Chapter 1 Problem 3

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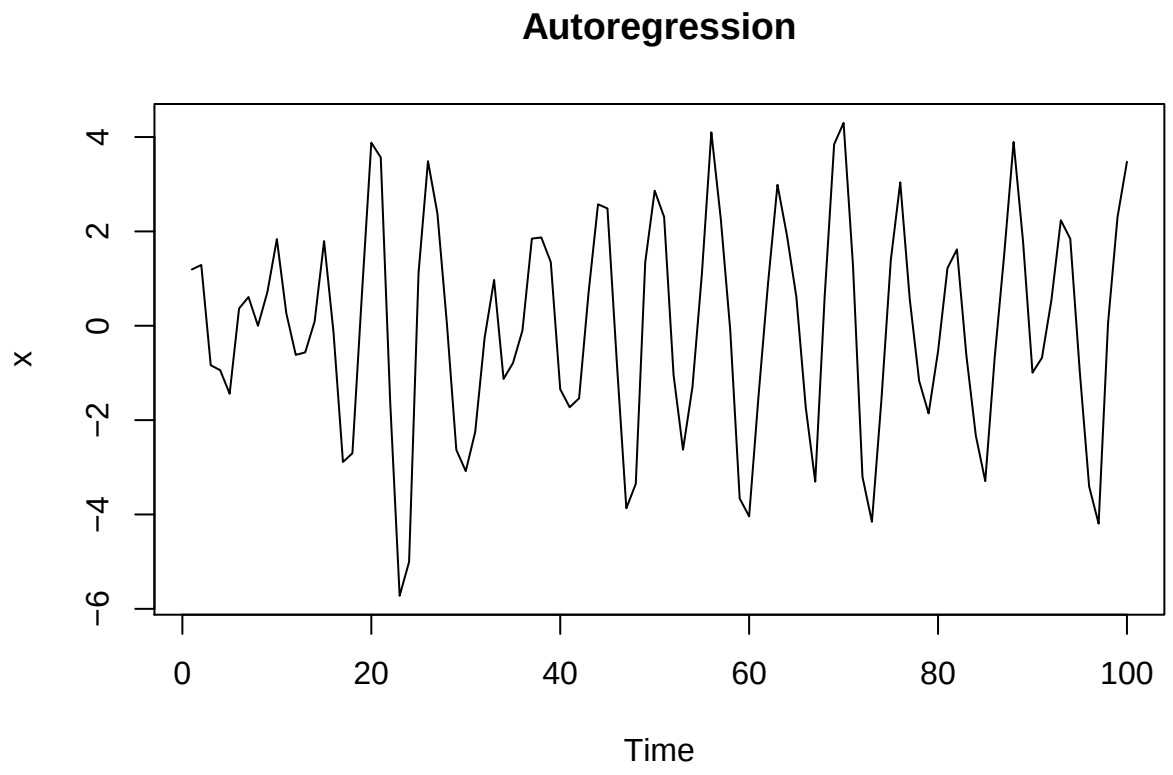
This problem studies white noise, moving average filters, and autoregression.

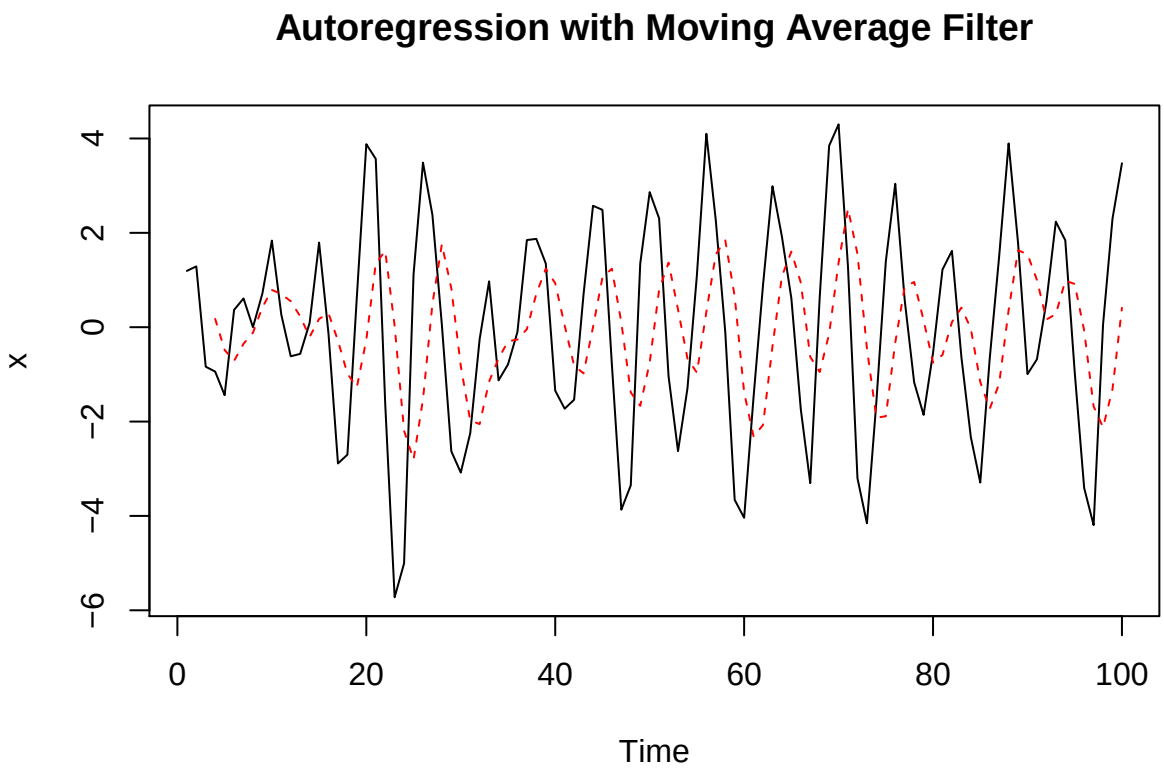
(a) Generate n = 100 observations from the autoregression $x_t = -.9x_t - 2 + w_t$ with $\sigma_w = 1$, using the method described in Example 1.10, page 13. Next, apply the moving average filter $v_t = (x_t + x_{t-1} + x_{t-2} + x_{t-3})/4$ to x_t, the data you generated. Now, plot x_t as a line and superimpose v_t as a dashed line. Comment on the behavior of x_t and how applying the moving average filter changes that behavior.

```
set.seed(89)
w=rnorm(150) #50 extra to avoid startup problems
x=filter(w,filter=c(1,-0.9),method="recursive")[-(1:50)]
plot.ts(x,main="Autoregression")
```



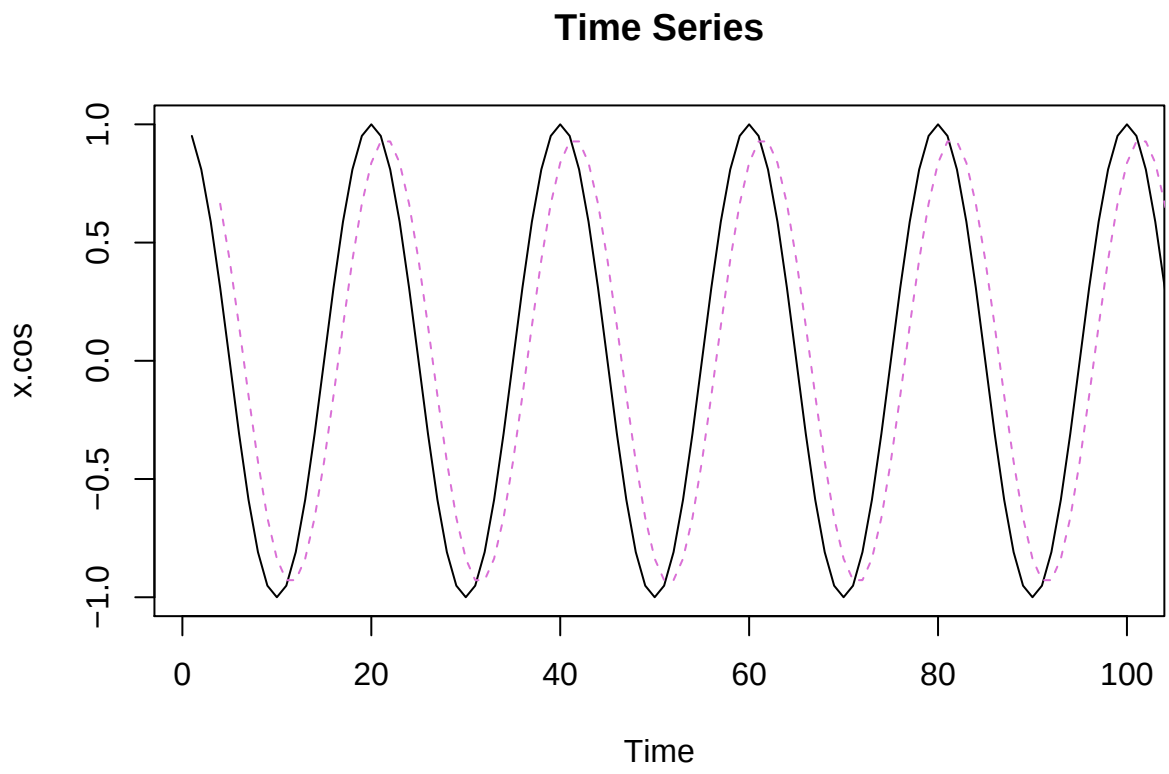
Now, we implement the moving average filter and superimpose it on the autoregression graph.

```
v=filter(x,sides=1,rep(1/4,4))
#sides=1 b/c we are only looking at past values
#rep(1/4,4) b/c we are averaging with 4 points
plot.ts(x,main="Autoregression with Moving Average Filter")
lines(v,col="red",lty="dashed")
```



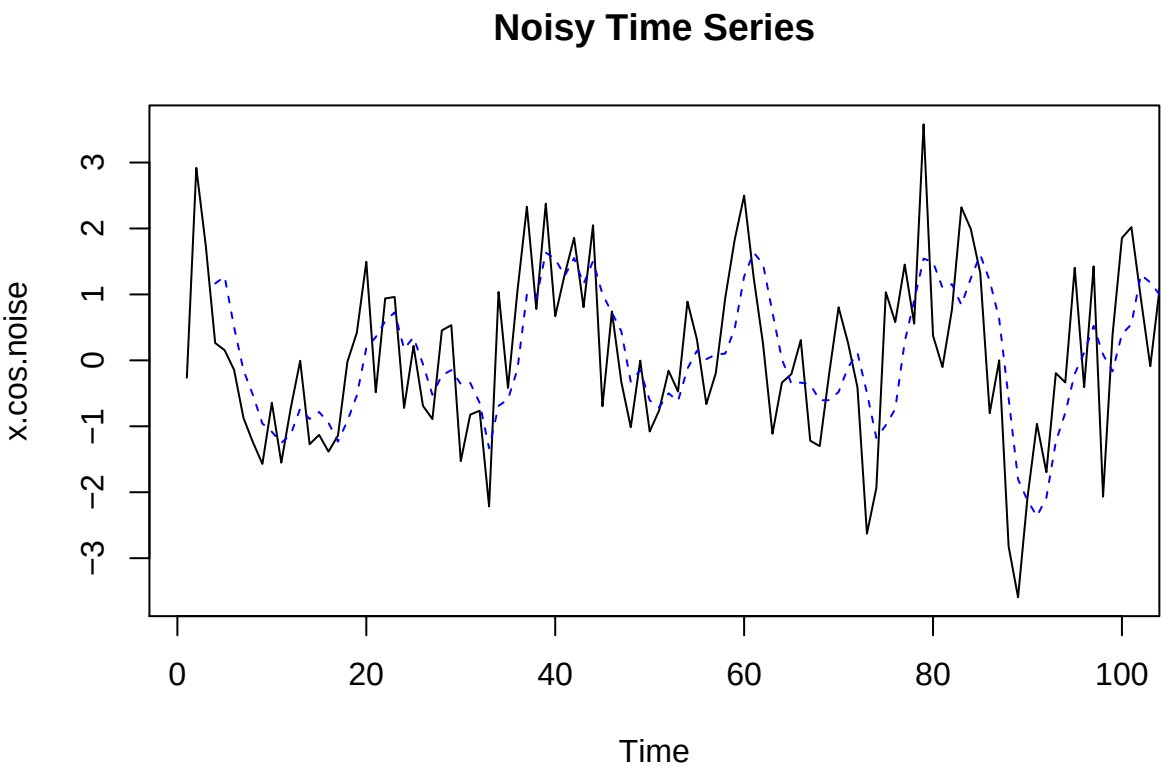
(b) Repeat but with $x_t = cos(2\pi t/4)$.

```
x.cos=ts(cos(2*pi*1:150*0.2/4))
v.cos=filter(x.cos,sides=1,rep(1/4,4))
plot.ts(x.cos,main="Time Series",xlim=c(1,100))
lines(v.cos,col="orchid",lty="dashed")
```



(c) Repeat but with added N(0,1) white noise; $x_t = cos(2\pi t/4) + w_t$.

```
noise=rnorm(150,0,1)
x.cos.noise=ts(cos(2*pi*1:150*0.2/4)+noise)
v.cos.noise=filter(x.cos.noise,sides=1,rep(1/4,4))
plot.ts(x.cos.noise,main="Noisy Time Series",xlim=c(1,100))
lines(v.cos.noise,col="blue",lty="dashed")
```



(d) Compare and contrast the models above.

When we move from part (a) to (b), we see that adding the moving average filter smoothens the time series, meaning there's not as many random fluctuations and we can more easily see a trend. We also see from parts (c) and (d) that white noise significantly roughens the time series.