

Cournot Duopoly: Question Set

Question 1

- a) People generally believe that oligopolies (and in particular duopolies) are always inefficient. Are they correct?
- b) Two firms A and B produce the same good. Each sets the price of the good. If the two firms set the same price, they share the market equally. The demand for the output of the industry is $Q = 90 - p$ where p is the price. Both firms have a marginal and average cost of £10. The two firms observe that they can both increase profits if they set up a cartel which produces the level of industry output that maximizes industry profits.
 - i. What is the output of the cartel?
 - ii. What is the industry price?
 - iii. Assume that the two firms share the market equally. What profit does each firm make?
- c) Suppose firm B believes that firm A will remain at its cartel level of output. What is the profit-maximizing level of output for firm B?
- d) Under what conditions is it difficult for firms to sustain a cartel?

- **Oligopoly:** A market structure characterized by a small number of firms, whose decisions are interdependent due to their significant market power, leading to strategic interactions in pricing and output decisions.
- **Cartel:** A formal agreement between firms within an oligopoly to restrict output, raise prices, and maximize collective profits by behaving like a monopoly.
- **Marginal Cost (MC):** The additional cost incurred from producing one more unit of a good. It is crucial in determining the optimal level of production.
- **Marginal Revenue (MR):** The additional revenue generated from selling one more unit of a good. In profit-maximizing firms, MR is set equal to MC.
- **Allocative Efficiency** Occurs when resources are distributed in a way that maximizes consumer satisfaction, where the price of goods equals the marginal cost of production ($P = MC$).
- **Productive Efficiency** Achieved when a firm produces goods at the lowest possible cost, meaning all resources are used optimally and no waste occurs.
- **Deadweight Loss:** A loss of economic efficiency that occurs when the equilibrium outcome is not achieved, typically due to market distortions like taxes, subsidies, or monopolistic practices.
- **Dynamic Efficiency:** Refers to the long-term improvement of products and processes, often driven by innovation and investment in research and development, resulting in growth over time.
- **Economies of Scale:** Cost advantages that firms experience as their production increases, leading to a decrease in average costs per unit due to spreading fixed costs over a larger output.
- **Residual Demand:** The remaining market demand that a single firm faces after accounting for the supply of competing firms.

Oligopolies typically form in industries where significant barriers to entry, such as high initial capital costs, economies of scale, or regulatory constraints, limit the number of firms that can successfully compete. Established companies in these industries may align their services and pricing strategies, creating a market structure that new entrants find difficult to penetrate—resulting in what is known as a “closed” or “collusion” oligopoly.

The smaller the number of established firms, the more likely an oligopoly is to form, and the greater the potential scope of the cartel. A duopoly is the more dangerous variant of an oligopoly. Oligopolies are regarded as inefficient market structures, as they are incentivised in pursuit of greater profit to produce less goods at a higher price and inferior quality, leading to a loss in consumer surplus, allocative inefficiency and a deadweight loss. This behavior is sustainable only if firms trust that others will neither undercut prices nor significantly differentiate their products.

However, this view is not universally correct, as the efficiency of oligopolies depends on various factors, including the potential for dynamic efficiency and the presence of economies of scale. Firms in an oligopoly may have higher profits, which can be reinvested into research and development, leading to innovation and long-term benefits for consumers. Additionally, in industries with high fixed costs, a small number of large firms can achieve economies of scale, reducing average costs and potentially leading to lower prices than would be possible with many small firms.

The theoretical framework of oligopolies often includes the possibility of collusion, where firms agree to act as a cartel to maximize joint profits. In a duopoly where two firms produce the same good, if both firms agree to set the same price and share the market equally, they can mimic the behaviour of a monopolist. Let the demand for a good be represented by the equation $Q = 90 - p$, where p is the price, and both firms have a marginal and average cost of £10. To maximize industry profits, the firms would set marginal revenue equal to marginal cost. The resulting industry output would be 40 units, sold at a price of £50 per unit. The total industry profit would be £1600, with each firm earning £800 if they share the market equally.

Marginal & Average Cost: £10

Unit Profit = $p - 10$

Units Sold: $Q = 90 - p$

Total Profit:

$$\text{Total Profit} = Q \times (p - 10) = (90 - p) \times (p - 10) = 100p - p^2 - 900$$

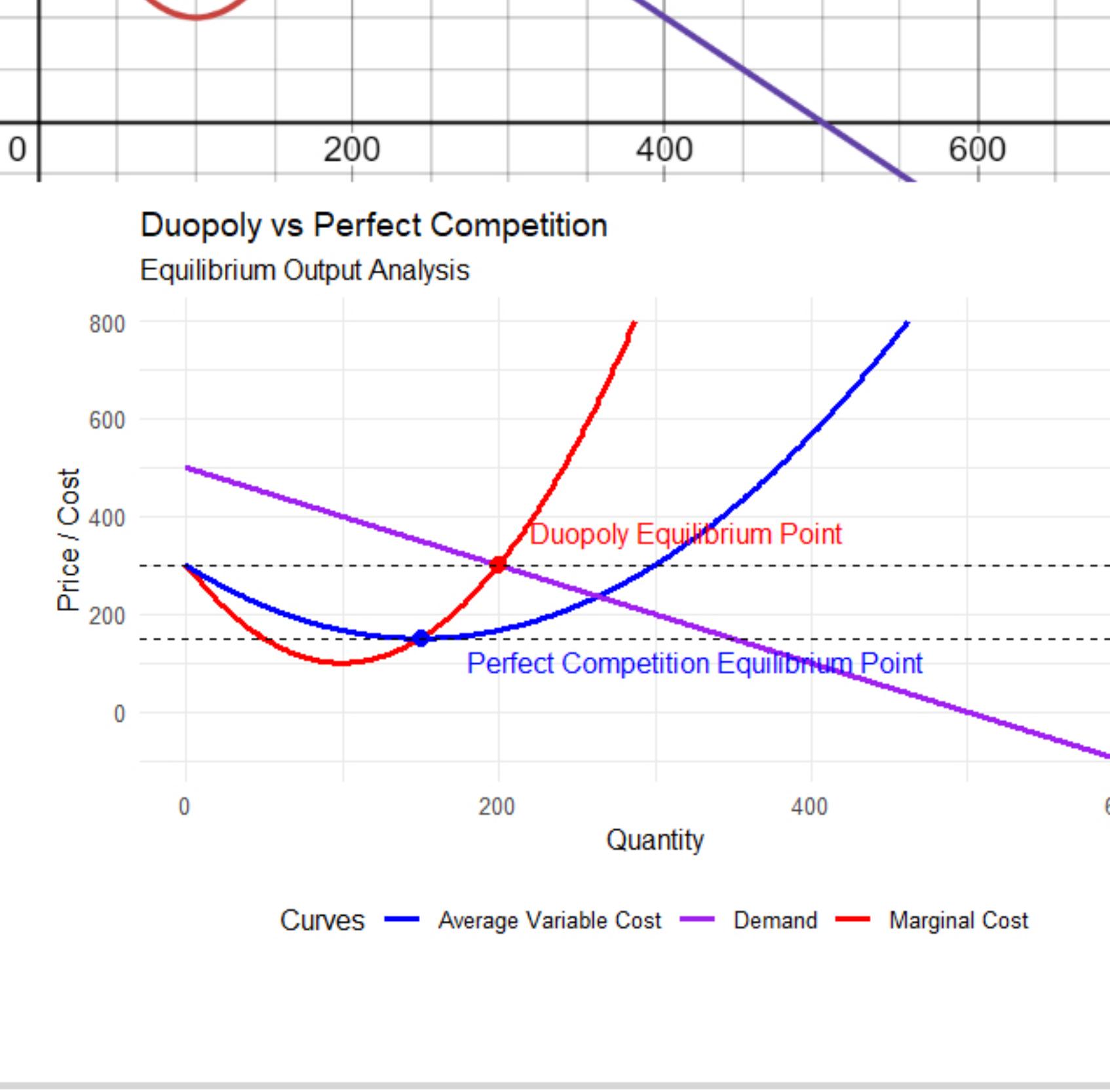
Maximize Total Profit:

$$\frac{d(100p - p^2 - 900)}{dp} = 100 - 2p = 0 \implies p = 50 \quad \text{and} \quad Q = 40$$

Profit Calculation:

$$\text{Unit Profit} = 50 - 10 = £40$$

$$\text{Total Profit} = 40 \times 40 = £1600$$



The stability of the cartel can be undermined if legal institutions undergo changes, such as shifts in government, new laws, or the enforcement of antitrust regulations that disallow their practices. Deregulation may also invite new entrants, including foreign firms, which could erode the cartel's market share. Additionally, informal institutions, such as consumer advocacy groups, might organize protests or boycotts, challenging the cartel's control. To mitigate these risks, the cartel should maintain competitive pricing and reinvest in research and development to ensure long-term stability.

The stability of the cartel can also be undermined if one firm cheats, triggering a breakdown of the agreement. If firm B believes that firm A will adhere to the cartel agreement, firm B has an incentive to deviate by increasing its output and lowering the price slightly to capture a larger market share. This behavior, known as cheating, can lead to higher profits for the deviating firm, at least in the short term.

Given the homogeneous nature of the products, a slight price undercut by firm B will allow it to capture the entire market share, effectively doubling its output and profits. This can be demonstrated in the below chart.



However, firm A might retaliate, kick starting a price war until the price is equal to the marginal cost, maximising productive and allocative efficiency but destroying profitability for both companies.

Question 2

What are the particular characteristics of monopolistic competition as a market structure?

Monopolistic competition is a market structure characterized by a large number of firms that produce differentiated products, meaning each firm offers a slightly unique version of a product that is not a perfect substitute for others. This differentiation can be based on factors such as branding, quality, features, or customer service. Each firm has some degree of pricing power due to product differentiation, allowing them to set prices above marginal cost. However, this pricing power is limited by the presence of close substitutes and low barriers to entry, which enable new firms to enter the market easily, thereby increasing competition.

The term “monopolistic competition” refers to the market power that individual firms possess due to product differentiation. While each firm can set its prices to some extent because its product is not a perfect substitute for others, competitive pressures from close substitutes prevent prices from being set too high without risking a loss of customers to competitors. This hybrid nature captures the essence of markets where firms have some monopoly power due to differentiation but still operate in a competitive environment. In the long run, economic profits tend to be zero as new entrants erode any short-term profits by increasing competition.

Question 3

If the market consists of only two producers that compete in prices and have the same production costs, then equilibrium output is the same as if the market was perfectly competitive.

Textbook Answer: A) True Under these assumptions firms compete à la Bertrand and, because they have the same marginal costs, they will end up setting the price equal to this. This is the same equilibrium we would expect in a perfectly competitive market, although firms reach this equilibrium for a different reason.

- **Answer: B) False**

In a market with two price-competing producers, firms undercut each other's prices until the price equals their marginal cost. If there were an increasing number of firms, as in perfect competition, the price would trend towards the lowest average cost. However, this level of optimisation may not be possible in a duopoly leading to a lower equilibrium output. Thus output in a duopoly is \leq output in perfect competition.

Question 4

In a tennis game, Serena is playing against Naomi. Serena and Naomi each choose between two actions: to play more in the front or in the back. If they both make the same choice, Serena wins. If they make different choices, Naomi wins. In this game, there are no pairs of actions that are best responses to each other.

- **Answer: A) True**

In each outcome Serena has an incentive to do whatever Naomi is doing and Naomi has an incentive to deviate.

Question 5

If two players have a dominant strategy, then there will not be any other Nash equilibria than the outcome in which both players play their dominant strategy.

- **Answer: A) True**

If both players have a dominant strategy—meaning that their best strategy does not depend on the other player's choice—then the outcome in which both players play their dominant strategy is the only Nash equilibrium. A Nash equilibrium occurs when each player's strategy is the best response to the other player's strategy. Since the dominant strategy is the best regardless of what the opponent does, the combination of both players playing their dominant strategies forms the unique Nash equilibrium, and no other Nash equilibria exist.

Question 6

Suppose both England and Norway fish in the North Sea. Both countries know that their fish supplies are being depleted and that this depletion could be slowed down if they both cut their fishing fleets in half. The matrix below shows the payoff for both countries (England's payoff is the first entry in each cell) with unchanged and halved fleets.

Matrix says:

	Norway: 10 boats	Norway: 5 boats
England: 10 boats	(300, 300)	(550, 250)
England: 5 boats	(250, 550)	(500, 500)

- **Option D:** The Nash equilibrium occurs when both countries employ 10 boats, as this is the best response for both under the given payoffs.

Question 7

Two gangsters have to choose whether to fight or make peace. The following matrix captures this strategic interaction.

	Fight	Truce
Fight	-5, -5	5, -10
Truce	-10, -5	0, 0

Mark the correct answer.

- **Option A:** Playing Fight strictly dominates playing Truce for the column player.

Question 8

An industry is a Cournot duopoly with two firms A and B. Industry inverse demand is $p = 10 - Q$, where $Q = q_A + q_B$. Both firms produce with a constant average and marginal cost of 4. Which of the following statements is correct?

- **Option A:** Firm A has a reaction function $q_A = 3 - \frac{q_B}{2}$.

In a Cournot duopoly, each firm chooses its output level q_A and q_B simultaneously, assuming the output of the other firm is fixed. The market price is determined by the total quantity $Q = q_A + q_B$ supplied to the market.

- Inverse Demand: $p = 10 - Q$, where $Q = q_A + q_B$
- Marginal Cost MC for both firms: $MC_A = MC_B = 4$
- q_B remains unknown

Deriving Firm A's Reaction Function:

$$TR_A = p \times q_A = (10 - Q) \times q_A = (10 - q_A - q_B) \times q_A$$

$$TR_A = 10q_A - q_A^2 - q_Aq_B$$

Profit is total revenue minus total cost, which function is then optimised for q_A

$$\pi_A = TR_A - TC_A = (10q_A - q_A^2 - q_Aq_B) - 4q_A = 6q_A - q_A^2 - q_Aq_B$$

$$\frac{\partial \pi_A}{\partial q_A} = 6 - 2q_A - q_B = 0$$

$$q_A = \frac{6 - q_B}{2}$$

Question 9

Two firms, A and B, produce the same product and compete by setting prices. They are the only firms that produce the product. Both firms have a marginal and average cost of £2. If both firms set the same price, they share the market equally. If they charge different prices, the firm charging the lower price takes the entire market. Which of the following statements is incorrect?

- **Option D:** Incorrect. In Bertrand competition with identical products and costs, firms make zero economic profits in equilibrium because price is driven down to marginal cost. The small number of firms does not guarantee profits; rather, the price competition ensures no firm makes profits.

In Cournot competition where firms choose quantities of production, an example might be oil, coal and natural gas in the energy market, while in a Bertrand model, where firms set prices, an example might be competition between food producers where consumers have preference for one type of food, but reduce their demand for it depending on the average price of substitutes.

A Dominant Strategy is a player's best strategy whatever the strategies adopted by rivals.

In Nash Equilibrium, each player chooses the best strategy, given the strategies being followed by other players.

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