

## 41. SÜDDEUTSCHES KOLLOQUIUM ÜBER DIFFERENTIALGEOMETRIE

### ABSTRACTS

JOSÉ BALADO-ALVES

#### **Construction of $r$ -harmonic submanifolds in spheres**

Polyharmonic submanifolds are a natural higher-order generalization of minimal surfaces. We provide a new construction method for polyharmonic submanifolds in spheres, obtaining a rich family of new examples. Moreover, we completely determine the index and nullity of these solutions in the biharmonic case. This is a joint work with Anna Siffert.

ANNEGRET BURTSCHER

TOBIAS DOTT

#### **Gromov-Hausdorff limits of closed surfaces**

We completely describe the Gromov-Hausdorff closure of the class of length spaces being homeomorphic to a fixed closed surface.

NADINE GROSSE

#### **On local boundary conditions for Dirac-type operators**

We discuss smooth local boundary conditions for Dirac-type operators, giving existence and non-existence results for local symmetric boundary conditions. We also give conditions when the boundary conditions are elliptic/regular/Shapiro-Lopatinski (i.e. in particular giving rise to self-adjoint Dirac operators with domain in  $H^1$ ). This is joint work with Hanne van den Bosch (Universidad de Chile) and Alejandro Uribe (University of Michigan).

JONAS HENKEL

#### **The Laplace-Beltrami spectrum on homogeneous spaces**

YINGXIANG HU

### **Curvature flows and their applications to geometric inequalities**

In this talk, we will first review the isoperimetric type inequalities, in particular, the Alexandrov-Fenchel type inequalities in Riemannian space forms. Then we will talk about the curvature flows with capillary boundary and their applications to isoperimetric type inequalities.

MOHAMMAD N. IVAKI

CHRISTIAN KETTERER

### **Stability of a Scalar curvature rigidity theorem for Riemannian tori**

One goal of this talk is to establish a nonlinear analogue of a splitting map to Euclidean space, as a harmonic map to a flat torus. Existence of such a map implies Gromov-Hausdorff closeness to a flat torus in any dimension. Furthermore, Gromov-Hausdorff closeness to a flat torus and a curvature bound imply the existence of a harmonic splitting map. Combining these results with Stern's inequality yields a new Gromov-Hausdorff stability theorem for Riemannian 3-tori under almost non-negative Scalar curvature. This is a joint work with Shouhei Honda, Ilaria Mondello, Raquel Perales and Chiara Rigoni.

TOBIAS KÖNIG

### **Quantitative stability of the total $Q$ -curvature near minimizing metrics**

Let  $(M, g)$  be a Riemannian manifold of dimension  $n$ , and  $1 \leq k < \frac{n}{2}$  an integer. We prove quantitative estimates for the total  $k$ -th order  $Q$ -curvature functional near minimizing metrics. More precisely, we show that in a generic closed Riemannian manifold, the distance to the minimizing set of metrics is controlled quadratically by the  $Q$ -curvature energy deficit. This extends recent work for  $k = 1$  by Engelstein, Neumayer and Spolaor (2021).

We also describe several instances of degenerate (or higher-order) stability. For instance, extending a classical example due to Schoen (1989), for every integer  $1 \leq k < \frac{n}{2}$  we exhibit an  $n$ -dimensional Riemannian manifold such that the distance to the minimizing set of metrics is controlled by a higher power of the  $Q$ -curvature energy deficit.

This is work in progress, joint with Joao Henrique Andrade (Sao Paulo), Jesse Ratzkin (Würzburg) and Juncheng Wei (Hong Kong).

ROB KUSNER

### **On Eigenspaces and Minimal Surfaces**

Sharp dimension bounds for (Steklov and Laplacian) eigenspaces on Riemannian surfaces are used to study existence and classification problems for embedded minimal surfaces in the round ball  $B^n$  (with free boundary) and sphere  $\mathbb{S}^n$  (compact without boundary). For instance, we prove the uniqueness of the critical catenoid in  $B^3$  and the critical Möbius band in  $B^4$  under the modest symmetry assumptions. Our main result constructs orientable free boundary minimal surfaces

embedded in  $B^3$  of every topological type, whose analog for closed minimal surfaces in  $\mathbb{S}^3$  was settled by Lawson in 1970. Our construction is based on equivariant optimization of Steklov or Laplace eigenvalues on surfaces, normalized by their boundary length or surface area, respectively. Our free boundary surfaces in  $B^n$  have area below  $2\pi$ , and if we fix the number of boundary components, the number of such surfaces grows at least linearly with respect to their genus; we prove analogous results for the number of embedded minimal surfaces in  $\mathbb{S}^n$  with area below  $8\pi$ . Geometrically, our surfaces are “doublings” of the equatorial  $B^2$  and  $\mathbb{S}^2$  respectively, in a precise sense to be described in the talk. Though we focus on the orientable case, if time permits we may also discuss the situation for nonorientable surfaces minimally embedded in  $B^4$  and  $\mathbb{S}^4$  (joint work with Peter McGrath and also with Misha Karpukhin, Peter McGrath, and Daniel Stern).

ELENA MÄDER-BAUMDICKER

ARTEM NEPECHIY

### **Locally homogeneous RCD spaces**

The goal of this talk is to demonstrate how existing results can be adapted to establish the following result: A locally metric measure homogeneous  $\text{RCD}(K, N)$  space is isometric to, after multiplying a positive constant to the reference measure, a smooth Riemannian manifold with the Riemannian volume measure.