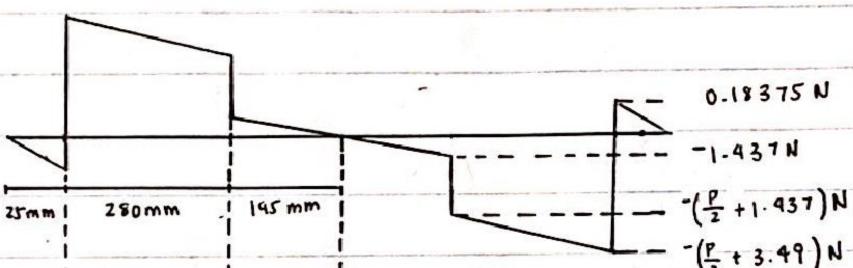
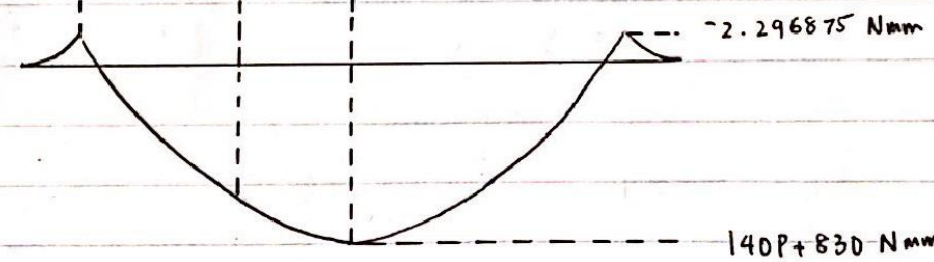
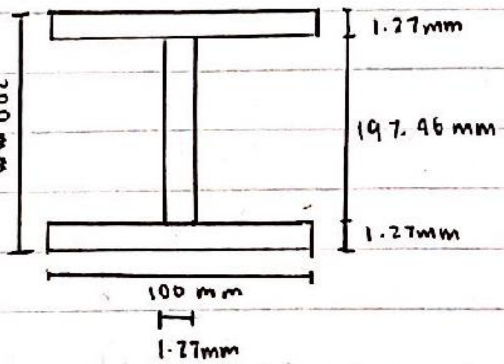
SFD:



BMD:



I Beam (One Layer)



Find I:

$\bar{y} = \frac{200}{2} = 100$ mm (symmetrical I beam)

$I = \sum I_{icm} + \sum A_i d_i^2$

$\Rightarrow I = 3.3226 \times 10^6$ mm^4

Flexural Tension:

$\sigma_t = 30$ MPa, $\sigma_t = \frac{My}{I} \Rightarrow M = \frac{\sigma_t I}{y} = 0.99678 \times 10^6$ N mm $= 0.99678 \times 10^3$ Nm

$M_{max} = (0.140P + 0.830) = 0.99678 \times 10^3$ Nm $\Rightarrow P = 7114$ N

Flexural Compression:

$\sigma_c = 6$ MPa, $\sigma_c = \frac{M(h-y)}{I} \Rightarrow M = \frac{\sigma_c I}{h-y} = 0.199356 \times 10^3$ Nm

$M_{max} = (0.140P + 0.830) = 0.199356 \times 10^3$ Nm $\Rightarrow P = 1418$ N

Shear Stress:

$Q = \sum A_i d_i = 18809.9$ mm^3, $\tau = 4$ MPa, $V = \frac{\tau I b}{Q}$, $b = 1.27$mm

$\Rightarrow V = 897.375$ N, $V = 0.5P \Rightarrow P = 1795$ N

Glue Failure:

$Q = \sum A_i d_i = 12538.71$ mm^3, $\tau = 2$MPa, $V = \frac{\tau I b}{Q}$, $b = 1.27$mm

$\Rightarrow V = 673.06$ N, $V = 0.5P \Rightarrow P = 1346$ N

Thin Wall Buckling - Flange Restrained

There is no restrained section on an I beam $\Rightarrow$ N/A

Flange Unrestrained

$$\sigma_c = \frac{0.425\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{99.365}\right)^2 = 0.23792 \text{ MPa}$$

$$0.140P + 0.830 = \frac{0.23792(3.3226\times10^6)}{(200-100)\times10^3} \Rightarrow P = \boxed{50.5N}$$

Web Flexural Tension

$$\frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{100}\right)^2 = 3.3164 \text{ MPa}$$

$$0.140P + 0.830 = \frac{3.3164(3.3226\times10^6)}{100\times10^3} \Rightarrow P = \boxed{781\ N}$$

Web Flexural Compression

$$\frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{100}\right)^2 = 3.3164 \text{ MPa}$$

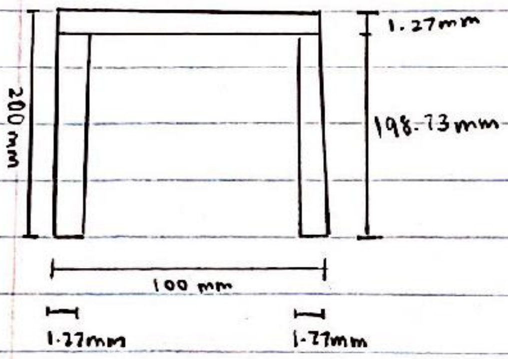
$$0.140P + 0.830 = \frac{3.3164(3.3226\times10^6)}{(200-100)\times10^3} \Rightarrow P = \boxed{781\ N}$$

Web Shear Buckling

$$\frac{5\pi^2 E}{12(1-\mu^2)}\left[\left(\frac{t}{h}\right)^2+\left(\frac{t}{a}\right)^2\right] = \frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\frac{1.27}{200}\right)^2+(0)^2\right] = 0.6909 \text{ MPa}$$

$$V = \frac{0.6909(3.3226\times10^6)(1.27)}{18809.9} = 155N,\ V = 0.5P \Rightarrow P = \boxed{310\ N}$$

Pi Beam (One Layer)



Find I:

$$\bar{y} = \frac{\Sigma A_i d_i}{\Sigma A_i} = 119.467 \text{ mm}$$

$$I = \Sigma I_{icm} + \Sigma A_i d_i^2$$

$$\Rightarrow I = 2.6759\times10^6 \text{ mm}^4$$

Flexural Tension:

$\sigma_t = 30$ MPa, $M = \frac{\sigma_t I}{y} = 0.67196\times10^3$ N·m

$M_{max} = (0.140P + 0.830) = 0.67196\times10^3$ Nm $\Rightarrow P = 4794$ N

Flexural Compression:

$\sigma_c = 6$ MPa, $M = \frac{\sigma_c I}{h-y} = 0.199365\times10^3$ N·m

$M_{max} = (0.140P + 0.830) = 0.199365\times10^3$ Nm $\Rightarrow P = 1418$ N

Shear Stress:

$Q = \Sigma A_i d_i = 18125.98 \text{ mm}^3$, $V = \frac{\tau I b}{Q}$, $\tau = 4$ MPa, $b = 2.54$ mm

$\Rightarrow V = 1500$ N, $V = 0.5P \Rightarrow P = 3000$N

Glue Failure:

$Q = \Sigma A_i d_i = 10147.046 \text{ mm}^3$, $V = \frac{\tau I b}{Q}$, $\tau = 2$ MPa, $b = 2.54$mm

$\Rightarrow V = 1339.7$ N, $V = 0.5P \Rightarrow P = 2679$ N

Thin Wall Buckling - Flange Restrained

$$\frac{4\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \left(\frac{4\pi^2\cdot 4000}{12(1-0.2^2)}\right)\left(\frac{1.27}{100-2.54}\right)^2 = 2.32767 \text{ MPa}$$

$$0.140P + 0.830 = \frac{2.32767(2.6759\times10^6)}{10^3(200-119.467)} \Rightarrow P = \boxed{547\ N}$$

Flange Unrestrained

$$\frac{0.425\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \frac{0.425\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{1.27}\right)^2 = 1456.45 \text{ MPa}$$

$$0.140P + 0.830 = \frac{1456.45(2.6759\times10^6)}{(200-119.467)10^3} \Rightarrow P = 345679 \text{ N}$$

Webs Flexural Tension

$$\frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{119.467}\right)^2 = 2.32365 \text{ MPa}$$

$$0.14P + 0.830 = \frac{2.32365(2.6759\times10^6)}{119.467\times10^3} \Rightarrow P = \boxed{366 \text{ N}}$$

Webs Flexural Compression

$$\frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{79.263}\right)^2 = 5.278675 \text{ MPa}$$

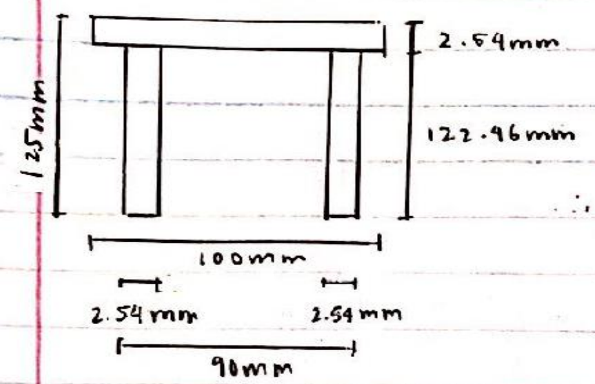
$$0.140P + 0.830 = \frac{5.278675(2.6759\times10^6)}{(200-119.467)\times10^3} \Rightarrow P = 1247 \text{ N}$$

Webs Shear Buckling

$$\frac{5\pi^2 E}{12(1-\mu^2)}\left[\left(\frac{t}{h}\right)^2 + \left(\frac{t}{a}\right)^2\right] = \frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\frac{1.27}{200}\right)^2 + \left(0\right)^2\right] = 0.690915 \text{ MPa}$$

$$V = \frac{0.690915(2.6759\times10^6)(2.54)}{18125.98} = 259 \text{ N}, \quad V = 0.5P \Rightarrow P = \boxed{518 \text{ N}}$$

Pi Beam (Doubling Leg Thickness, Minimizing Height)



Find I:

$$\bar{y} = \frac{\Sigma A_i d_i}{\Sigma A_i} = 79.35 \text{ mm}$$

$$I = \Sigma I_{i,cm} + \Sigma A_i d_i$$
$$= 1.482104\times10^6 \text{ mm}^4$$

Flexural Tension:

$\sigma_t = 30$ MPa, $M = \frac{\sigma_t I}{y} = 0.560342\times10^3$ Nm $= 0.14P + 0.830 \Rightarrow P = 3997$ N

Flexural Compression:

$\sigma_c = 6$ MPa, $M = \frac{\sigma_c I}{h-y} = 0.194800\times10^3$ Nm $= 0.14P + 0.830 \Rightarrow P = 1386$ N

Shear Stress:

$Q = \Sigma A_i d_i = 15993 \text{ mm}^3$, $\tau = 4$ MPa, $V = \frac{\tau I b}{Q}$, $b = 5.08$ mm

$\Rightarrow V = 1883.1$ N, $V = 0.5P \Rightarrow P = 3766$ N

Glue Failure:

$Q = \Sigma A_i d_i = 11272.52 \text{ mm}^3$, $\tau = 2$ MPa; $V = \frac{\tau I b}{Q}$, $b = 5.08$ mm

$\Rightarrow V = 1335.8$ N, $V = 0.5P \Rightarrow P = 2672$ N

Thin Wall Buckling - Flange Restrained

$$\frac{4\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \frac{4\pi^2(4000)}{12(1-0.2^2)}\left(\frac{2.54}{84.92}\right)^2 = 11.441 \text{ MPa}$$

$$0.140P + 0.830 = \frac{11.441(1.482104\times10^6)}{(125-79.35)\times10^3} \Rightarrow P = 2838 \text{ N}$$

Flange Unrestrained

$$\frac{0.425\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \frac{0.425\pi^2(4000)}{12(1-0.2^2)}\left(\frac{2.54}{7.54}\right)^2 = 165.28\text{MPa}$$

$$0.140P + 0.830 = \frac{165.28(1.482104\times10^6)}{(125-79.35)\times10^3} \Rightarrow P = 38323\text{N}$$

Webs Flexural Tension

$$\frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{2.54}{79.35}\right)^2 = 21.068\text{ MPa}$$

$$0.140P + 0.830 = \frac{21.068(1.482104\times10^6)}{79.35\times10^3} \Rightarrow P = 2805\text{N}$$

Webs Flexural Compression

$$\frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{2.54}{43.11}\right)^2 = 71.379\text{ MPa}$$

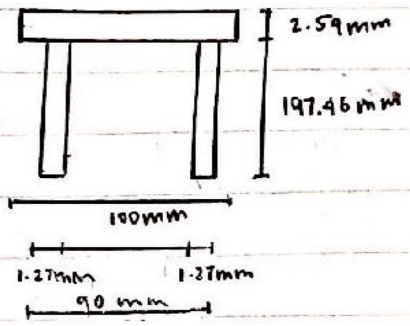
$$0.140P + 0.830 = \frac{71.379(1.482104\times10^6)}{(125-79.35)\times10^3} \Rightarrow P = 16547\text{ N}$$

Webs Shear Buckling

$$\frac{5\pi^2 E}{12(1-\mu^2)}\left[\left(\frac{t}{h}\right)^2 + \left(\frac{t}{a}\right)^2\right] = \frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\frac{2.54}{125}\right)^2 + (\ 0\ )^2\right] = 7.075\text{ MPa}$$

$$V = \frac{7.075(1.482104\times10^6)(5.08)}{15993} = 3330.7\text{ N},\quad V = 0.5P \Rightarrow P = 6661\text{N}$$

Pi Beam (Double Top Flange and Move Legs Closer)



Find I:

$$\bar{y} = \frac{\Sigma A_i d_i}{\Sigma A_i} = 132.348\text{mm}$$

$$I = \Sigma I_{icm} + \Sigma A_i d_i^2 = 3.315873\times10^6\text{ mm}^4$$

Flexural Tension:

$\sigma_t = 30\text{MPa}$, $M = \frac{\sigma_t I}{y} = 0.751626\times10^3\text{ N}\cdot\text{m} = 0.14P + 0.830 \Rightarrow P = 5363\text{ N}$

Flexural Compression:

$\sigma_c = 6\text{ MPa}$, $M = \frac{\sigma_c I}{h-y} = 0.294082\times10^3\text{ Nm} = 0.14P + 0.830 \Rightarrow P = 2095\text{ N}$

Shear Stress:

$Q = \Sigma A_i d_i = 22246\text{ mm}^3$, $\tau = 4\text{MPa}$, $V = \frac{\tau I b}{Q}$, $b = 2.54\text{mm}$

$\Rightarrow V = 1514.4\text{ N}$, $V = 0.5P \Rightarrow P = 3029\text{N}$

Glue Failure:

$Q = \Sigma A_i d_i = 16861\text{ mm}^3$, $\tau = 2\text{MPa}$, $V = \frac{\tau I b}{Q}$, $b = 2.54\text{mm}$

$\Rightarrow V = 999\text{ N}$, $V = 0.5P \Rightarrow P = 1998\text{N}$

Thin Wall Buckling - Flange Restrained

$$\frac{4\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \frac{4\pi^2(4000)}{12(1-0.2^2)}\left(\frac{2.54}{87.46}\right)^2 = 11.5615\text{ MPa}$$

$$0.140P + 0.830 = \frac{11.5615(3.315873\times10^6)}{(200-132.348)\times10^3} \Rightarrow P = 4042\text{ N}$$

Flange Unrestrained

$$\frac{0.425\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \frac{0.425\pi^2(4000)}{12(1-0.2^2)}\left(\frac{2.54}{6.27}\right)^2 = 239.017 \text{ MPa}$$

$$0.140P + 0.830 = \frac{239.017(3.315873\times10^6)}{(200-132.348)\times10^3} \Rightarrow P = 83673 \text{ N}$$

Webs Flexural Tension

$$\frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{132.348}\right)^2 = 1.89335 \text{ MPa}$$

$$0.140P + 0.830 = \frac{1.89335(3.315873\times10^6)}{132.348\times10^3} \Rightarrow P = \boxed{333 \text{ N}}$$

Webs Flexural Compression

$$\frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{65.112}\right)^2 = 7.82247 \text{ MPa}$$

$$0.140P + 0.830 = \frac{7.82247(3.315873\times10^6)}{(200-132.348)\times10^3} \Rightarrow P = 2733 \text{ N}$$

Webs Shear Buckling

$$\frac{5\pi^2 E}{12(1-\mu^2)}\left[\left(\frac{t}{h}\right)^2+\left(\frac{t}{a}\right)^2\right] = \frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\frac{1.27}{200}\right)^2+\left(0\right)^2\right] = 0.690915 \text{ MPa}$$

$$V = \frac{0.690915(3.315873\times10^6)(2.54)}{22246} = 261.58\text{N}, \quad V = 0.5P \Rightarrow P = \boxed{523\text{N}}$$

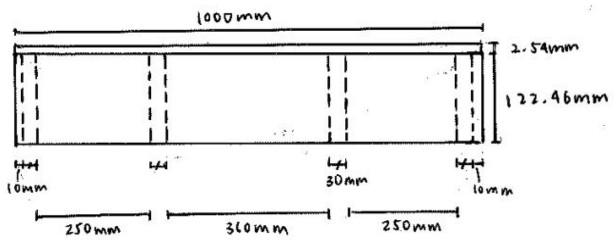
Pi Beam (Adding Diaphragms for Support)

Webs Shear Buckling.

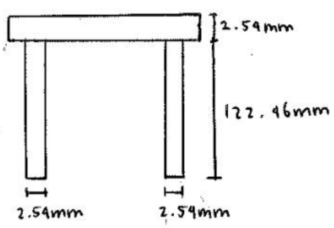
$$\frac{5\pi^2 E}{12(1-\mu^2)}\left[\left(\frac{t}{h}\right)^2+\left(\frac{t}{a}\right)^2\right] = \frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\frac{2.54}{125}\right)^2+\left(\frac{2.54}{360}\right)^2\right] = 7.92795 \text{ MPa}$$

$$V = \frac{7.92795(1.482109\times10^6)(5.08)}{15993} = 3732.3\text{ N}, \quad V = 0.5P \Rightarrow P = 7465 \text{ N}$$

Finding Matboard Used (Concept I):



Cross Section:



Webs:

4 panels: 122.46mm x 1000mm

Top flange:

2 panels: 100mm x 1000mm

Diaphragms:

8 panels: 122.46mm x 100mm

Total area of matboard used:

$4(122.46\times1000) + 2(100\times1000) + 8(122.46\times100)$

$= 787808 \text{ mm}^2$

Strength to Weight Ratio:

$$\text{Weight} = \frac{757808\,mm^2}{826008\,mm^2} \times 750g = 715.3g$$

Failure Load = 1386 N (Flexural Compression)

$\Rightarrow$ Strength to Weight Ratio:

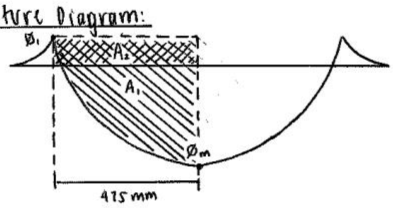
$$\frac{1386N}{715.3g} = 1.938\ N/g$$

End of Design 1

Concept II - Finding the SFD and BMD

Refer to SFD and BMD for concept I to sketch the curvature diagram for the beam with only supports (using the one layer pi beam cross section):

Curvature Diagram:



$$\varnothing_m = \frac{M_m}{EI}$$

$$\Rightarrow \varnothing_1 = \frac{-2.297}{4000(1482103)} = -3.875 \times 10^{-10}$$

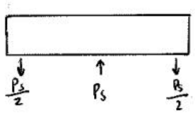
$$\Rightarrow \varnothing_m = \frac{140P + 830.02}{4000(1482103)} = 2.36151 \times 10^{-8} P + 1.4 \times 10^{-7}$$

$$A_1 = (2.36151 \times 10^{-8} P + 1.40387 \times 10^{-7})(475)(2/3)$$
$$= (7.478115 \times 10^{-6} P + 4.4456 \times 10^{-5})\ rad$$
$$d_1 = \tfrac{5}{8}(475) = 296.875\,mm$$
$$A_2 = (-3.875 \times 10^{-10})(475) = 1.840625 \times 10^{-7}\ rad$$
$$d_2 = 237.5\ mm$$

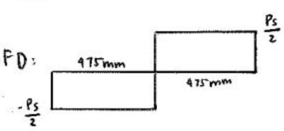
$$\Delta = A_1 d_1 + A_2 d_2$$

$$\therefore \Delta = -4.37148 \times 10^{-5} + 2.22 \times 10^{-3} P + 1.31978 \times 10^{-2}$$
$$\Delta = (2.22 \times 10^{-3} P + 0.013154)\ mm$$

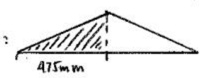
Curvature Diagram for only two loads:



$\Rightarrow$ SFD:

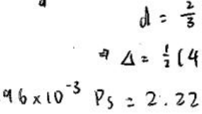


BMD:



$$M_{max} = 475\left(\frac{P_s}{2}\right) = 237.5 P_s$$

$\Rightarrow \varnothing$:



$$\varnothing_{max} = \frac{237.5 P_s}{4000(1482103)} = 4.00613 \times 10^{-8} P_s\ rad/mm$$

$$d = \tfrac{2}{3}(475) = 316.67$$

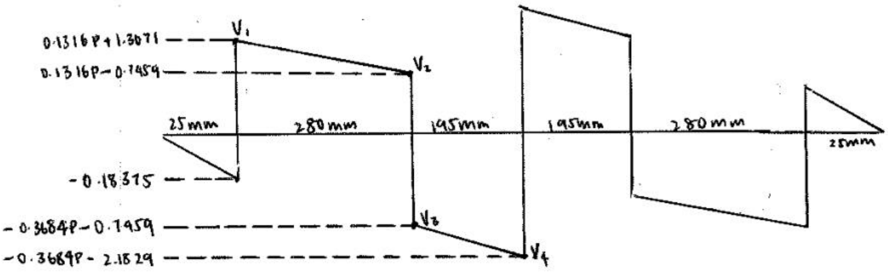
$$\Rightarrow \Delta = \tfrac{1}{2}(475)(4.00613 \times 10^{-8} P_s)(316.67) = 3.01296 \times 10^{-3}\ P_s$$

$$\therefore 3.01296 \times 10^{-3} P_s = 2.22 \times 10^{-3} P + 0.013154$$

$$\Rightarrow P_s = (0.7368 P + 4.3658)\ N$$

Combining Calculations from Concept I and Supports:

SFD (In Newtons):



$V_1 = 0.5P + 3.49 - \frac{1}{2}(0.7368P + 4.3658) = (0.1316P + 1.3071)\,N$

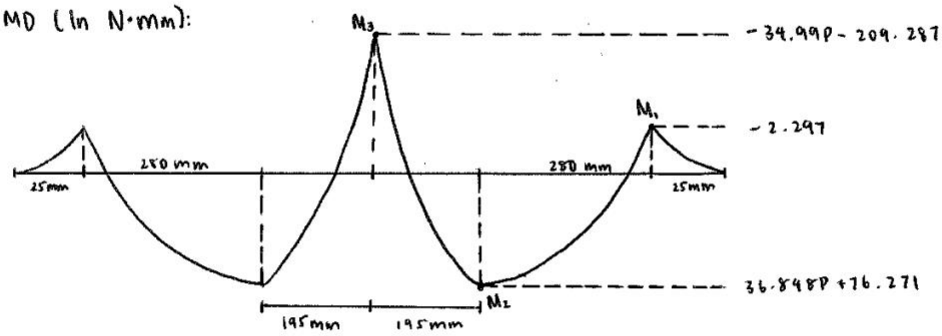
$V_2 = V_1 - 2.053 = (0.1316P - 0.7459)\,N$

$V_3 = V_2 - 0.5P = (-0.3684P - 0.7459)\,N$

$V_4 = V_3 - 1.437 = (-0.3684P - 2.1829)\,N$

$V_{max} = (0.3684P + 2.1829)\,N$

BMD (In N·mm):



$M_1 = 25 \times (-0.18375) = -2.297\ Nmm$

$M_2 = 280(0.1316P - 0.7459) + 190(1.3071 + 0.7459) - M_1 = (36.848P + 76.271)\ Nmm$

$M_3 = M_2 - 195(0.3684P + 0.7459) - \frac{195}{2}(2.1829 - 0.7459) = (-34.99P - 209.287)\ Nmm$

Iteration 1 (+ve moment)

Flexural Tension: $0.67196 \times 10^6\ Nmm = 36.848P + 76.271 \Rightarrow P = 18235\ N$

Flexural Compression: $0.199365 \times 10^6 = 36.848P + 76.271 \Rightarrow P = 5409\ N$

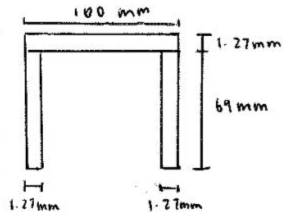
Flange Restrained: $\frac{2.32767(2.6759\times10^6)}{(200-119.467)} = 36.848P + 76.271 \Rightarrow P = 2097\ N$

Flange Unrestrained: $\frac{1456.45(2.6759\times10^6)}{(200-119.467)} = 36.848P + 76.271 \Rightarrow P = 1313392\ N$

Webs Flexural Tension: $\frac{2.32365(2.6759\times10^6)}{119.467} = 36.848P + 76.271 \Rightarrow P = 1410\ N$

Webs Flexural Compression: $\frac{5.278675(2.6759\times10^6)}{(200-119.467)} = 36.848P + 76.271 \Rightarrow P = 4758\ N$

Iteration 2: Minimizing Height and Adding Diaphragms (+ve moment)



Find I: $\bar{y} = \frac{\Sigma A_i d_i}{\Sigma A_i} = 49.26\ mm$

$I = \Sigma I_{i,cm} + \Sigma A_i d_i^2$

$= 0.160456 \times 10^6\ mm^4$

$\rightarrow \sigma_t = 30\ MPa,\ \sigma_c = 6\ MPa,\ h = 70.27\ mm$

Flexural Tension:

$M = \frac{\sigma_t I}{y} = 97719.85\ Nmm = 36.848P + 76.271 \Rightarrow P = 2650\ N$

Flexural Compression:

$M = \frac{\sigma_c I}{h-y} = 45822.75\ Nmm = 36.848P + 76.271 \Rightarrow P = 1242\ N$

Shear Stress:

$Q = \Sigma A_i d_i = 3082\ mm^3,\ V = \frac{\tau I b}{Q},\ \tau = 4\ MPa,\ b = 2.54\ mm \Rightarrow V = 529\ N = 0.5P \Rightarrow P = 1058\ N$

Glue Failure:

$Q = \Sigma A_i d_i = 2581\ mm^3,\ V = \frac{\tau I b}{Q},\ \tau = 2\ MPa,\ b = 2.54\ mm \Rightarrow V = 315\ N = 0.5P \Rightarrow P = \boxed{630\ N}$

Flange Restrained:

$\frac{4\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{100-2.54}\right)^2 = 2.32767\ MPa$, $\frac{2.32767(0.160456\times10^6)}{70.27-49.26} = 36.848P + 76.271 \Rightarrow P = \boxed{480\ N}$

Flange Unrestrained:

$\frac{0.425\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{1.27}\right)^2 = 1456.45\ MPa$, $\frac{1456.45(0.160456\times10^6)}{70.27-49.26} = 36.848P + 76.271 \Rightarrow P = 301900\ N$

Webs Flexural Tension:

$\frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{49.26}\right)^2 = 13.667\ MPa$, $\frac{13.667(0.160956\times10^6)}{49.26} = 36.848P + 76.271 \Rightarrow P = 1206\ N$

Webs Flexural Compression:

$\frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{19.79}\right)^2 = 85.11\ MPa$, $\frac{85.11(0.160456\times10^6)}{70.27-49.26} = 36.848P + 76.271 \Rightarrow P = 17649\ N$

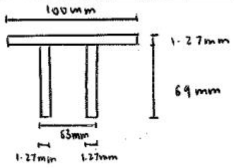
Iteration 2 (+ve moment)

Webs Shear Buckling

$$\frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\frac{1.27}{70.27}\right)^2+\left(\frac{1.27}{360}\right)^2\right]=5.8101\ MPa$$

$$V=\frac{5.8101\,(0.160456\times10^6)(2.54)}{3082}=768.3N=0.5P\Rightarrow P=1537\ N$$

Iteration 3 - Minimizing Distance Between Webs (+ve moment)



Find I: $\bar{y}=\frac{\Sigma A_i d_i}{\Sigma A_i}=49.26mm$

$I=\Sigma I_{i\,cm}+\Sigma A_i d_i^2$

$=0.160456\times10^6\ mm^4$ (remains constant from iteration 2)

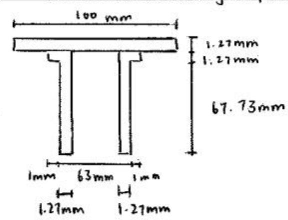
Flange Restrained:

$$\frac{4\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{60.46}\right)^2=6.0484\ MPa,\quad \frac{6.0484(0.160456\times10^6)}{70.27-49.26}=36.848P+76.271\Rightarrow P=1252N$$

Flange Unrestrained:

$$\frac{0.425\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{19.77}\right)^2=6.0102MPa,\quad \frac{6.0102(0.160456\times10^6)}{70.27-49.26}=36.848P+76.271\Rightarrow P=1244N$$

Iteration 4 - Adding Top Tab (+ve moment)



find I: $\bar{y} = \frac{\Sigma A_i d_i}{\Sigma A_i} = 49.42mm$

$I = \Sigma I_{i\,cm} + \Sigma A_i d_i^2$

$= 161375.5 = 0.1613755 \times 10^6 \; mm^4$

$\rightarrow \sigma_t = 30$ MPa, $\sigma_c = 6$ MPa, $h = 70.27$ mm

Flexural Tension

$M = \frac{\sigma_t I}{y} = 97961.7$ Nmm $= 36.848P + 76.271 \Rightarrow P = 2656$ N

Flexural Compression

$M = \frac{\sigma_c I}{h - y} = 46439$ Nmm $= 36.848P + 76.271 \Rightarrow P = 1258$ N

Shear Stress:

$Q = \Sigma A_i d_i = 3102\, mm^3$, $V = \frac{\tau I b}{Q}$, $\tau = 4$ MPa, $b = 2.54$ mm $\Rightarrow V = 528.6$ N $= 0.5P \Rightarrow P = 1057$ N

Glue Failure:

$Q = \Sigma A_i d_i = 2567\, mm^3$, $V = \frac{\tau I b}{Q}$, $\tau = 2$MPa, $b = 5.08$mm $\Rightarrow V = 638.7$ N $= 0.5P \Rightarrow P = 1277$ N

Flange Restrained:

$\frac{4\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{60.46}\right)^2 = 6.04837$ MPa, $\frac{6.04837(0.1613755 \times 10^6)}{70.27 - 49.42} = 36.848P + 76.271 \Rightarrow P = 1268$ N

Flange Unrestrained:

$\frac{0.425\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{19.77}\right)^2 = 6.0102$ MPa, $\frac{6.0102(0.1613755 \times 10^6)}{70.27 - 49.42} = 36.848P + 76.271 \Rightarrow P = 1260$ N

Webs Flexural Tension:

$\frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{49.42}\right)^2 = 13.579$ MPa, $\frac{13.579(0.1613755 \times 10^6)}{49.42} = 36.848P + 76.271 \Rightarrow P = 1201$ N

Webs Flexural Compression:

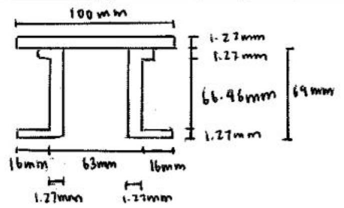
$\frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{19.58}\right)^2 = 86.5$ MPa, $\frac{86.5(0.1613755 \times 10^6)}{70.27 - 49.42} = 36.848P + 76.271 \Rightarrow P = 18173$ N

Webs Shear Buckling:

$\frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\frac{1.27}{70.27}\right)^2 + \left(\frac{1.27}{360}\right)^2\right] = 5.8101$ MPa

$V = \frac{5.8101(0.1613755 \times 10^6)(2.54)}{3102} = 767.7$ N $= 0.5P \Rightarrow P = 1535$ N

Iteration 5 - Adding Bottom Tab (+ve moment)



Find I: $\bar{y} = \frac{\Sigma A_i d_i}{\Sigma A_i} = 43.68mm$

$I = \Sigma I_{i,cm} + \Sigma A_i d_i^2$

$= 246730\ mm^4 = 0.246730 \times 10^6\ mm^4$

$\rightarrow \sigma_t = 30\ MPa,\ \sigma_c = 6\ MPa,\ h = 70.27mm$

Flexural Tension:

$M = \frac{\sigma_t I}{y} = 169457.4 Nmm = 36.848P + 76.271 \Rightarrow P = 4597\ N$

Flexural Compression:

$M = \frac{\sigma_c I}{h - y} = 55674.3\ Nmm = 36.848P + 76.271 \Rightarrow P = 1509\ N$

Shear Stress:

$Q = \Sigma A_i d_i = 4172.76\ mm^3,\ V = \frac{\tau I b}{Q},\ \tau = 4MPa,\ b = 2.54 \Rightarrow V = 600.7\ N = 0.5P \Rightarrow P = 1201\ N$

Glue Failure:

$Q = \Sigma A_i d_i = 3296\ mm^3,\ V = \frac{\tau I b}{Q},\ \tau = 2MPa,\ b = 2.54 \Rightarrow V = 760.5\ N = 0.5P \Rightarrow P = 1521\ N$

Flange Restrained:

$\frac{4\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{60.46}\right)^2 = 6.04837MPa,\quad \frac{6.04837(0.246730 \times 10^6)}{70.27 - 43.68} = 36.848P + 76.271 \Rightarrow P = 1521N$

Flange Unrestrained:

$\frac{0.425\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{19.77}\right)^2 = 6.0102\ MPa,\quad \frac{6.0102(0.246730 \times 10^6)}{70.27 - 43.68} = 36.848P + 76.271 \Rightarrow P = 1512\ N$

Webs Flexural Tension:

$\frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{43.68}\right)^2 = 17.382\ MPa,\quad \frac{17.382(0.246730 \times 10^6)}{43.68} = 36.848P + 76.271 \Rightarrow P = 2662\ N$

Webs Flexural Compression:

$\frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{25.32}\right)^2 = 51.73\ MPa,\quad \frac{51.73(0.246730 \times 10^6)}{70.27 - 43.68} = 36.848P + 76.271 \Rightarrow P = 13028\ N$

Webs Shear Buckling:

$\frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\frac{1.27}{70.27}\right)^2 + \left(\frac{1.27}{360}\right)^2\right] = 5.8101\ MPa$

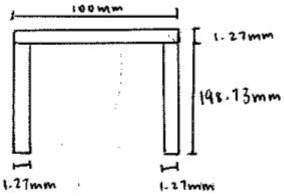
$V = \frac{5.8101\ (0.246730 \times 10^6)(2.54)}{4172.76} = 872.6\ N = 0.5P \Rightarrow P = 1745\ N$

* Input the actual distance between diaphragms, a :

$\frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\frac{1.27}{70.27}\right)^2 + \left(\frac{1.27}{280}\right)^2\right] = 5.949\ MPa$

$V = \frac{5.949\ (0.246730 \times 10^6)(2.54)}{4172.76} = 893.5N = 0.5P \Rightarrow P = 1787\ N$

Iteration 1 (-ve moment)



From Pi Beam (One Layer) from concept 1, we know:

$\bar{y} = 119.47\text{mm}$

$I = 2.676 \times 10^6\ \text{mm}^4$

$\rightarrow \sigma_t = 30\ \text{MPa}$, $\sigma_c = 6\ \text{MPa}$, $h = 200\ \text{mm}$

Flexural Tension:

$M = \frac{\sigma_t I}{h-y} = 996895.6\ \text{Nmm} = 34.99P + 209.287 \Rightarrow P = 28484\ \text{N}$

Flexural Compression:

$M = \frac{\sigma_c I}{y} = 134393.6\ \text{Nmm} = 34.99P + 209.287 \Rightarrow P = 3835\ \text{N}$

Webs Flexural Tension:

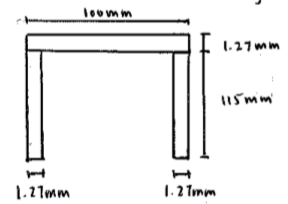
$\frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{79.26}\right)^2 = 5.279\ \text{MPa}$, $\frac{5.279(2.676\times10^6)}{200-119.47} = 34.99P + 209.287 \Rightarrow P = 5007\ \text{N}$

Webs Flexural Compression:

$\frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{119.47}\right)^2 = 2.3235\ \text{MPa}$, $\frac{2.3235(2.676\times10^6)}{119.47} = 34.99P + 209.287 \Rightarrow P = 1482\ \text{N}$

Shear Stress, Glue Failure, and Webs Shear Buckling is the same as the calculated values for the Pi Beam (One Layer) from concept I

Iteration 2: Minimizing Height (-ve moment)



Find I: $\bar{y} = \frac{\Sigma A_i d_i}{\Sigma A_i} = 75.12\ mm$

$I = \Sigma I_{i\,cm} + \Sigma A_i d_i^2$

$= 621088\ mm^4 = 0.621088 \times 10^6\ mm^4$

$\rightarrow \sigma_t = 30MPa,\ \sigma_c = 6MPa,\ h = 116.27\ mm$

Flexural Tension:

$M = \frac{\sigma_t I}{h - y} = 452798\ Nmm = 34.99P + 209.287 \Rightarrow P = 12934\ N$

Flexural Compression:

$M = \frac{\sigma_c I}{y} = 49608\ Nmm = 34.99P + 209.287 \Rightarrow P = 1412\ N$

Shear Stress:

$Q = \Sigma A_i d_i = 7166\ mm^3,\ V = \frac{\tau I b}{Q},\ \tau = 4\ MPa,\ b = 2.54\ mm \Rightarrow V = 880.6\ N = 0.5P \Rightarrow P = 1761\ N$

Glue Failure:

$Q = \Sigma A_i d_i = 5146\ mm^3,\ V = \frac{\tau I b}{Q},\ \tau = 2MPa,\ b = 2.54\ mm \Rightarrow V = 613\ N = 0.5P \Rightarrow P = 1226\ N$

Webs Flexural Tension:

$$\frac{6\pi^2 (4000)}{12(1-0.2^2)} \left(\frac{1.27}{39.88}\right)^2 = 20.85\ MPa\ ,\quad \frac{20.85\,(0.621088 \times 10^6)}{116.27 - 75.12} = 34.99P + 209.287 \Rightarrow P = 8987\ N$$

Webs Flexural Compression:

$$\frac{6\pi^2 (4000)}{12(1-0.2^2)} \left(\frac{1.27}{75.12}\right)^2 = 5.877\ MPa\ ,\quad \frac{5.877\,(0.621088 \times 10^6)}{75.12} = 34.99P + 209.287 \Rightarrow P = 1383\ N$$

Webs Shear Buckling

$$\frac{5\pi^2(4000)}{12(1-0.2^2)} \left[\left(\frac{1.27}{116.27}\right)^2 + \left(0\right)^2\right] = 2.044\ MPa$$

$$V = \frac{2.044\,(0.621088 \times 10^6)(2.54)}{7166} = 450\ N = 0.5P \Rightarrow P = \boxed{900N}$$

Iteration 3: Adding Diaphragms (-ve moment)

Besides Webs Shear Buckling, the other failure loads remain the same from iteration 2.

Webs Shear Buckling:

$$\frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\frac{1.27}{116.27}\right)^2+\left(\frac{1.27}{180}\right)^2\right] = 2.8973 \text{ MPa}$$

$$V = \frac{2.8973(0.621088\times10^6)(2.54)}{7166} = 637.8\text{ N} = 0.5P \Rightarrow P = 1276\text{ N}$$

* The 'a' value between diaphragms was calculated when finding the middle support to be 166.67mm. This was then plugged into the equation to find the actual webs shear buckling force:

$$\frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\frac{1.27}{116.27}\right)^2+\left(\frac{1.27}{166.67}\right)^2\right] = 3.0392 \text{ MPa}$$

$$V = \frac{3.0392(0.621088\times10^6)(2.54)}{7166} = 669.1\text{ N} = 0.5P \Rightarrow P = 1338\text{ N}$$

Negative moment is where the support is placed, but global minimum force for positive or negative moment is the failure stress.

$\therefore$ the bridge failure force: 1201N due to maximum shear on the cross section

Force on the middle support:

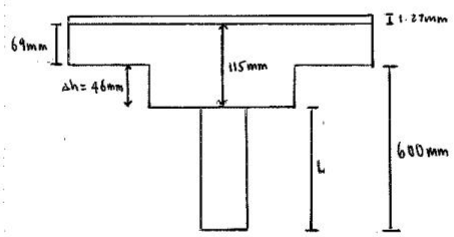
$$P_s = 0.1368P + 4.3658N$$

$$\Rightarrow P_s = 889.2626N$$

We know the compression strength is 6MPa:

$$A \geq \frac{F}{\sigma_c} = \frac{889.2626}{6} = 148.2104\ mm^2$$

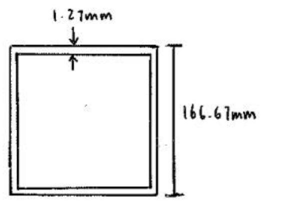
$$I \geq \frac{FL^3}{\pi^2 E} = \frac{889.2626(554)^3}{\pi^2(4000)} = 3830007\ mm^4$$



$\therefore L = 600 - 46$

$= 554\ mm$

We will use a square cross section:



$$I = \frac{b^4}{12}$$

$$\Rightarrow b = \sqrt[4]{3830007(12)} = 82mm$$

$$A = bh = b^2$$

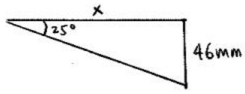
$$\Rightarrow b = \sqrt{148.2104} = 12.174\ mm$$

$$I = \frac{b^4}{12} - \frac{(b-2t)^4}{12}$$

$$\Rightarrow b = 166.67mm$$

$\therefore$ the value of b we use is 166.67mm so $A \geq 148.2104mm^2$ and $I \geq 3830007\ mm^4$

Finding Matboard Used (Concept II):



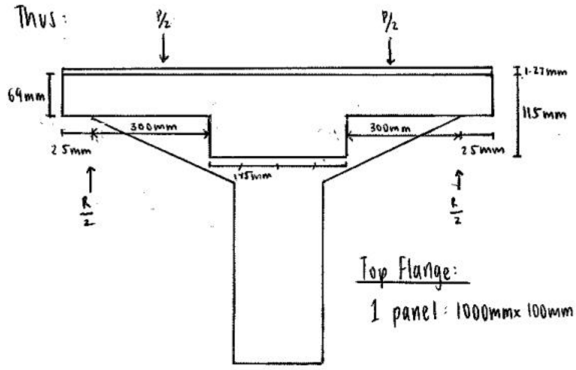
$\frac{46}{x} = \tan 25$

$\Rightarrow x = 98.647\text{mm}$

∴ minimum height change will be at 98.647mm through the span where the space constraint is passed

We are then allowed to switch heights at halfway where the moment is approximately 300 mm into the span

Thus:



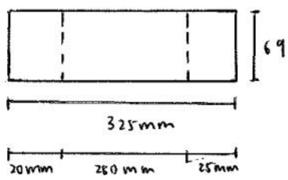
Top Flange:
1 panel: 1000mm x 100mm

Web height (+ve moment)
$= 1 + 69 + 16 = 86\text{mm}$
∴ 2 panels: 325mm x 86mm

Webs (-ve moment):
2 panels: 350 mm x 115 mm

Middle support:
4 panels: 554 mm x 166.67 mm

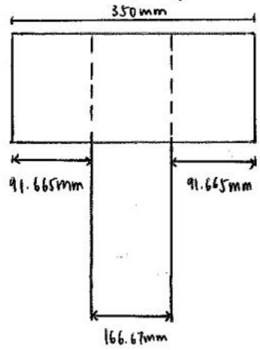
Section A Diaphragms:



total diaphragms for section A:
4 panels: 100 mm x 69 mm

Diagram for the right side. Diagram for the left side is the same, but mirrored.

Section B Diaphragms:



total diaphragms for section B:
2 panels: 100mm x 115mm

∴ Total area of Matboard used:

$$4(100\times69) + 2(100\times115) + 2(325\times86) + 2(350\times115) + 4(554\times166.67) + (1000\times100)$$
$$= 663290.72\text{ mm}^2$$