41. SÜDDEUTSCHES KOLLOQUIUM ÜBER DIFFERENTIALGEOMETRIE

ABSTRACTS

José Balado-Alves

Construction of r-harmonic submanifolds in spheres

Polyharmonic submanifolds are a natural higher-order generalization of minimal surfaces. We provide a new construction method for polyharmonic submanifolds in spheres, obtaining a rich family of new examples. Moreover, we completely determine the index and nullity of these solutions in the biharmonic case. This is a joint work with Anna Siffert.

Annegret Burtscher

The many faces of globally hyperbolic Lorentzian manifolds

The notion of global hyperbolicity was introduced by Jean Leray in 1952 to obtain global uniqueness of solutions to nonlinear wave equations. Globally hyperbolic Lorentzian manifolds subsequently turned out to be the right geometric setting not only for the well-posedness of the initial value formulation for the Einstein equations in General Relativity but also for the singularity theorems of Penrose and Hawking and several splitting results in Lorentzian geometry. In this talk, we review the rich history and omnipresence of global hyperbolicity in General Relativity and Lorentzian geometry and discuss similarities and differences to complete Riemannian manifolds. In this vein we also present a surprising new characterization of globally hyperbolic spacetimes resembling aspects of the Hopf-Rinow theorem (joint work with L. García-Heveling) that makes use of ideas and tools from metric geometry.

Tobias Dott

Gromov-Hausdorff limits of closed surfaces

We completely describe the Gromov-Hausdorff closure of the class of length spaces being homeomorphic to a fixed closed surface.

Nadine Grosse

On local boundary conditions for Dirac-type operators

We discuss smooth local boundary conditions for Dirac-type operators, giving existence and non-existence results for local symmetric boundary conditions. We also give conditions when the boundary conditions are elliptic/regular/Shapiro-Lopatinski (i.e. in particular giving rise to self-adjoint Dirac operators with domain in H^1). This is joint work with Hanne van den Bosch (Universidad de Chile) and Alejandro Uribe (University of Michigan).

Jonas Henkel

The Laplace-Beltrami spectrum on homogeneous spaces

The spectrum of the Laplace-Beltrami operator can be computed using Freudenthal's formula for normal homogeneous spaces $(G/K, g)$, i.e. the metric g is the restriction of a biinvariant metric of G. We extend this formula to naturally reductive homogeneous spaces and provide naturally reductive realizations of deformations of normal homogeneous metrics along commuting subalgebras. This method can be applied to 3-Sasaki manifolds together with their canonical deformation, which are positive 3-(α, δ)-Sasaki manifolds. We obtain a formula to compute the spectrum of 3-(α, δ)-Sasaki manifolds. Our approach has been inspired by Wilking's normal homogeneous realization of the Aloff-Wallach manifold $W^{1,1} = SU(3)/S^1$. We compute its full 3-(α, δ)-Sasaki spectrum and related its spectrum to its geometry.

Yingxiang Hu

Curvature flows and their applications to geometric inequalities

In this talk, we will first review the isoperimetric type inequalities, in particular, the Alexandrov-Fenchel type inequalities in Riemannian space forms. Then we will talk about the curvature flows with capillary boundary and their applications to isoperimetric type inequalities.

Mohammad N. Ivaki

Existence and uniqueness results for curvature problems

I will discuss various existence and uniqueness results for a class of curvature problems. In particular, I discuss kinematic curvature problems related to area measures and curvature measures.

Christian Ketterer

Stability of a Scalar curvature rigidity theorem for Riemannian tori

One goal of this talk is to establish a nonlinear analogue of a splitting map to Euclidean space, as a harmonic map to a flat torus. Existence of such a map implies Gromov-Hausdorff closeness to a flat torus in any dimension. Furthermore, Gromov-Hausdorff closeness to a flat torus and a curvature bound imply the existence of a harmonic splitting map. Combining these results with Stern's inequality yields a new Gromov-Hausdorff stability theorem for Riemannian 3-tori under almost non-negative Scalar curvature. This is a joint work with Shouhei Honda, Ilaria Mondello, Raquel Perales and Chiara Rigoni.

Tobias König

Quantitative stability of the total Q-curvature near minimizing metrics

Let (M, g) be a Riemannian manifold of dimension n, and $1 \leq k < \frac{n}{2}$ an integer. We prove quantitative estimates for the total k-th order Q-curvature functional near minimizing metrics. More precisely, we show that in a generic closed Riemannian manifold, the distance to the minimizing set of metrics is controlled quadratically by the Q-curvature energy deficit. This extends recent work for $k = 1$ by Engelstein, Neumayer and Spolaor (2021).

We also describe several instances of degenerate (or higher-order) stability. For instance, extending a classical example due to Schoen (1989), for every integer $1 \leq k < \frac{n}{2}$ we exhibit an n-dimensional Riemannian manifold such that the distance to the minimizing set of metrics is controlled by a higher power of the Q-curvature energy deficit.
This is work in progress, joint with Joao Henrique Andrade (Sao Paulo), Jesse Ratzkin (Würzburg) and Juncheng Wei (Hong Kong).

Rob Kusner

On Eigenspaces and Minimal Surfaces

Sharp dimension bounds for (Steklov and Laplacian) eigenspaces on Riemannian surfaces are used to study existence and classification problems for embedded minimal surfaces in the round ball B^n (with free boundary) and sphere $\mathbb{S}^n$ (compact without boundary). For instance, we prove the uniqueness of the critical catenoid in B^3 and the critical Möbius band in B^4 under the modest symmetry assumptions. Our main result constructs orientable free boundary minimal surfaces embedded in B^3 of every topological type, whose analog for closed minimal surfaces in $\mathbb{S}^3$ was settled by Lawson in 1970. Our construction is based on equivariant optimization of Steklov or Laplace eigenvalues on surfaces, normalized by their boundary length or surface area, respectively. Our free boundary surfaces in B^n have area below 2π, and if we fix the number of boundary components, the number of such surfaces grows at least linearly with respect to their genus; we prove analogous results for the number of embedded minimal surfaces in $\mathbb{S}^n$ with area below 8π. Geometrically, our surfaces are "doublings" of the equatorial B^2 and $\mathbb{S}^2$ respectively, in a precise sense to be described in the talk. Though we focus on the orientable case, if time permits we may also discuss the situation for nonorientable surfaces minimally embedded in B^4 and $\mathbb{S}^4$ (joint work with Peter McGrath and also with Misha Karpukhin, Peter McGrath, and Daniel Stern).

Elena Mäder-Baumdicker

Willmore Klein bottles and interesting minimal surfaces in the 5-sphere

After a short introduction to the Willmore functional and its properties I will report on an existence theorem of a Willmore minimizer in higher codimension for surfaces of Klein bottle type. The conjectured minimizer is a special one, it carries a metric which is critical for the first normalized eigenvalue of the Laplacian and it is a so-called bipolar minimal surface (going back to Lawson 1970). I will explain, what bipolar surfaces are and that they inherit interesting topological properties in some cases. This talk is based on work with Patrick Breuning, Jonas Hirsch and Melanie Rothe.

Artem Nepechiy

Locally homogeneous RCD spaces

The goal of this talk is to demonstrate how existing results can be adapted to establish the following result: A locally metric measure homogeneous RCD(K,N) space is isometric to, after multiplying a positive constant to the reference measure, a smooth Riemannian manifold with the Riemannian volume measure.