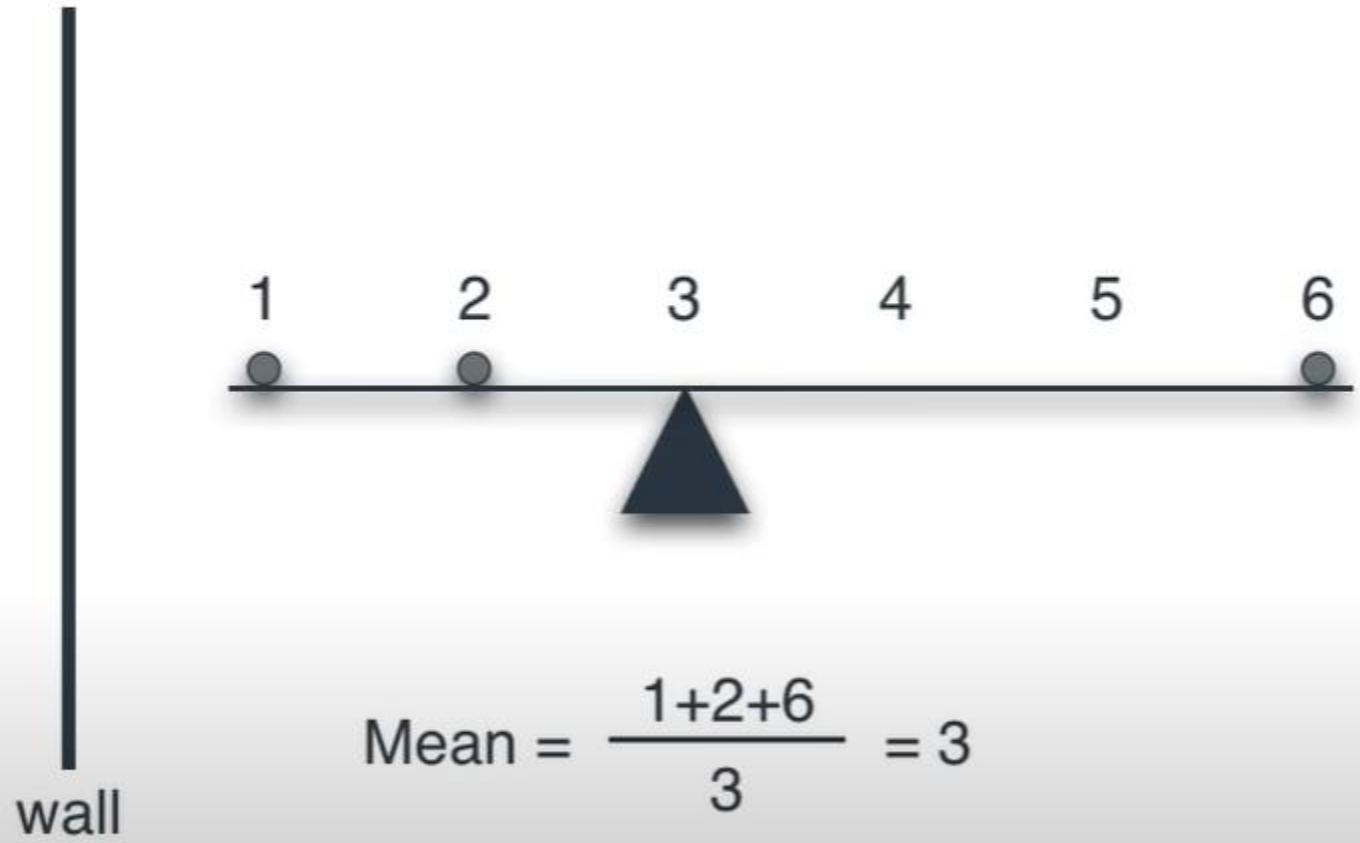
size feature



Housing Data

5 dimensions	2 dimensions
Size	
Number of rooms	Size feature
Number of bathrooms	
Schools around	Location feature
Crime rate	

Mean



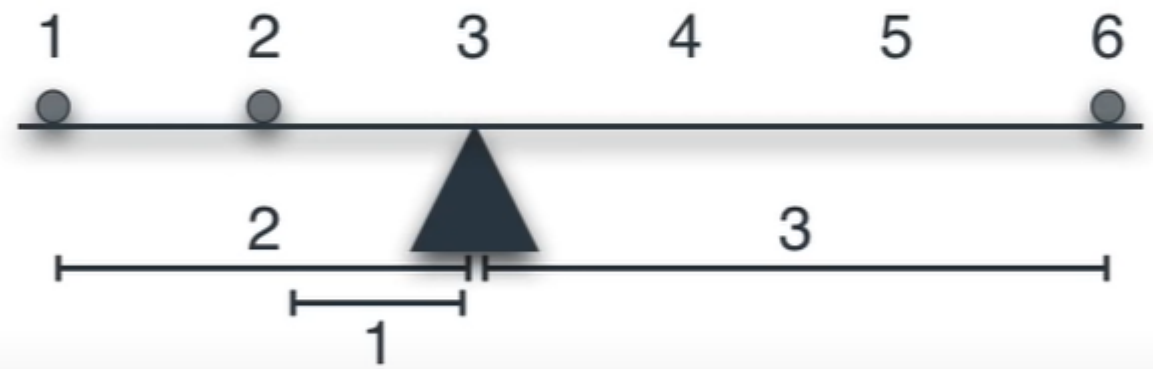
$$\text{Mean} = \frac{1+2+6}{3} = 3$$

Variance

$$\text{Variance} = \frac{1^2+0^2+1^2}{3} = 2/3$$

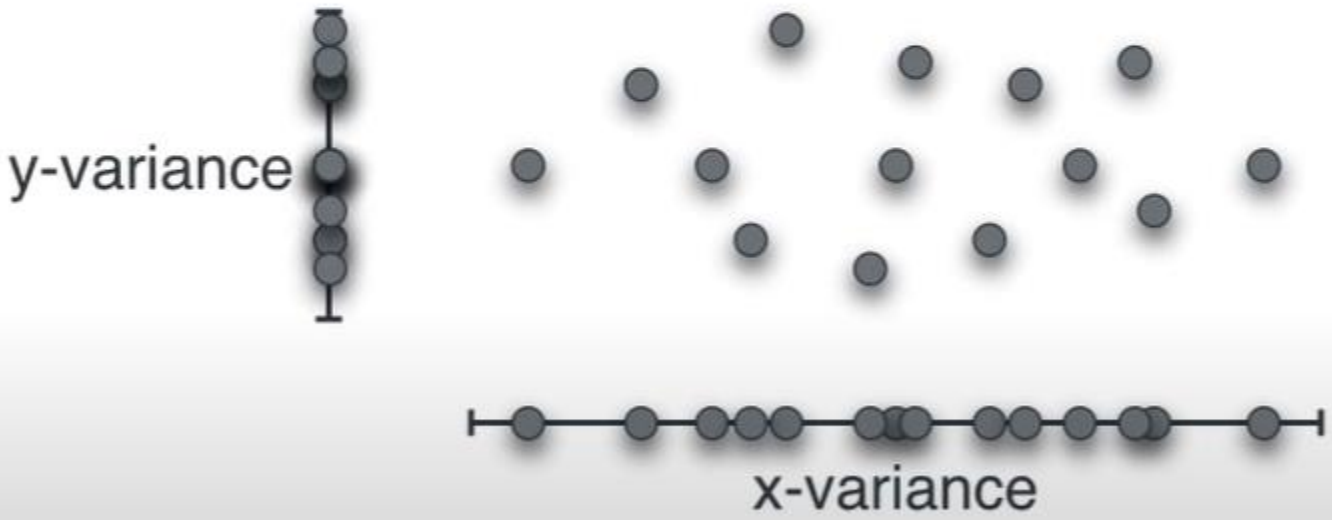
$$\text{Variance} = \frac{5^2+0^2+5^2}{3} = 50/3$$

Mean

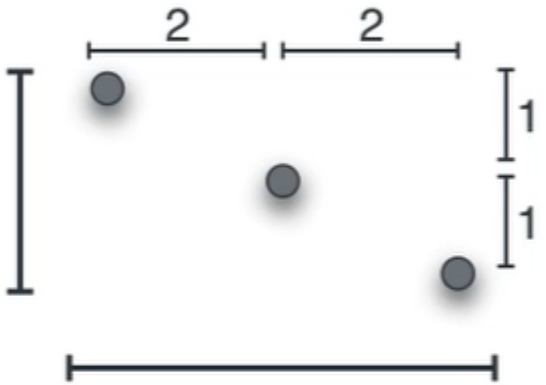


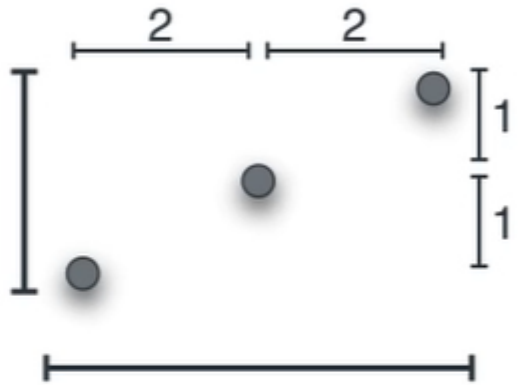
$$\text{Variance} = \frac{2^2+1^2+3^2}{3} = 14/3$$

Variance?



Variance?

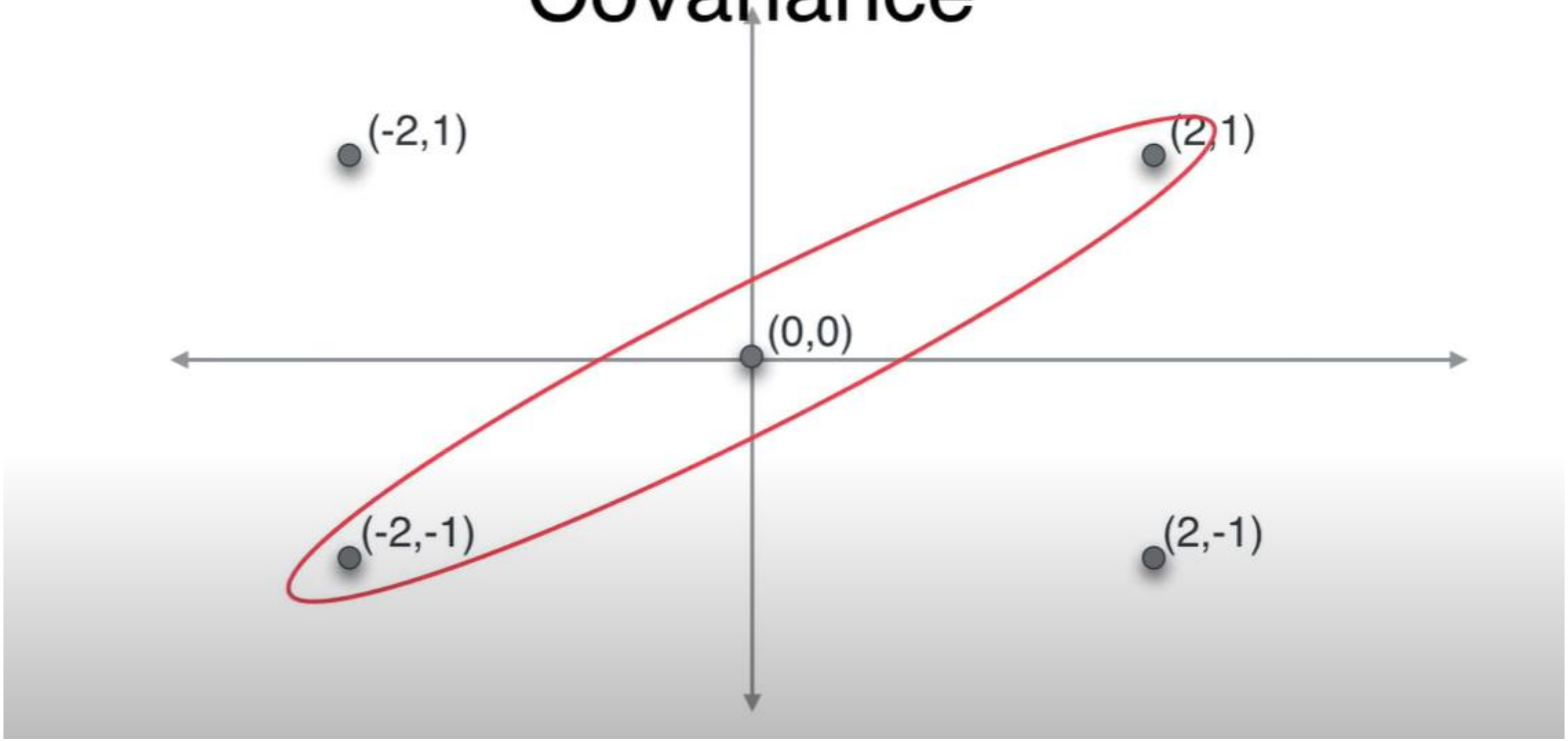




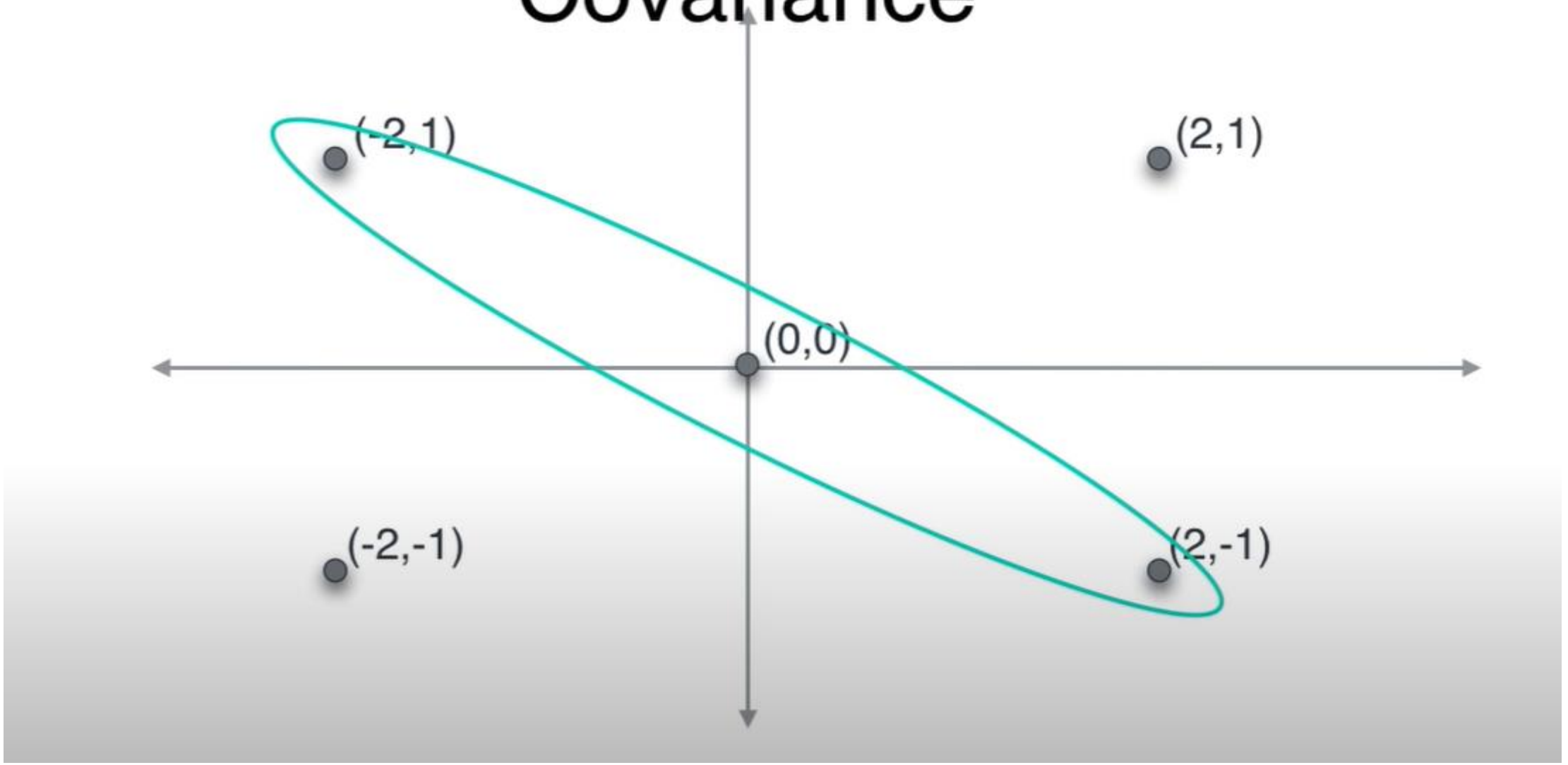
$$\text{x-variance} = \frac{2^2+0^2+2^2}{3} = 8/3$$

$$\text{y-variance} = \frac{1^2+0^2+1^2}{3} = 2/3$$

Covariance

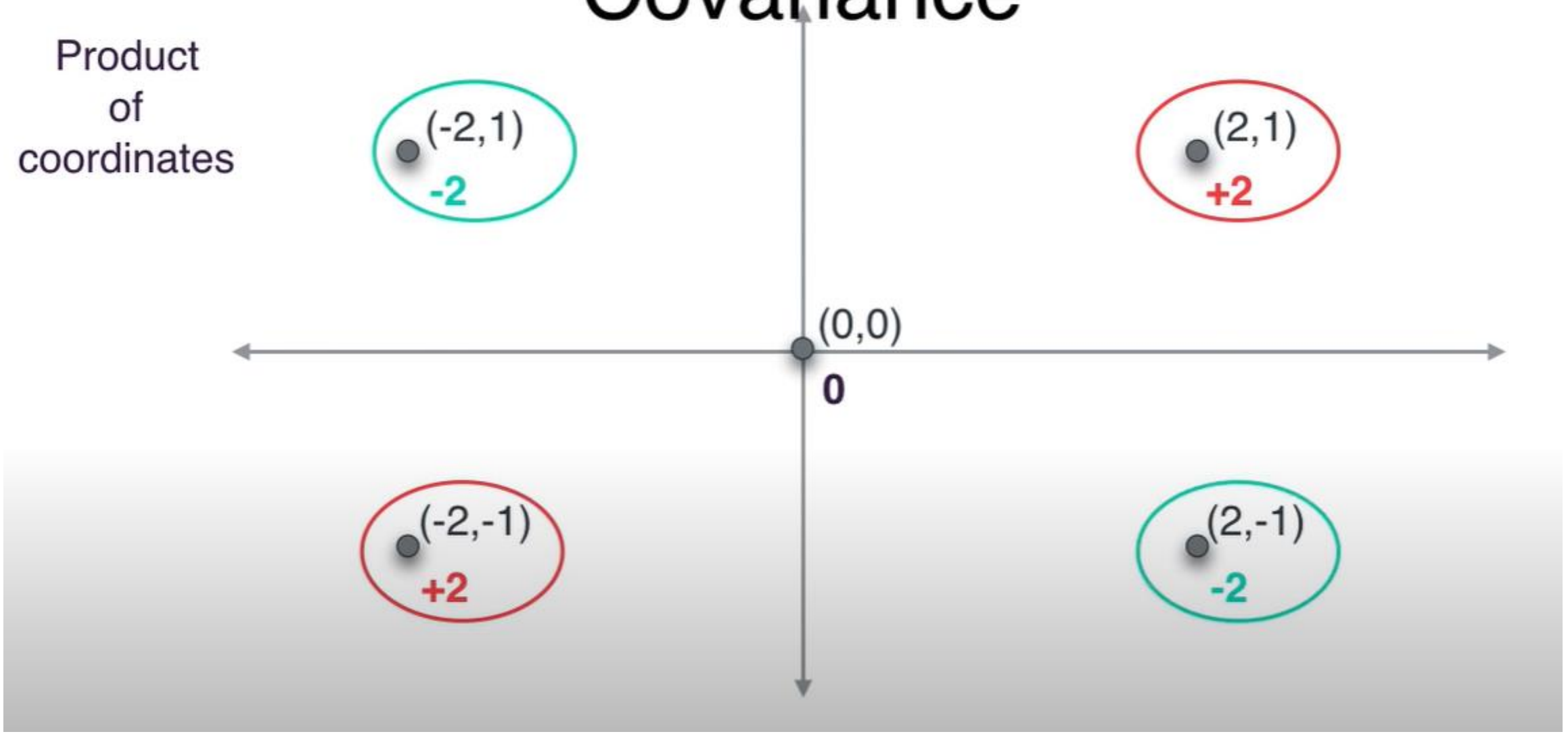


Covariance

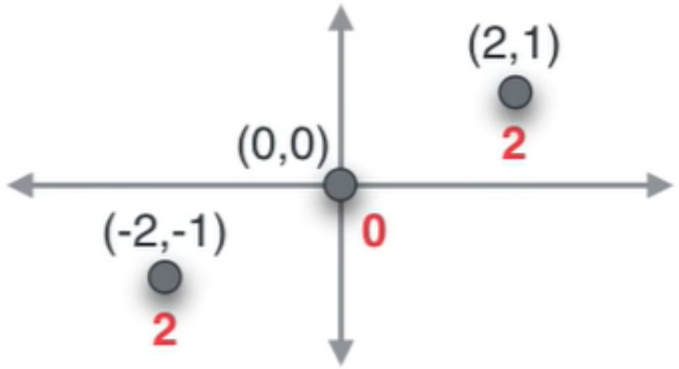


Covariance

Product of coordinates



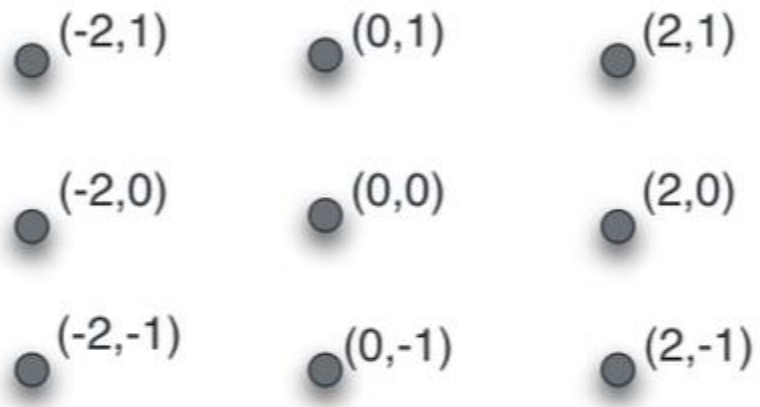
Covariance



$$\text{covariance} = \frac{(-2) + 0 + (-2)}{3} = -4/3$$

$$\text{covariance} = \frac{2 + 0 + 2}{3} = 4/3$$

Covariance



$$\text{covariance} = \frac{-2 + 0 + 2 + 0 + 0 + 0 + 2 + 0 + -2}{9} = 0$$

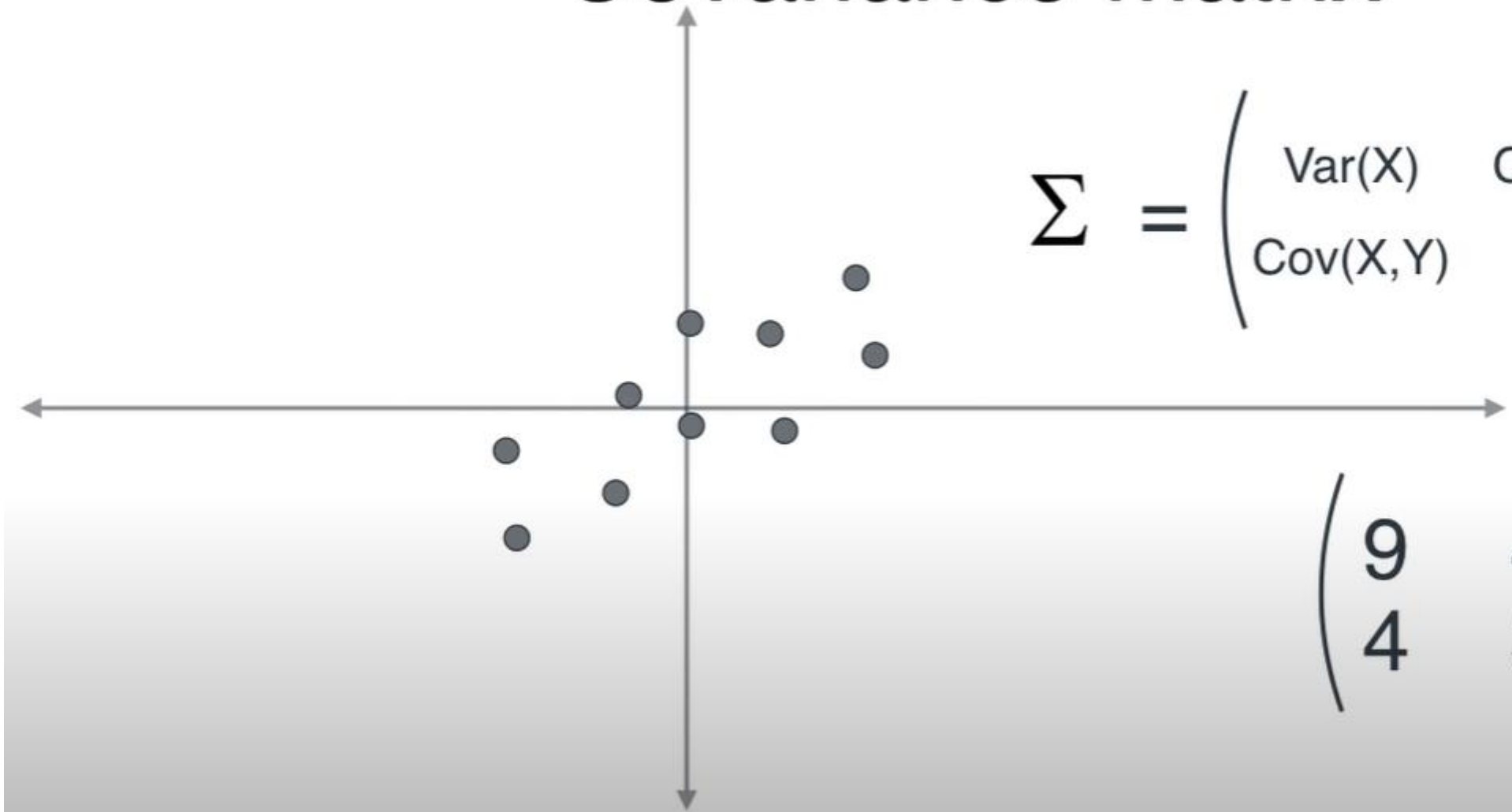
Covariance

negative
covariance

covariance zero
(or very small)

positive
covariance

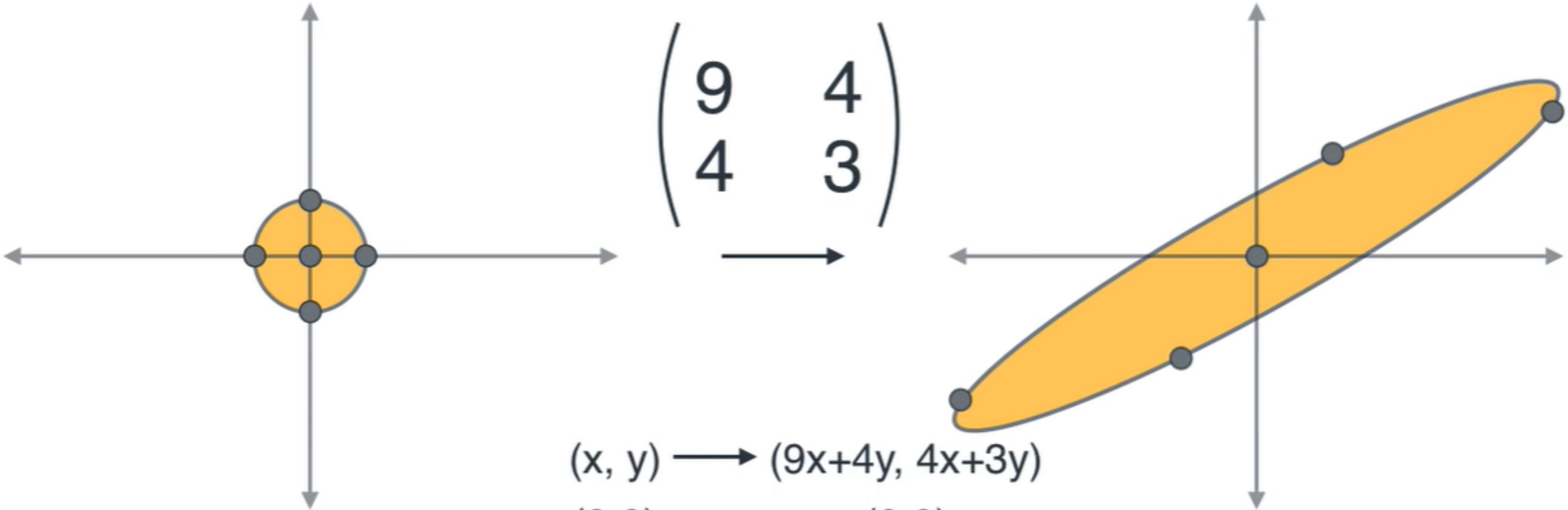
Covariance matrix



$$\Sigma = \begin{pmatrix} \mathrm{Var}(X) & \mathrm{Cov}(X,Y) \\ \mathrm{Cov}(X,Y) & \mathrm{Var}(Y) \end{pmatrix}$$

$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

Linear Transformations

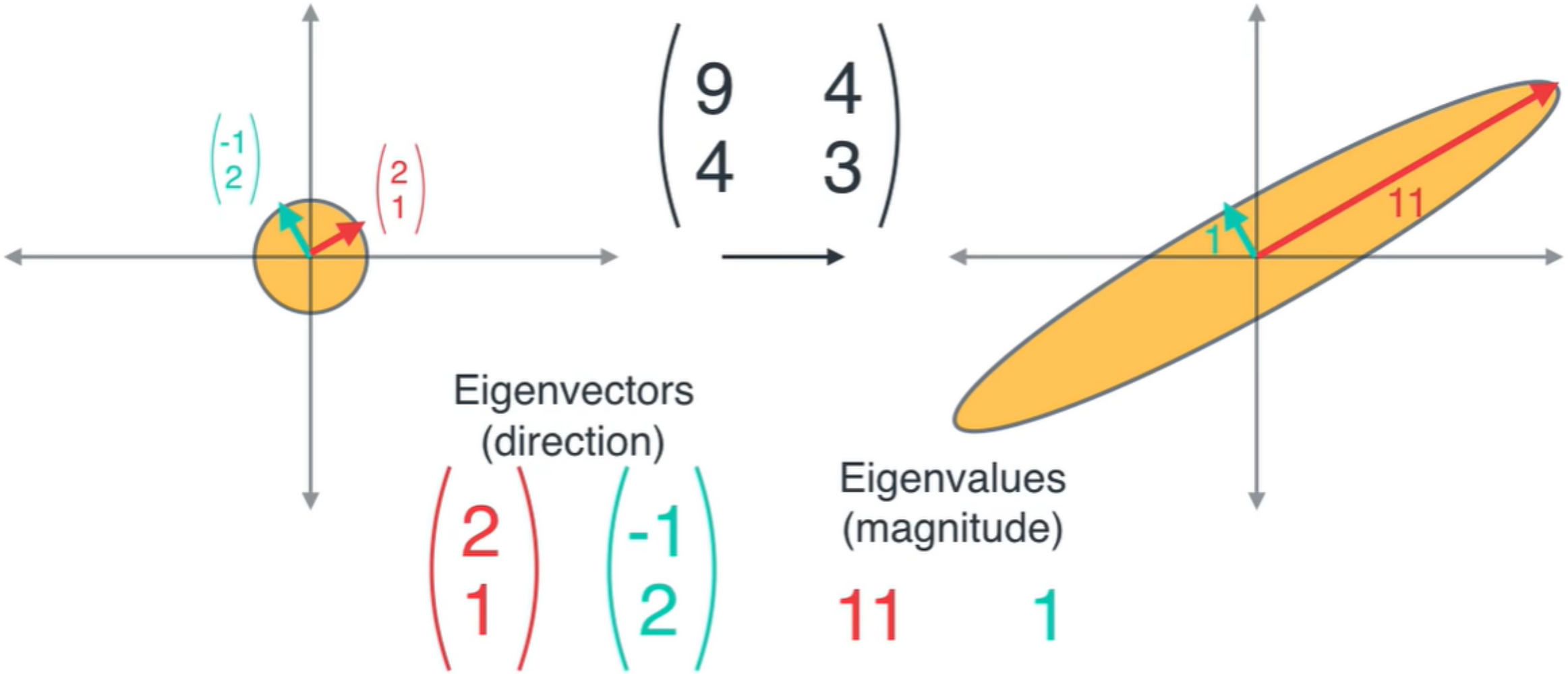


$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

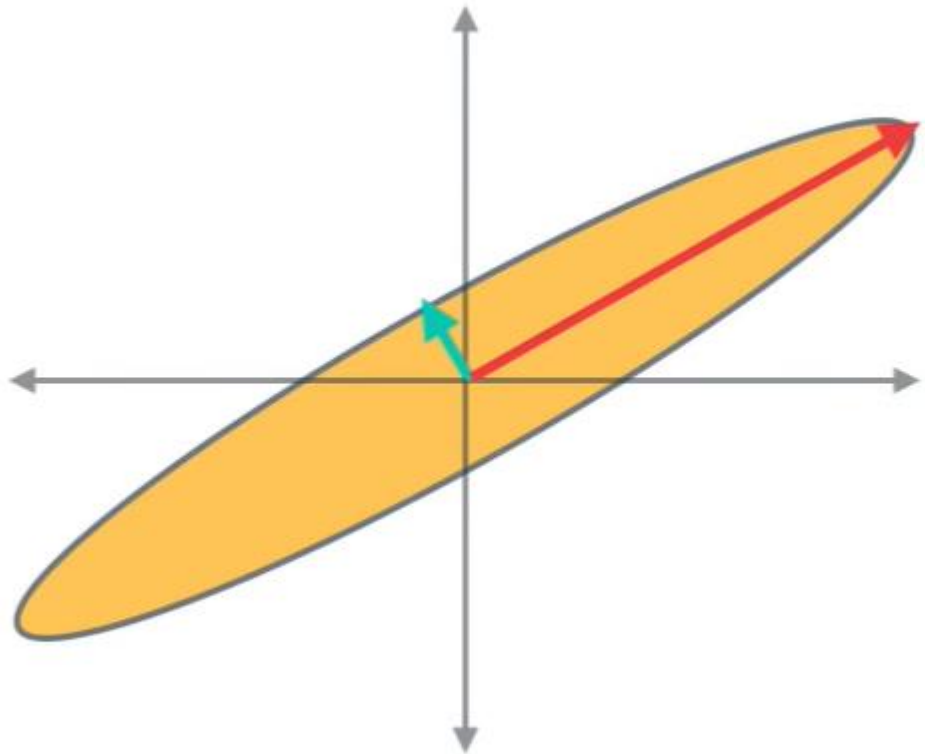
$(x, y) \longrightarrow (9x+4y, 4x+3y)$

(0,0)	(0,0)
(1,0)	(9,4)
(0,1)	(4,3)
(-1,0)	(-9,-4)
(0,-1)	(-4,-3)

Linear Transformations



Linear Transformations



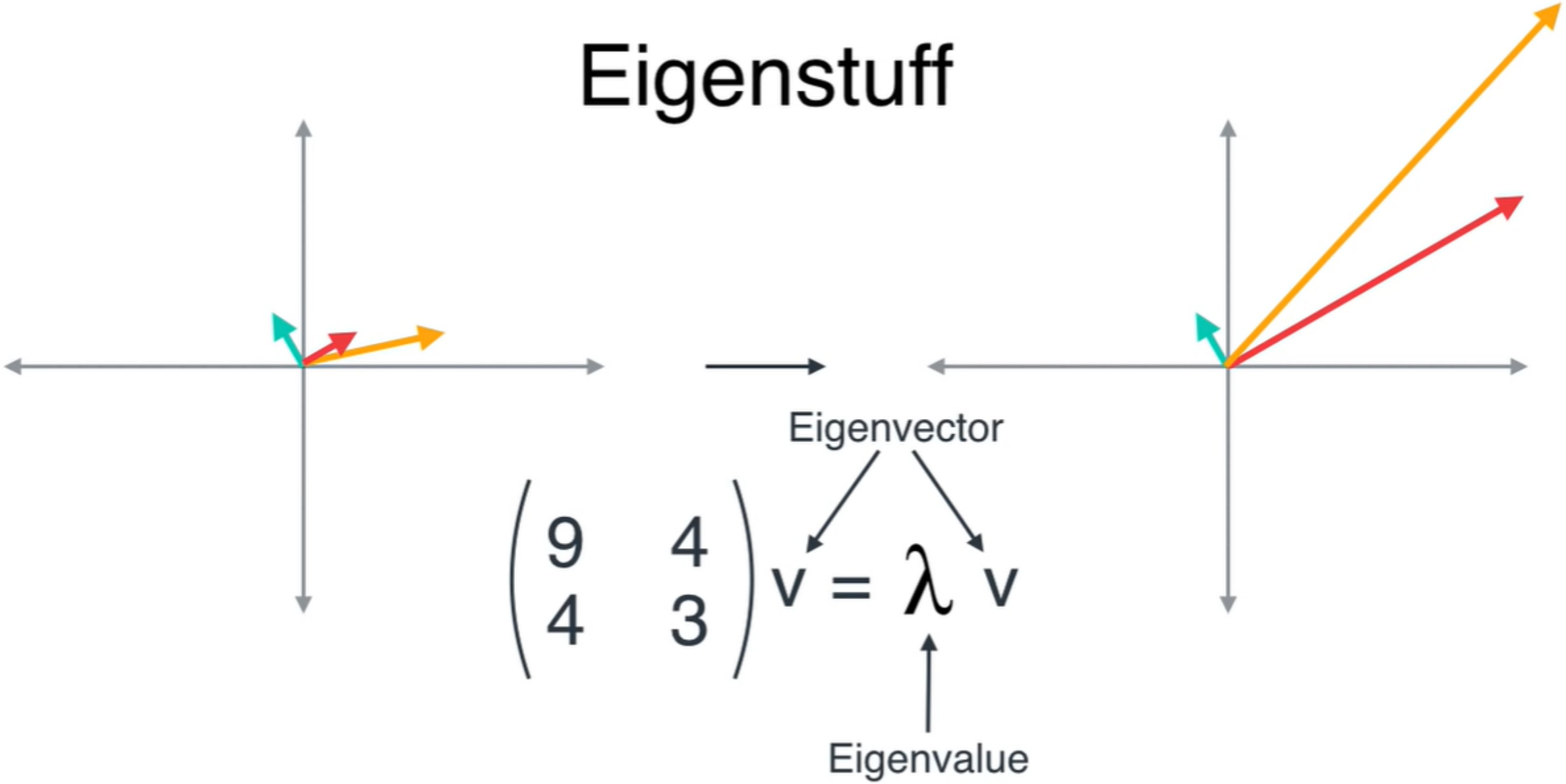
Eigenvectors
(direction)

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Eigenvalues
(magnitude)

11 1

Eigenstuff


Eigenvector
$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix} v = \lambda v$
Eigenvalue

Eigenvalues

$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

Characteristic Polynomial

$$\begin{vmatrix} x-9 & -4 \\ -4 & x-3 \end{vmatrix} = (x-9)(x-3) - (-4)(-4) = x^2 - 12x + 11$$

$$= (x-11)(x-1)$$

Eigenvalues 11 and 1

Eigenvalues

$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

Characteristic Polynomial

$$\begin{vmatrix} x-9 & -4 \\ -4 & x-3 \end{vmatrix} = (x-9)(x-3) - (-4)(-4) = x^2 - 12x + 11$$

$$= (x-11)(x-1)$$

Eigenvalues 11 and 1

Eigenvalue

고유값

자료행렬을 요약하는 수치로서, 특성치라고도 한다. 각 고유값은 그에 대응하는 고유벡터가 있다. A는 m×n 행렬이고, $\vec{x}$는 R^n의 영벡터가 아닌 벡터이다. 스칼라 λ에 대하여 $A\vec{x}$가 $\vec{x}$의 스칼라 λ배, 즉 $A\vec{x}=\lambda\vec{x}$일 때, λ를 A의 고유값(eigenvalue of A)이라 하고, $\vec{x}\,(\vec{x}\neq\vec{0})$를 λ에 대응하는 A의 고유 벡터(eigenvector of A)라 한다.

예를 들어, 벡터 $x=\begin{pmatrix}1\\2\end{pmatrix}$는 $Ax=\begin{pmatrix}3&0\\8&-1\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}3\\6\end{pmatrix}=3\begin{pmatrix}1\\2\end{pmatrix}=3x$이므로 고유치 λ = 3에 대응하는 행렬 $A=\begin{pmatrix}3&0\\8&-1\end{pmatrix}$의 고유 벡터이다. A가 실수의 n×n 대칭행렬이면 A의 고유값은 실수이다. A는 n×n 행렬일 때 A의 고유값이 λ이기 위한 필요충분조건은 det(A - λI) = 0이다. det(A - λI)는 n차 다항식이 된다. 이때 det(A - λI) = 0을 행렬 A의 특성방정식(characteristic equation of A)라고 한다.

n×n 행렬 A가 서로 다른 고유값을 가지면 A는 대각화가 가능한 행렬이다. 대각행렬과 닮은 행렬을 "대각화 가능 행렬(diagonalizable matrix)"이라 한다. n×n 행렬 A가 대각행렬 D와 닮았을 때 A는 '대각화 가능하다(be diagonalizable)'라고 하고 A를 대각화 가능 행렬이라 한다. n×n 행렬 A가 대각화 가능 행렬이기 위한 필요충분조건은 A가 n 개의 일차독립인 고유벡터를 갖는 것이다.

행렬 A의 서로 다른 고유값 $\lambda_1, \lambda_2, \cdots, \lambda_m$에 대응하는 고유벡터가 $\vec{x_1}, \vec{x_2}, \cdots, \vec{x_m}$일 때 $\vec{x_1}, \vec{x_2}, \cdots, \vec{x_m}$는 일차독립이다. SVD(Singular Value Decomposition, 특이값 분해), Pseudo-Inverse, 선형연립방정식의 풀이, PCA(Principal component analysis, 주성분분석) 등의 주요 응용이 eigenvalue, eigenvector를 그 밑바탕에 깔고 있다.

https://terms.naver.com/entry.naver?docId=3475938&cid=58439&categoryId=58439

Eigenvalues

$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

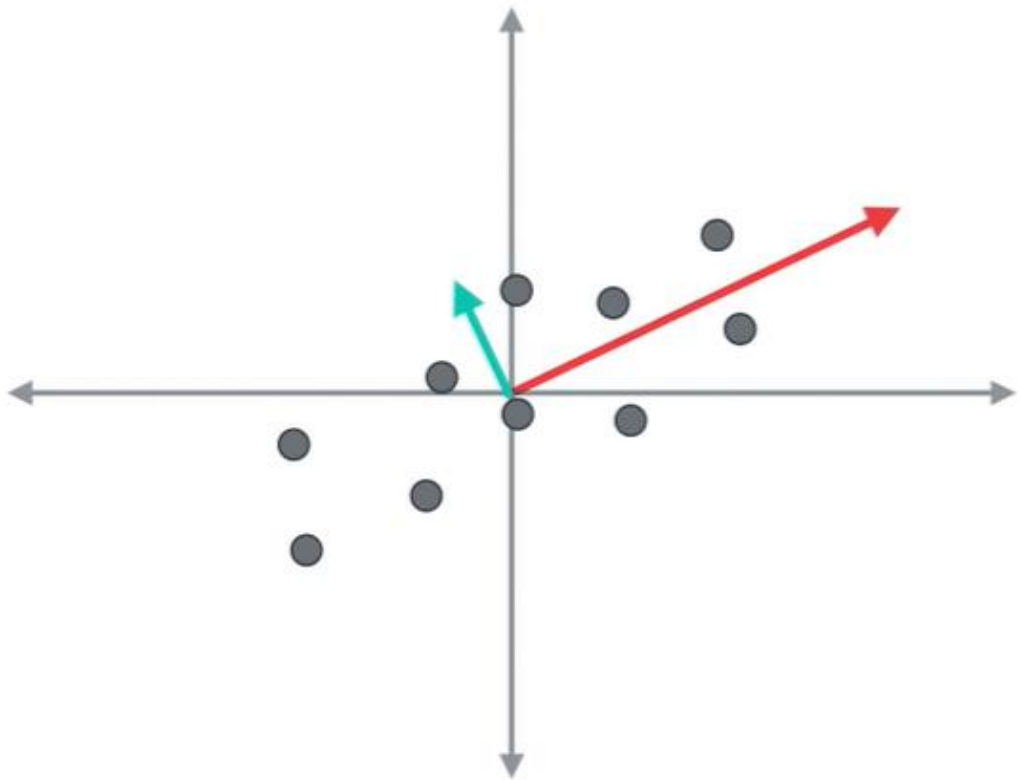
Characteristic Polynomial

$$\begin{vmatrix} x-9 & -4 \\ -4 & x-3 \end{vmatrix} = (x-9)(x-3) - (-4)(-4) = x^2 - 12x + 11$$

$$= (x-11)(x-1)$$

Eigenvalues 11 and 1

Principal Component Analysis (PCA)



$$\Sigma = \begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

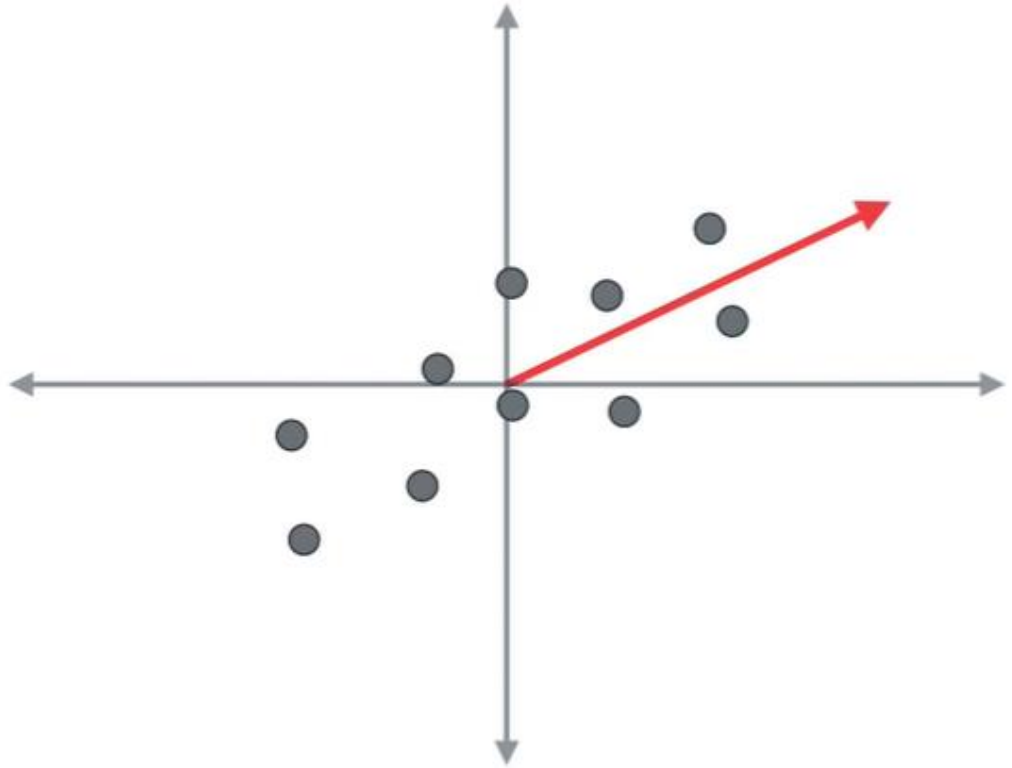
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Eigenvectors (direction)

11 1

Eigenvalues (magnitude)

Principal Component Analysis (PCA)



$$\Sigma = \begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

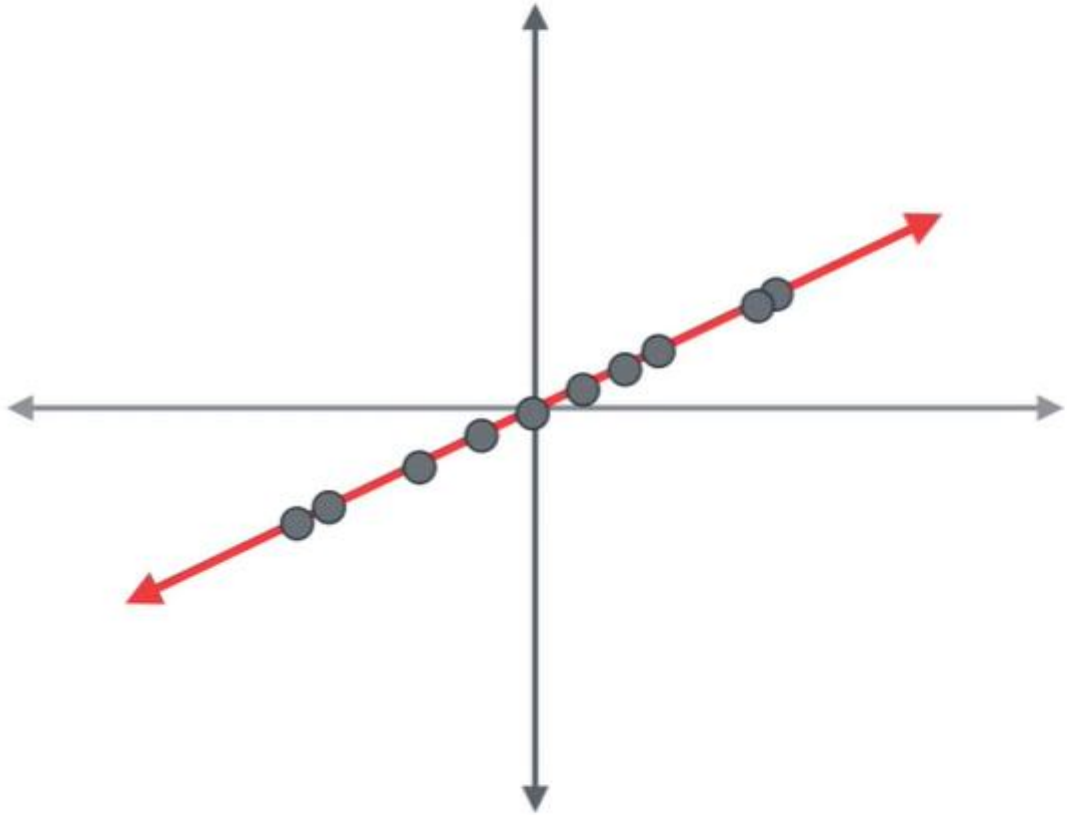
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Eigenvectors
(direction)

11

Eigenvalues
(magnitude)

Principal Component Analysis (PCA)



$$\Sigma = \begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

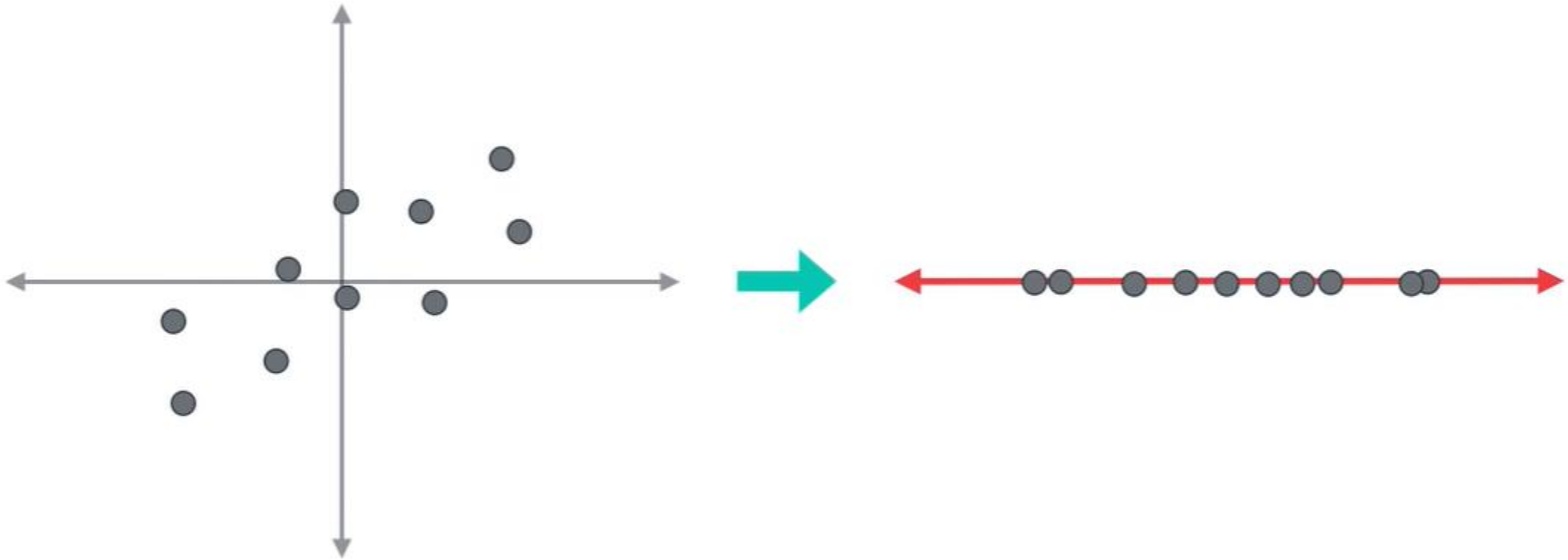
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Eigenvectors (direction)

11

Eigenvalues (magnitude)

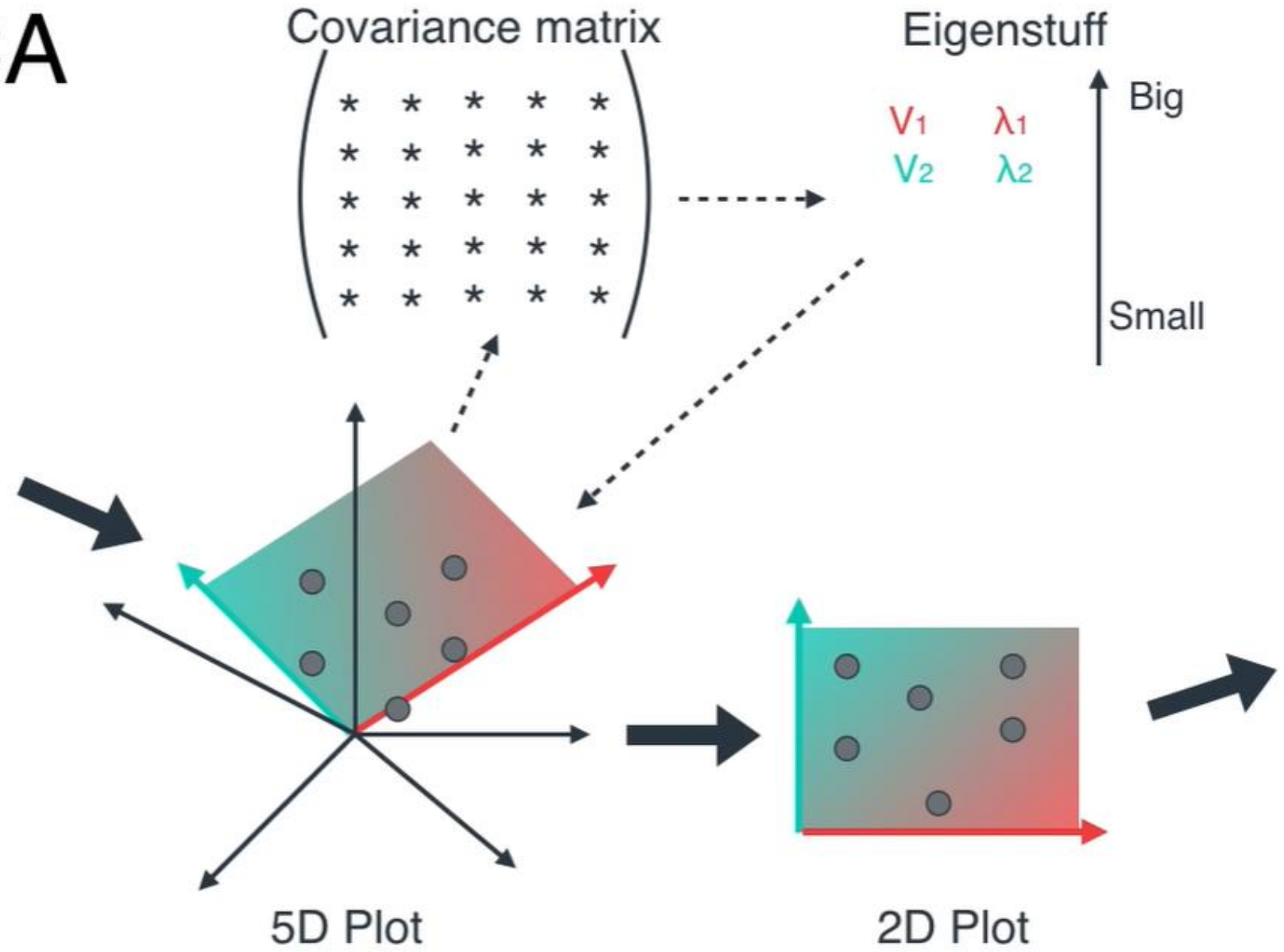
Principal Component Analysis (PCA)



PCA

Large Table

X1	X2	X3	X4	X5
*	*	*	*	*
*	*	*	*	*
*	*	*	*	*
*	*	*	*	*
*	*	*	*	*
*	*	*	*	*
*	*	*	*	*
*	*	*	*	*
*	*	*	*	*
*	*	*	*	*
*	*	*	*	*
*	*	*	*	*



Small Table

W1	W2
*	*
*	*
*	*
*	*
*	*
*	*
*	*
*	*
*	*
*	*
*	*
*	*

목표변수

1.매출

2.팁

3.요일별 팀 Size 수

4.Tip rate

5.요일별 방문 예상 팀 수

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