ibs_ring Simulation Program

Michael Ehrlichman, D. Sagan

October 21, 2022

Contents

1 Introduction

`ibs_ring` is a program for simulating intra-beam scattering (IBS) in a storage ring. The source code for this program lives in the `bsim` directory in the standard Bmad[1] distribution.

Many formulas for calculating IBS rates are implemented. They are:

1. Modified Piwinski with Zotter's integral.
2. Modified Piwinski with constant Coulomb Log.
3. Completely Integrated Modified Piwinski (CIMP).
4. Bjorken & Mtingwa's formula.
5. Bane's approximation of Bjorken & Mtigwa's formula.
6. Kubo & Oide's generalization of Bjorken & Mtingwa's formula.

These formula's are described in the following section.

2 Methods for calculating IBS growth rates

2.1 Modified Piwinski with Zotter's integral

This formula is selected by setting `ibs_formula='mpzt'`.

Piwinski's original formula for calculating IBS rates is Ref. [2]. The original formula contained a numerically difficult triple integral. In Ref. [4], the triple integral is reduced to a single integral that is much easier to evaluate. No approximations are applied to obtain the single integral, it is exact.

The original formula included Twiss β and dispersion η, but neglicted the derivatives of the lattice functions Twiss α and η'. In Ref. [3], the derivatives of the lattice functions are included. This form of Piwinski's original formula with derivatives of the lattice functions is usually called "Modified Piwinski".

2.2 Modified Piwinski with constant Coulomb Log

This formula is selected by setting `ibs_formula='mpxx'`.

Piwinski's origional IBS formula typically gives a considerablly different growth rate than Bjorken & Mtingwa's, Kubo & Oide's, and approximations of Piwinski. This is because Piwinski's formula is unique in how it treats the Coulomb Logarithm.

In the `'mpxx'` IBS formula, Piwinski's formula has been rederived and the log term pulled out of the integral. This allows the Coulomb Logarithm to be treated the same as it is in other IBS formulas. This derivation is available in Ref. [6].

2.3 Completely Integrated Modified Piwinski (CIMP)

This formula is selected by setting `ibs_formula='cimp'`.

In Ref. [5], a high energy approximation of the Modified Piwinski formula is obtained. This formula contains one integral which can be easily and quickly tabulated. This formula is very fast and, for ILC and CesrTA, returns growth rates similar to those obtained from the more general IBS formulas.

2.4 Bjorken & Mtingwa's formula

This formula is selected by setting `ibs_formula='cimp'`.

In Ref. [7], the authors take a distinct approach to calculating IBS growth rates. In Ref. [8], Bjorken & Mtingwa's formula is compared to the Modified Piwinski formula. It is found that, for high energy beams, after some modifications to Piwinski's formula, the two formulas are algebraically similar and give similar results.

2.5 Bane's approximation of Bjorken & Mtingwa's formula

This formula is selected by setting `ibs_formula='bane'`.

In Ref. [9], a high energy approximation of Bjorken & Mtingwa's formula is obtained. The formula is simpler and numerically easier to evaluate, but does not give sensible results when vertical dispersion is zero.

2.6 Kubo & Oide's generalization of Bjorken & Mtingwa's formula

This formula is selected by setting `ibs_formula='kubo'`.

In Ref. [10], a generalization of Bjorken & Mtingwa's formula is derived. This formula is unique in that it is based on the 6×6 matrix of the second order moments of the beam distribution (the beam sigma matrix), rather than on Twiss parameters.

This formula should be able to handle arbitrary coupling conditions, though that has not been tested in experiment.

When `ibs_formula='kubo'`, it may be helpful to set `use_t6_cache=.true.`. This causes `ibs_ring` to pre-compute the one-turn maps. This greatly speeds up `ibs_ring` on larger accelerators. With `use_t6_cache=.true.`, the simulation is $\mathcal{O}(n)$. With `use_t6_cache=.false.`, the simulation is $\mathcal{O}(n^2)$. This setting has no effect for other IBS calculation methods.

3 Setting vertical dispersion

Vertical dispersion is zero in an ideal flat storage ring. Realistically, storage rings are not ideal and have misalignments which result in vertical dispersion. One method to simulate the effect of misalignments is to use a lattice with a realistic distribution of element misalignments. Since it is often the case that obtaining such a lattice is not possible, `ibs_ring` allows the vertical dispersion η_b and derivative of vertical dispersion η_b' to be specified in the parameters file.

The following setting can be used to set the vertical dispersion.

```
set_dispersion = <logical>
       eta_set = <float>
      etap_set = <float>
```

The interpretation of these parameters depends on the IBS formula being used. These differences are described below. Note that the two interpretations are not equivalent. Results obtained from the Twiss-based IBS formulas using a particular `eta_set` and `etap_set` are not necessarily comparable to results obtained from the `kubo` formula using the same `eta_set` and `etap_set`.

3.1 For all IBS calculation formulas except `kubo`

If `set_dispersion=.true.`, then `ele%b%eta` and `ele%b%etap` will be overwritten with `eta_set` and `etap_set` for every element.

3.2 For `kubo` IBS formula

If `set_dispersion=.true.`, then the one-turn matrix $\mathbf{T}_6$ will be replaced with $\tilde{\mathbf{T}}_6$, where $\tilde{\mathbf{T}}_6 = \mathbf{T}_6\mathbf{W}$ and

$$\mathbf{W} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\eta_b \\ 0 & 0 & 0 & 1 & 0 & -\eta_b' \\ 0 & 0 & \eta_b' & -\eta_b & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{1}$$

where η_b and η_b' are set by `eta_set` and `etap_set`.

4 Potential well distortion (pwd)

Intrabeam scattering does not directly change the bunch length. It changes the energy spread, which in turn results in a change in bunch length.

If `do_pwd=.false.`, then the simulation maintains the ratio between energy spread and bunch length. i.e.,

$$\frac{\sigma_{p,IBS}}{\sigma_{z,IBS}} = \frac{\sigma_{p0}}{\sigma_{z0}}, \tag{2}$$

where σ_{p0} and σ_{z0} are the zero-current energy spread and bunch length, and $\sigma_{p,IBS}$ and $\sigma_{z,IBS}$ are the energy spread and bunch length after including IBS effects.

If `do_pwd=.true.`, then the the effect of potential well distortion is simulated as a current-dependent defocusing rf voltage V_{pwd},

$$V_{pwd} = \frac{-LNec^2}{\sqrt{2\pi}\sigma_z^3 E_0}, \tag{3}$$

where e is electric charge, c is the speed of light, σ_z is the bunch length, and E_0 is the beam energy. L is inductance and is a parameter set using `inductance = <float>`. Typical values of L are tens of nH.

5 Methods for determining equilibrium emittances

5.1 Derivatives method

This method is selected by setting `eqb_method='der'`.

This method finds the equilibrium beam size using differential equations to evolve the emittance through time. The time step is hard coded to $\frac{\tau}{10}$, where τ is the horizontal damping time.

The differential equations are,

$$\frac{d\epsilon_a}{dt} = -\left(\epsilon_a - \epsilon_{a0}\right)\frac{2}{\tau_a} + \epsilon_a \frac{2}{T_a} \tag{4}$$

$$\frac{d\epsilon_b}{dt} = -\left(\epsilon_b - \epsilon_{b0}\right)\frac{2}{\tau_b} + \epsilon_b \frac{2}{T_b} \tag{5}$$

$$\frac{d\sigma_p}{dt} = -\left(\sigma_p - \sigma_{p0}\right)\frac{1}{\tau_z} + \sigma_p \frac{1}{T_z}, \tag{6}$$

where ϵ_{a0}, ϵ_{b0}, and σ_{p0} are the zero-current emittances and energy spread, τ_a, τ_b, and τ_z are the damping times, and T_a, T_b, and T_z are the IBS growth rates given the IBS formulas. The factors of 2 come about because τ_a, τ_b, T_a, and T_b are betatron growth rates. i.e. they are the rates of change the beam sizes, rather than the emittances.

5.2 Relaxation of equilibrium equations

This method is selected by setting `eqb_method='rlx'`.

The solutions to differential equations (4), (5), and (6) are,

$$\epsilon_a = \frac{1}{1 - \frac{\tau_a}{T_a}} \epsilon_{a0} \tag{7}$$

$$\epsilon_b = \frac{1}{1 - \frac{\tau_b}{T_b}} \epsilon_{b0} \tag{8}$$

$$\sigma_p = \frac{1}{1 - \frac{\tau_z}{T_z}} \sigma_{p0}. \tag{9}$$

Note that T_a, T_b, and T_z are functions of ϵ_a, ϵ_b, and σ_p.

In Ref. [5], a method for approximating the effect of transverse coupling replaces Eqn. 8 with,

$$\epsilon_b = \left((1 - r_\epsilon) \frac{1}{1 - \frac{\tau_b}{T_b}} + r_\epsilon \frac{1}{1 - \frac{\tau_a}{T_a}} \right) \epsilon_{b0}, \tag{10}$$

where r_ϵ describes the amount of vertical emittance that is due to transverse mode coupling. $0 < r_\epsilon < 1$. $r_\epsilon = 0$ describes a situation where there is no mode coupling and ϵ_{b0} is determined entirely by physics in the vertical plane. $r_\epsilon = 1$ describes a situation where ϵ_{b0} is determined entirely by coupling from the horizontal plane.

In the `ibs_ring` simulation r_ϵ is set by `ratio`.

6 Coulomb Logarithm

The IBS growth rates are directly proportional to a quantity that has come to be called the Coulomb Logarithm The Coulomb Logarithm is,

$$\log \frac{b_{max}}{b_{min}}, \tag{11}$$

where b_{max} is the largest impact parameter for particle-on-particle collisions within a bunch, and b_{min} is the smallest impact parameter. The IBS growth rates blow up as b_{min} goes to zero or b_{max} gets large.

b_{max} is set to the smallest of either the beam height or mean interparticle distance.

In machines with little damping, such as proton rings and linacs, b_{min} is set to the impact parameter associated with a scattering angle of $\frac{\pi}{2}$.

It was proposed in [11], that in machines with strong damping, such as light sources, damping rings, and circular lepton colliders, b_{min} should be set according to the damping rate. This is called the *Tail Cut* because the idea is to exclude from the calculation of the rise time rare, large angle scattering events that populate non-gaussian tails of the beam distribution.

The Tail Cut was also applied in Ref. [10], albeit using a different formulation. The formulation

there is,

$$b_{min1} = \frac{r_e}{(p_\perp \gamma)^2} \tag{12}$$

$$b_{min2} = \sqrt{\frac{vol}{N \pi p_\perp c \tau_a}} \tag{13}$$

$$b_{min} = max\left(b_{min1}, b_{min2}\right), \tag{14}$$

where r_e is the classical electron radius, $p_\perp$ is the average transverse momentum of particles in the bunch, γ is the relativistic factor of the bunch centroid, vol is the volume of the bunch envelop, N is the number of particles in the bunch, c is the speed of light, and τ_a is the horizontal damping rate.

In `BMAD`, the method used for calculating the Coulomb Log is set using `clog_to_use = <integer>`.

For all IBS formulas except for `kubo`, the following options are available,

1 Classic Coulomb Log (no Tail Cut, $\frac{\pi}{2}$ scattering angle).

2 Integral-based Tail Cut given in Ref. [11].

3 Tail Cut Eqn. 14, as prescribed in Ref. [12].

4 Tail Cut Eqn. 14, similar to that in Ref. [10].

For the `kubo` IBS formulas, the following options are available,

1 Tail Cut disabled (no Tail Cut, $\frac{\pi}{2}$ scattering angle).

else Tail Cut applied as described in Ref. [10].

7 Beam size calculations

`ibs_ring` calculates beam size by computing the sigma matrix $\mathbf{\Sigma}$ of a beam matched to the machine optics. The beam envelop projected into the horizontal, vertical, and longitudinal planes are the 11, 33, and 55 elements of the sigma matrix. This method of calculating beam sizes naturally takes into account arbitrary coupling conditions.

$\mathbf{\Sigma}$ is obtained from,

$$\mathbf{\Sigma S} = \mathbf{N D N}^\dagger, \tag{15}$$

where $\mathbf{S}$ is the symplectic matrix, $\mathbf{N}$ is formed from the eigenvectors of the one-turn transfer matrix at a particular element, and $\mathbf{D}$ is,

$$\mathbf{D} = \begin{pmatrix} 0 & \epsilon_a & 0 & 0 & 0 & 0 \\ -\epsilon_a & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \epsilon_b & 0 & 0 \\ 0 & 0 & -\epsilon_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \epsilon_c \\ 0 & 0 & 0 & 0 & -\epsilon_c & 0 \end{pmatrix}. \tag{16}$$

This calculation and **N** are discussed in detail in Ref. [13] and Ref. [14].

The `ibs_ring` parameters `x_view`, `y_view`, and `z_view` are the element indexes where the horizontal, vertical, and longitudinal beam sizes are calculated.

8 Current range

The `ibs_ring` simulation is designed to generate beam size versus current graphs. The current range, in mA, is set by `low_current`, `high_current`, and `delta_current`,

`high_current` is the highest current and `low_current` is the lowest current. `delta_current` is the approximate step size. The actual step size is adjusted by the simulation such that an current range is spanned by equally sized steps.

9 `ibs_ring` output files

9.1 `emittance.dat`

Properties of beam envelop at equilibrium. By column:

`current` Current in Amps.

`emit_a` a-mode emittance.

`emit_b` b-mode emittance.

`sigE/E` Energy spread.

`sigma_x` Size of horizontal projection of beam envelop at element `x_view`.

`sigma_y` Size of vertical projection of beam envelop at element `y_view`.

`sigma_z` Size of longitudinal projection of beam envelop at element `z_view`.

9.2 `ibs_rates.out`

IBS growth rate at each element at equilibrium. Useful in for answering question "Where in the lattice is the IBS growth coming from?". By column:

`ele ix` Element index

`s` Element location in meters.

`inv_Ta` $\frac{1}{T_a}$, where T_a is the a-mode IBS growth time.

`inv_Tb` $\frac{1}{T_b}$, where T_b is the b-mode IBS growth time.

`inv_Tz` $\frac{1}{T_z}$, where T_z is the longitudinal IBS growth time.

9.3 `rad_int.out`

Beam properties at zero current, determined either by radiation integrals or set in parameters file. Also contains beam properties at high current (`high_current`).

10 Parallelization

The `ibs_ring` code has been parallelized with OpenMP. Enabling OpenMP requires setting the environment variable `ACC_ENABLE_OPENMP=1` before compillation. The entire BMAD distribution, including packages such as LAPACK and forest, must be compiled with `ACC_ENABLE_OPENMP=1`, otherwise the resulting code will not be thread safe.

If `ibs_ring` is compiled with OpenMP enabled, then the environment variable `OMP_NUM_THREADS=<integer>` will set the number of cores used for computation.

Signs of the code not being thread safe include unstable lattice errors. Thread safety can be diagnosed by setting `OMP_NUM_THREADS=1`.

11 Parameter file summary

The input for the `ibs_ring` program uses Fortran90 namelist syntax: The data begins with the string `&parameters` and ends with a slash `/`. Everything outside this is ignored. The input parameters are:

```
&parameters
  lat_file = <lattice-file-name>
  granularity = <real>   ! -1 for element-by-element.
  ptc_calc = <logical>  ! If true, use PTC for rad. int. calculations.
  b_emit = <real>  ! Zero current vertical emittance.  Set to -1 for rad int.
  a_emit = <real>  ! Zero current horizontal emittance.  Set to -1 for rad int.
  energy_spread = <real>  !Zero current energy spread.  Set to -1 for rad int.
  fake_3HC = <real> ! If greater than zero, reduce rates by this factor.
                    ! IBS rates scale with 1/sigma_z
  high_current = <real>  ! highest current
  delta_current = <real> ! step size
  low_current = <real>   ! lowest current
  ibs_formula = <type>  ! 'cimp', 'bjmt', 'bane', 'mpzt', 'mpxx', or 'kubo'
  clog_to_use = <int>   ! 1=no tail cut, 2=raubenheimer, 3=kubo, 4=kubo w/vertical
  eqb_method = <type>   ! 'der' derivatives or 'rlx' for Wolski's relaxation.
                        ! 'der' is default.
  ratio = <real>  ! "Coupling parameter r" for relaxation eqb_method
```

```
  x_view = <int>  ! ix of element where projection is taken
                  ! for horizontal beam size calculation.
  y_view = <int>  ! ix of element where projection is taken
                  ! for vertical beam size calculation.
  z_view = <int>  ! ix of element where projection is taken
                  ! for longitudinal beam size calculation.
  do_pwd = <logical>  ! Apply potential well distortion to bunch length
  inductance = <real>  ! A PWD parameter akin to inductance
  set_dispersion = <logical> ! Assign a vertical dispersion (eta_set and etap_set).
  eta_set = <real> ! If set_dispersion, then at every element,
                   ! set vertical dispersion to eta_set.
  etap_set = <real> ! If set_dispersion, then at every element,
                    ! set vertical dispersion' to etap_set.
/
```

`lat_file`
: Bmad lattice file describing the ring.

`ibs_formula`
: Name of the algorithm to use for the calculation. Possibilities are:

```
'cimp'
'bjmt'
'bane'
'mpzt'
'mpxx'
'kubo'
```

`eqb_method`
: Method used for finding the equilibrium solution. Possibilities are:

```
'der'
'rlx'
```

'der' finds the equilibrium emittances using differential equations. The differential is with respect to time.

'rlx finds the equilibrium emittances by iterating to find the solution to an analytic form for the equilibrium emittance. The 'rlx' method includes the controversial 'coupling parameter' which says that the vertical emittance has contributions from both the vertical and horizontal IBS rates and damping rates.

In the limit that the coupling parameter is zero, 'der' and 'rlx' are equivalent. In fact, the equations used for 'rlx' are the equilibrium solution to the 'der' method's differential equations.

'der' is much faster and robust. However, it does not allow for the 'coupling parameter', which has its uses. Also, the two different methods can be useful in diagnosing the code.

`clog_to_use`
: Logarithmic cutoff to use. Possibilities are:

```
1  ! Classic, no tail cut.
2  ! Raubenheimer.
3  ! Bane.
4  ! Oide.
```

`inductance`
: Longitudinal inductance for PWD calc. Effects bunch length vs. current.

`set_dispersion`
: If true, then apply eta_set and etap_set. If false, then do not.

`eta_set`
: Used only if ibs_formula set to 'kubo'. Applies x-pz coupling to each element of lattice when calculating IBS rates.

`etap_set`
: Used only if ibs_formula set to 'kubo'. Applies px-pz coupling to each element of lattice when calculating IBS rates.

`a_emit`
: Zero current horizontal emittance. If set to -1 then value is obtained from an evaluation of the radiation integrals.

`b_emit`
: Zero current vertical emittance. If set to -1 then value is obtained from an evaluation of the radiation integrals.

`energy_spread`
: Zero current energy spread. If set to -1 then value is obtained from an evaluation of the radiation integrals.

`ratio`
: "Coupling parameter r" hack (§5.2) for including coupling.

`granularity`
: Step size along lattice in meters to evaluate the various integrals. Set to -1 for one step per element.

`x_view`
: Index of element where projection is taken for horizontal beam size calculation.

`y_view`
: Index of element where projection is taken for vertical beam size calculation.

`z_view`
: Index of element where projection is taken for longitudinal beam size calculation.

`high_current`
: Largest current per bunch in mA.

`low_current`
: Smallest current per bunch in mA.

`delta_current`
: mA step size.

References

[1] D. Sagan, "Bmad: A Relativistic Charged Particle Simulation Library" Nuc. Instrum. & Methods Phys. Res. A, **558**, pp 356-59 (2006). The Bmad web site: http://www.lepp.cornell.edu/~dcs/bmad

[2] Piwinski, A., "Intra-Beam Scattering". Proceedings of the 9th International Conference on High Energy Accelerators, Stanford, CA, 1974. p. 405.

[3] Martini, M., CERN PS/84-9 (AA) (1984).

[4] Evans, L. and Zotter, B., "Intrabeam Scattering in the SPS". CERN-SPS-80-15, CERN, 1980.

[5] Kubo, K., Mtingwa, S. K. and Wolski, A., "Intrabeam Scattering Formulas for High Energy Beams," Phys. Rev. ST Accel. Beams, 8, 2005.

[6] Ehrlichman, M., "Thesis: Normal Mode Analysis of Single Bunch, Charge Density Dependent Behavior in Electron/Positron Beams". Cornell University (2013).

[7] Bjorken, J., Mtingwa, S., "Intrabeam Scattering", Particle Accelerators. 13. pp. 115-143. (1983).

[8] Bane, K., "An Accurate, Simplified Model of Intrabeam Scattering", SLAC-AP-141, arXiv:physics/0205058. (2002).

[9] Bane, K. in *Proceedings of the 8th European Particle Accelerator Conference, Paris, France, 2002*, p. 1443, (2002).

[10] Kubo, K., Katsunobu, O., "Intrabeam Scattering in Electron Storage Rings", Phys. Rev. ST Accel. Beams 4, 124401 (2001).

[11] Raubenheimer, T. O., "The Core Emittance with Intrabeam Scattering in e^+/e^- Rings", Part. Accel. 45, pp. 111-118, (1994).

[12] Bane, K. L. F, Hayano, H., Kubo, K., Naito, T., Okugi, T., Urakawa, J., "Intrabeam scattering analysis of measurements at KEK's ATF damping ring", SLAC-PUB-9227 (also KEK-Preprint 2002-26), (2002).

[13] Ehrlichman, M., et. al., "Measurement and Compensation of Horizontal Crabbing at the Cornell Electron Storage Ring Test Accelerator", Submitted to Phys. Rev. ST Accel. Beams (2013).

[14] Wolski, A., "Alternative approach to general coupled linear optics", Phys. Rev. ST Accel. Beams 9, 024001 (2006).