Positron Converter Model

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Background

The positron converter provides CESR with its positrons. The converter is a slab of heavy metal (usually tungsten), which is bombarded with electrons whose energy is on the order of $\sim 100\,\mathrm{MeV}$. As the incident electrons pass through the converter, they emit photons via Bremmstrahlug, which in turn decay to e^+e^- pairs:

$$e^- + Z \to e^- + Z + \gamma \to e^- + Z + e^+ + e^-$$

The production of positrons in the converter is a stochastic process, the details of which are computationally expensive to simulate. As such, it is desirable to have a model for the properties of the produced positrons (their energy, radial displacement, and direction of motion) in terms of probability distributions.

Coordinate System

Figure 1 illustrates the coordinate system we use to describe the converter. The incoming electron beam is taken to be along the z-axis. Outgoing positrons will exit the target with some displacement r off of the z-axis, and have some energy E_+ and momentum $\mathbf{p}_+$. It is convenient to define a coordinate system (x', y', z) with $\hat{\mathbf{x}}'$ pointing along the direction of $\hat{\mathbf{r}}$, and $\hat{\mathbf{y}}'$ taken perpendicular to $\hat{\mathbf{x}}'$ so that (x', y', z) is a right-handed coordinate system. This defines p'_x and p'_y, the components of the outgoing positron momentum in the primed coordinate system. We then define the "transverse momenta" $\frac{dx'}{ds}$ and $\frac{dy'}{ds}$ by

$$\frac{dx'}{ds} = \frac{p'_x}{p_z}$$
$$\frac{dy'}{ds} = \frac{p'_y}{p_z}$$

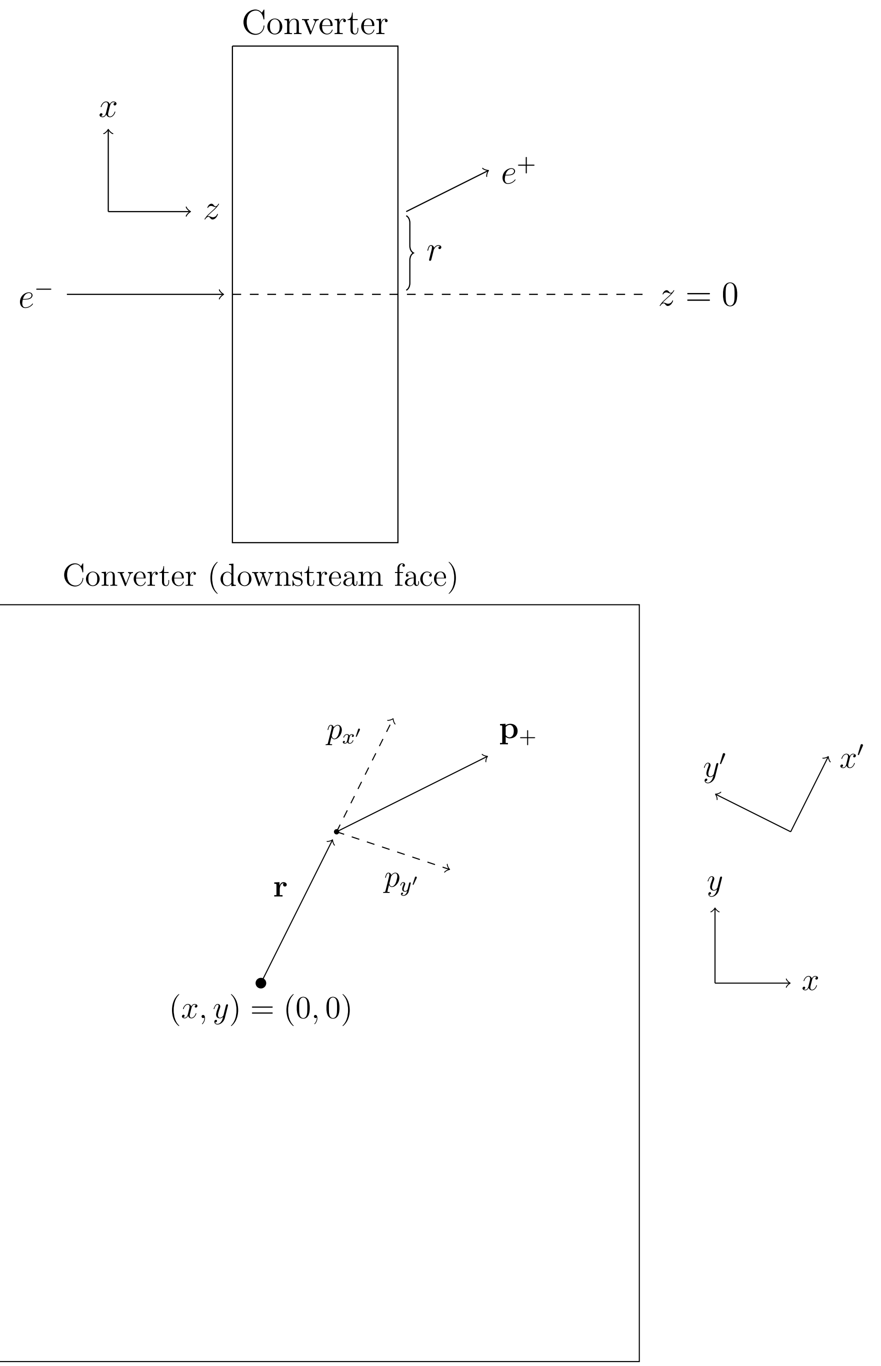

Figure 1: Coordinates used to describe the positrons exiting the converter.

The Model

Given a positron converter of thickness T and incoming electrons of energy E_-, we wish to predict E_+, r, $\frac{dx'}{ds}$, and $\frac{dy'}{ds}$ for the outgoing positrons. We do this in two steps:

a. First, we pull E_+ and r from a two-dimensional probability distribution $P(E_+, r)$, which we model as

$$\begin{aligned} P(E_+, r) &= P_3(E_+)P_3(r)e^{-\alpha E_+}e^{-\beta r} \\ &= C(E_+^3 + A_E E_+^2 + B_E E_+ + C_E)(r^3 + A_r r^2 + B_r r + C_r)e^{-\alpha E_+}e^{-\beta r} \end{aligned}$$

b. For outgoing positrons of any given (E_+, r), $\frac{dx'}{ds}$ and $\frac{dy'}{ds}$ are normally distributed. However, the parameters of these distributions (μ and σ) depend on E_+ and r, with

$$\begin{aligned} \mu_{dx'/ds}(E_+, r) &= A_x E_+^{\alpha_x} r^{\beta_x} \\ \sigma_{dx'/ds}(E_+, r) &= (a_x + b_x r)(1 - e^{-k_x E_+}) \\ \mu_{dy'/ds}(E_+, r) &= 0 \\ \sigma_{dy'/ds}(E_+, r) &= (a_y + b_y r)(1 - e^{-k_y E_+}) \end{aligned}$$

Note that these functional forms are empirically derived, and are not heavily motivated by the underlying physical processes that occur in the converter.

Obtaining the Model Coefficients

Using the Geant4[1] software package developed at CERN, we have developed a program for simulating the production of positrons in the converter. Using this program, one can generate data on the number of positrons N_+ produced at given E_+ and r values, as well as data on $\mu_{dx'/ds}$, $\sigma_{dx'/ds}$, $\mu_{dy'/ds}$, $\sigma_{dy'/ds}$ vs E_+ and r. With this data in hand, one can fit the models in the previous section to the data, yielding the fit parameters needed to simulate the converter in *Bmad*. We have included an *R* script which performs this fitting and outputs the fit parameters.

Note that each of the fit parameters will change for different values of the thickness T and incoming electron energy E_-. As such, it is advised that the simulation and fitting process be performed at several points over the range of E_- and T values of interest. For each choice of (E_-, T), one set of fit parameters is produced. *Bmad* then interpolates between the discrete set of (E_-, T) values simulated to model the converter at any electron energy and target thickness in the range of interest.

References

[1] S. Agostinelli et al. "Geant4—a simulation toolkit". In: *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 506.3 (2003), pp. 250–303. ISSN: 0168-9002. DOI: `https://doi.org/10.1016/S0168-9002(03)01368-8`. URL: `http://www.sciencedirect.com/science/article/pii/S0168900203013688`.