02612 Constrained Optimization - Exercise at Lecture 03

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1 Quadratic Optimization, Sensitivity and Duality

Consider the problem

$$\min_x \quad f(x) = 3x_1^2 + 2x_1x_2 + x_1x_3 + 2.5x_2^2 + 2x_2x_3 + 2x_3^2 - 8x_1 - 3x_2 - 3x_3 \tag{1.1a}$$

$$s.t. \quad x_1 + x_3 = 3 \tag{1.1b}$$

$$x_2 + x_3 = 0 \tag{1.1c}$$

in the form

$$\min_x \quad f(x) = \frac{1}{2}x'Hx + g'x \tag{1.2a}$$

$$s.t. \quad A'x = b \tag{1.2b}$$

1. What are H, g, A, b
2. Write the KKT optimality conditions.
3. Make a function `[x,lambda]=EqualityQPSolver(H,g,A,b)` for solution of equality constrained convex quadratic programs. `hint:` Consider and discuss which factorization to use when you factorize the KKT-matrix.
4. Test your program on the above problem.
5. Generate random convex quadratic programs (consider how this can be done) and test you program.
6. Write the sensitivity equations for the equality constrained convex QP.
7. Make a function that returns the sensitivities of the solution with respect to g and b. Test your program and discuss how you can verify that the sensitivities you compute are correct.
8. Write the dual program of the equality constrained QP.
9. What is the optimality conditions of the dual QP? How are they related to the primal QP? How are the variables in the primal and the dual problem related?
10. Make a function that solves the dual QP. Is there any advantages in solving the dual QP instead of the primal QP for the equality constrained convex quadratic program?