

Chapter 1 Problem 2

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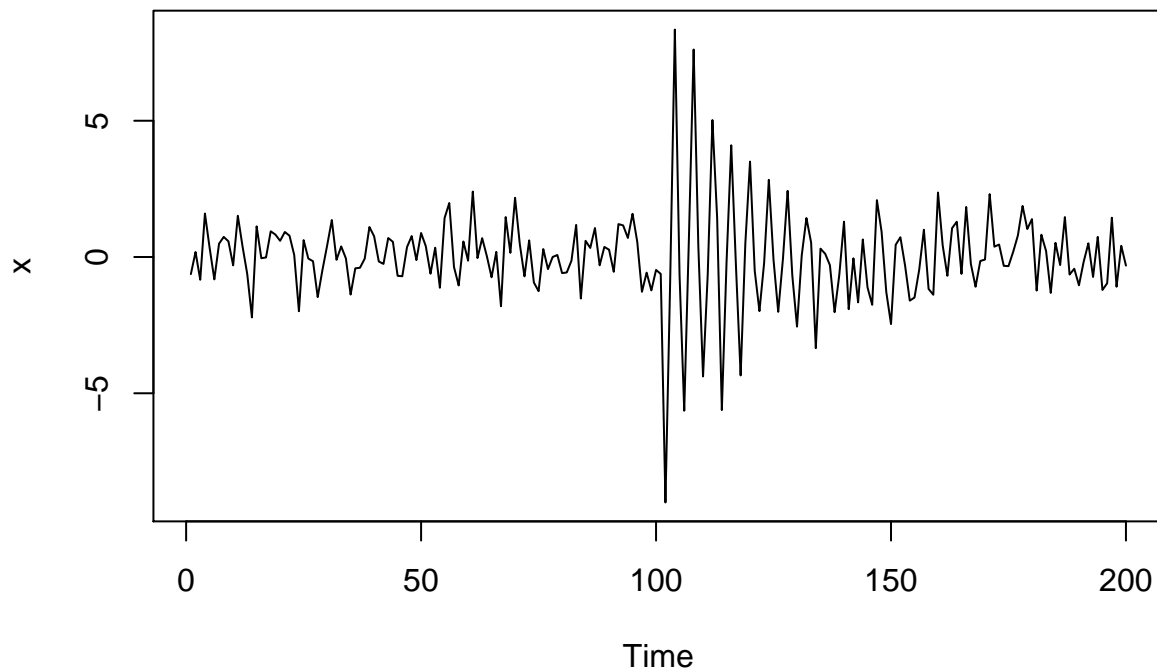
Consider a signal-plus-noise model of the general form $x_t = s_t + w_t$, where w_t is Gaussian white noise with $\sigma_w^2 = 1$. Simulate and plot $n=200$ observations from two models. Compare the general appearance of the series (a) and (b) with the earthquake series and the explosion series shown in Figure 1.7. In addition, plot (or sketch) and compare the signal modulators (a) $\exp\{-t/20\}$ and (b) $\exp\{-t/200\}$, for $t = 1, 2, \dots, 100$.

a.)

$$s_t = 0; ; t = 1, \dots, 100$$

$$s_t = 10e^{\frac{(100t)}{20}} \cos\left(\frac{2\pi t}{4}\right); ; t = 101, \dots, 200$$

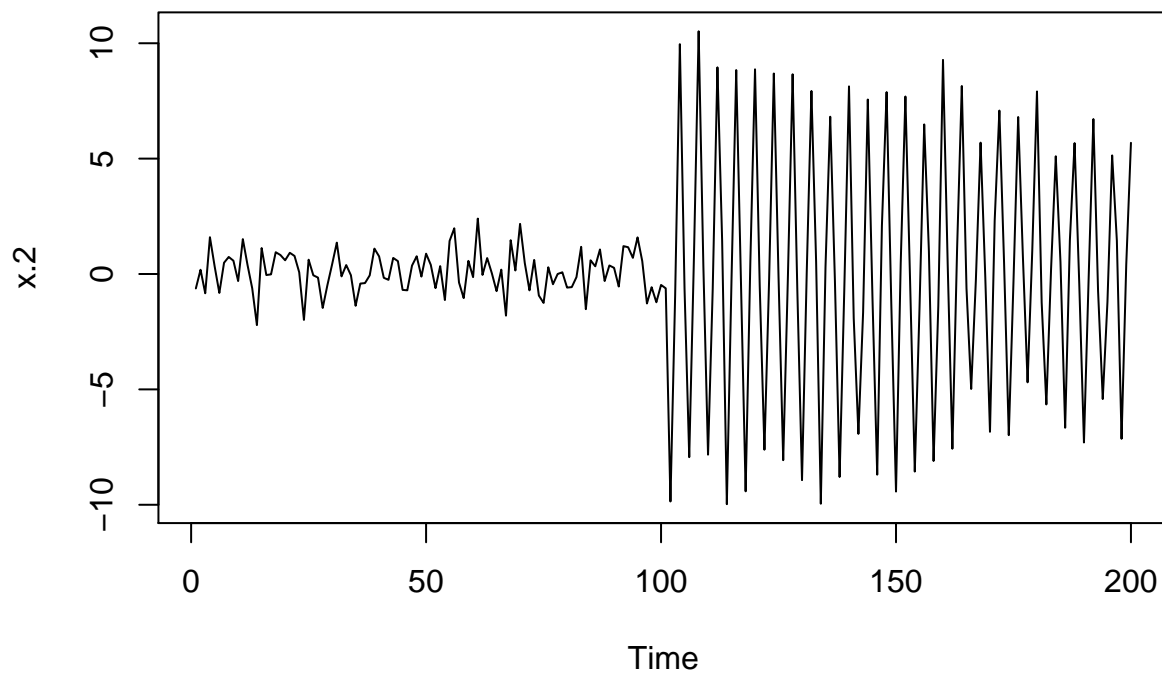
```
setwd("~/Time Series Analysis and Its Applications")
set.seed(1)
s=c(rep(0,100),10*exp(-(1:100)/20)*cos(2*pi*1:100/4))
x=ts(s+norm(200,0,1))
plot(x)
```



b.)

$$s_t = 0; ; t = 1, \dots, 100$$
$$s_t = 10e^{\frac{(100t)}{200}} \cos\left(\frac{2\pi t}{4}\right); ; t = 101, \dots, 200$$

```
set.seed(1)
s.2=c(rep(0,100),10*exp(-(1:100)/200)*cos(2*pi*1:100/4))
x.2=ts(s.2+rnorm(200,0,1))
plot(x.2)
```



We compare the graphs above to Figure 1.7, shown below:

We look at how the oscillation damps (decreases in amplitude while remaining sinusoidal) in our two models above and in the earthquake/explosion plots. The model in (a) resembles the explosion time series plot in that they both have a burst and quickly dampens afterwards. Model (b) resembles the earthquake time series in that after a burst, the oscillations do not dampen.

```
t=rnorm(1000,0,1)
y1=exp(-t/20)
y2=exp(-t/200)
plot(t,y1,xlim=c(-5,5),ylim=c(0,2))
```

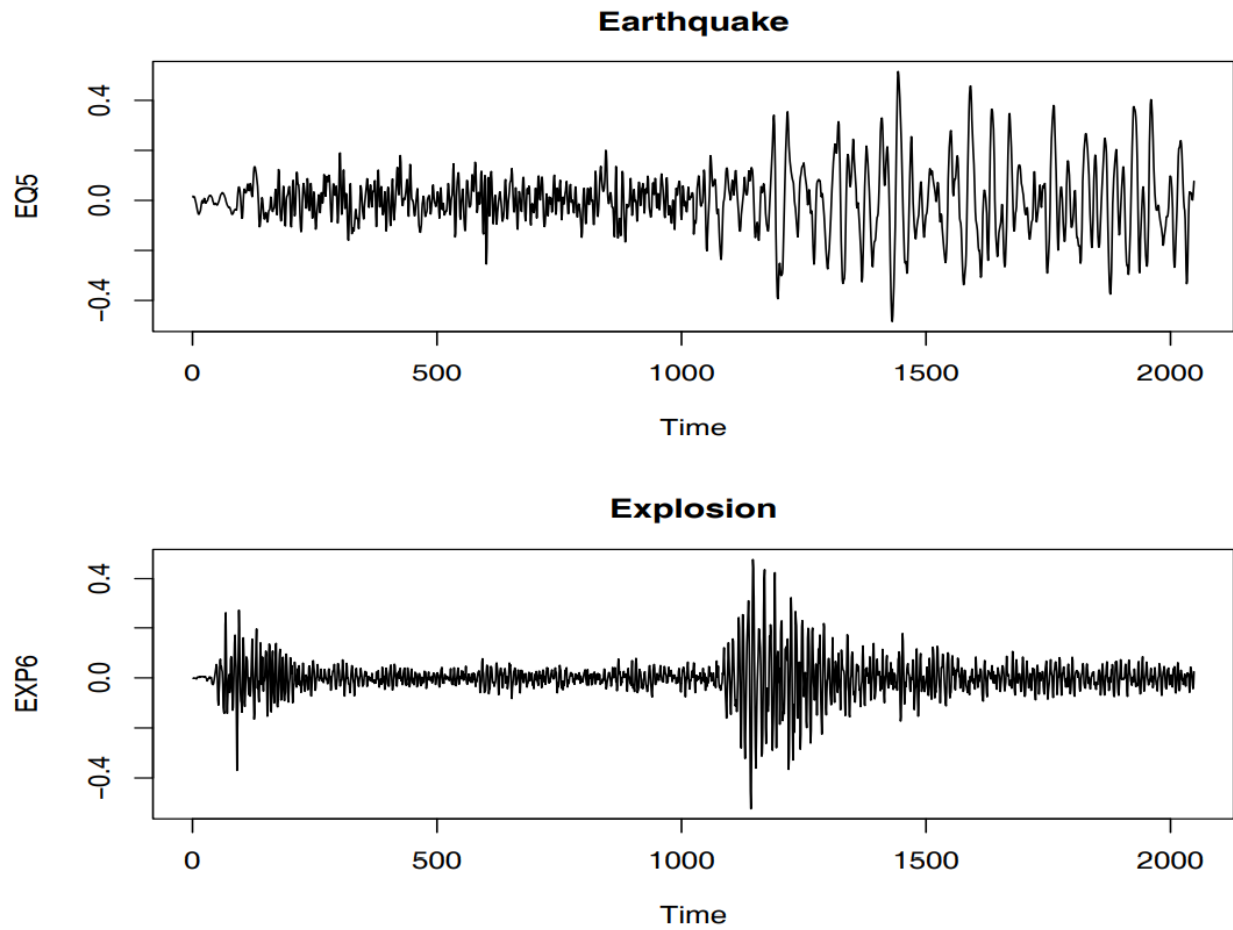
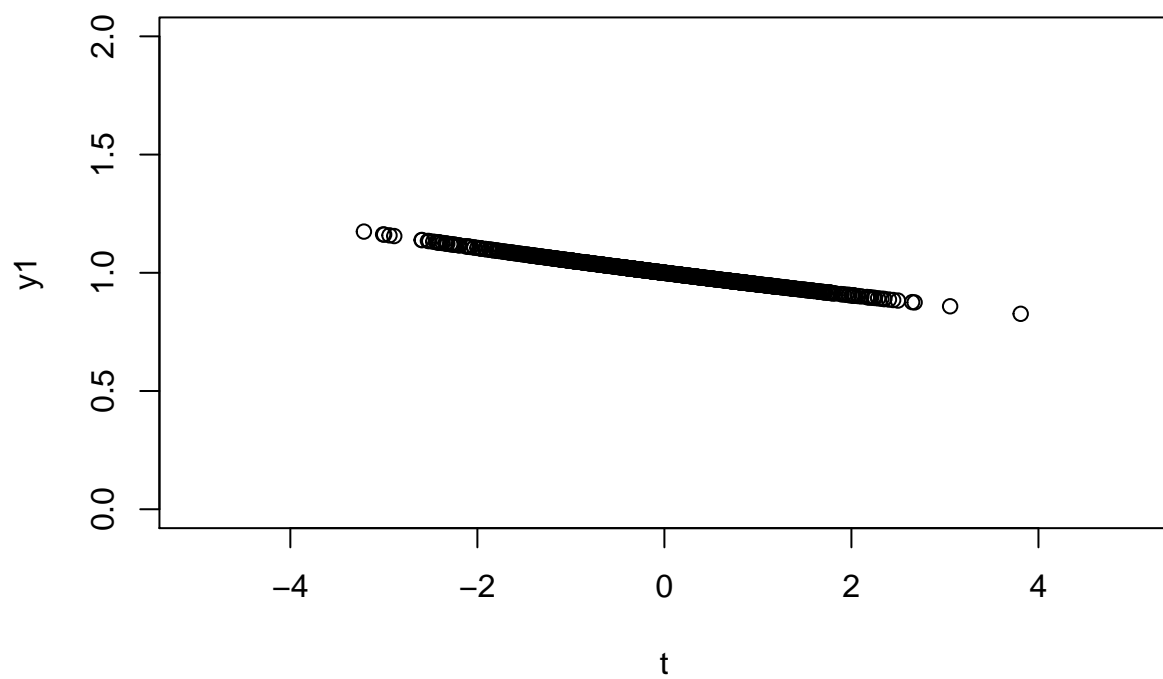
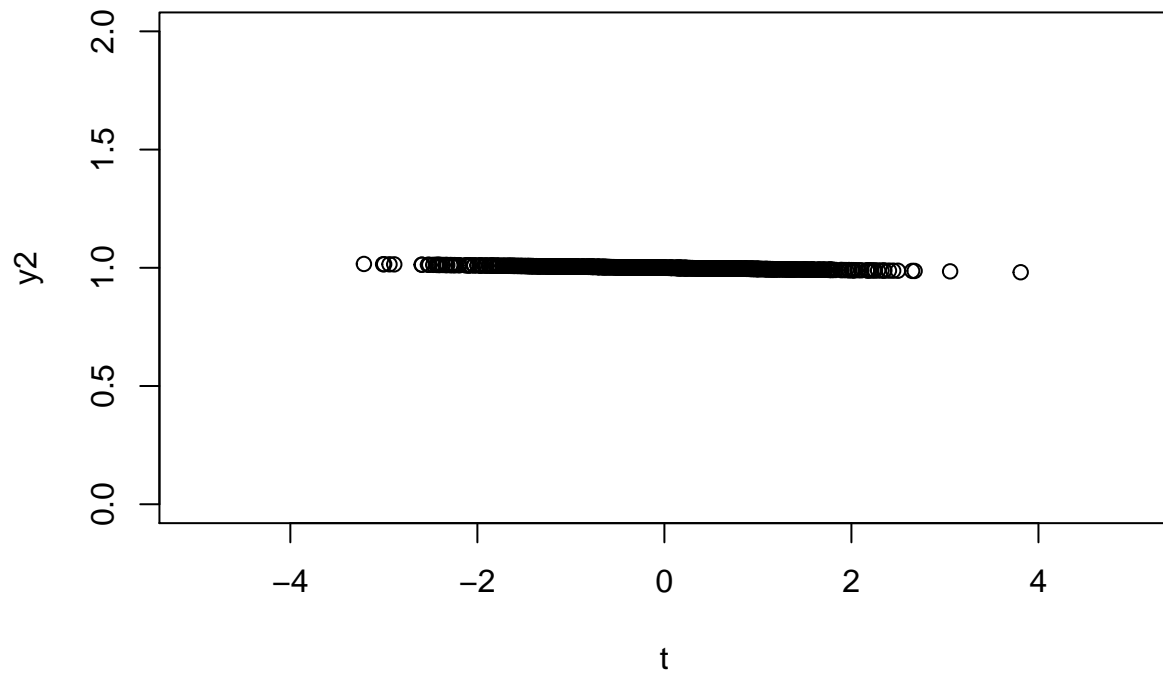


Figure 1: Arrival phases from an earthquake (top) and explosion (bottom) at 40 points per second.



```
plot(t,y2,xlim=c(-5,5),ylim=c(0,2))
```



We see that the signal modulator $\exp(-t/20)$ decreases faster than $\exp(-t/200)$, which could explain the faster damping rate of model (a) than model (b).