Jump-Diffusion Model Report

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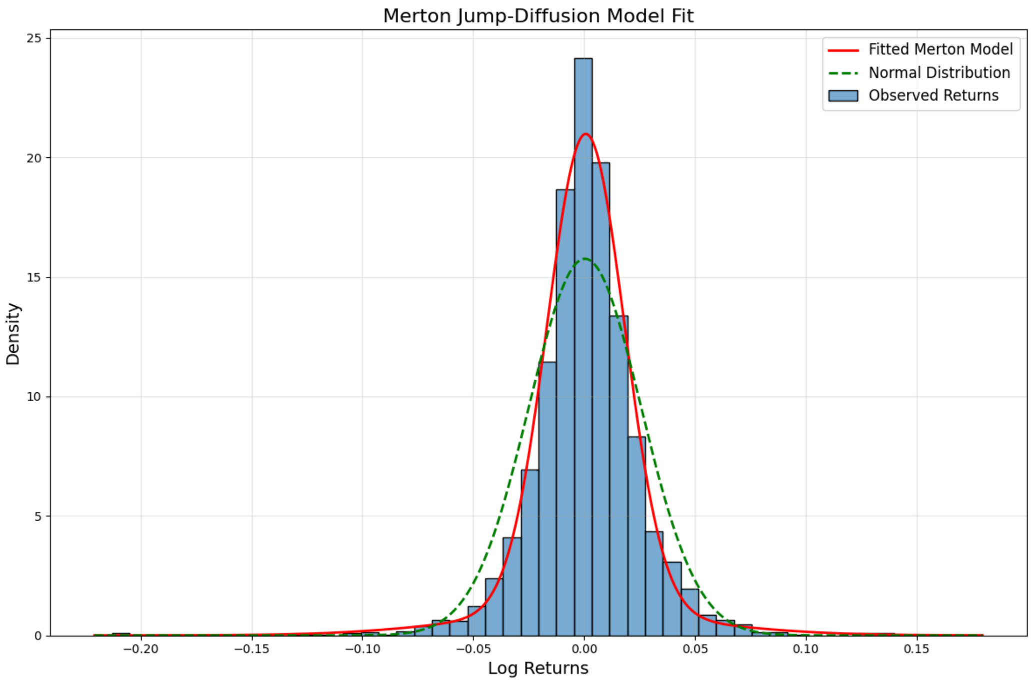
Introducing the Merton Jump-Diffusion model for financial time series analysis. This is an extension of the standard geometric Brownian motion model that accounts for sudden jumps in asset prices.



Looking at the figure above, the Merton Jump-Diffusion model appears to fit the observed returns better than the normal distribution. The model captures the nature of the return distribution. In comparison to the normal distribution, it has a higher peak around zero and fatter tails, which is typical for financial returns.

The model shows particularly good fit in the tails of the distribution, which can be crucial for risk management. The normal distribution underestimates the extreme cases, while our Merton model accounts for these jumps, through the jump component.

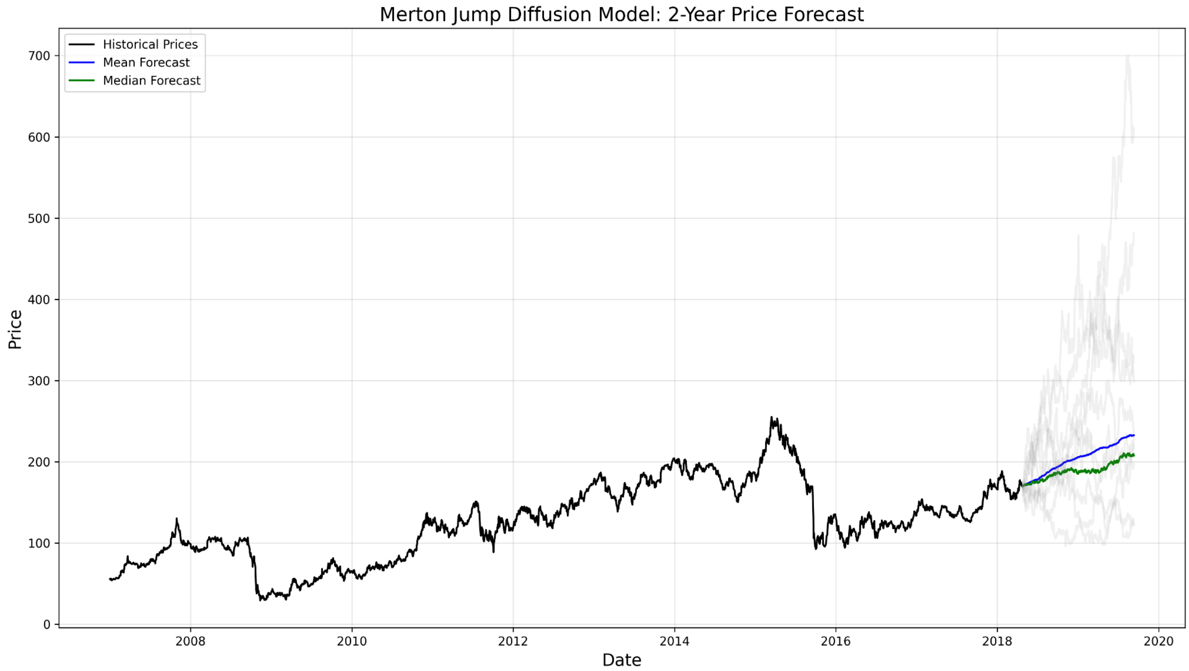
Parameter Interpretation

The estimated parameters are the following (estimation ± standard error):

- gamma (0.238989 ± 0.101394): this is the drift parameter, a positive drift indicates an overall upward trend in returns, the standard error would be considered high, which generates uncertainty.
- sigma (0.276186 ± 0.005657): this represents the diffusion volatility, it estimates the small random fluctuations in the price. A small standard error suggests this estimate can be considered reliable.
- lambda (33.775186 ± 2.031949): this high jump parameter indicates frequent jumps, approximately 33 jumps per year (trading year). This can be seen as an overestimate at first, but we will develop more about lambda further into the report.

- alpha (-0.003033 ± 0.003118): the negative mean jump size indicates that jumps tend to be slightly downward.
- beta (0.050050 ± 0.002621): this represents the standard deviation of jump sizes, with a small standard error indicating a reliable estimate.

Overall, the optimization was successful and worked with a reasonable number of function evaluations (336). The standard errors for most parameters are relatively small in comparison to the parameter value, which indicates an overall good precision. Furthermore, the highly significant lambda parameter, with a p-value ≈ 0, confirms that including jumps significantly improves the model fit in comparison to a normal diffusion model. However, the alpha parameter, with a p-value ≈ 0.330676, suggests that while jumps are frequent, their size isn't far from 0.



This price forecast plot shows a 2 year-price forecast using the model and the estimated values. The mean forecast shows a moderate upward trend, which aligns with the positive drift parameter estimated by the model. The median, slightly below the mean, is consistent with the negative alpha parameter, indicating the downward bias in jumps. The forecast is also displaying uncertainty through the multiple grey paths, which captures the volatility and jumps in the asset price.

Here, the high lambda parameter estimation comes to sense. From historical prices, we can observe that there are multiple jumps throughout the years, so it is reasonable that the model estimated a high lambda parameter. This also explains why the forecast paths (grey lines) show multiple small deviations rather than dramatic jumps, which we saw from the p-value of alpha, being close to 0.

The forecast also appears to show a wide probability of potential outcomes as the prediction extends, which is realistic since it reflects increasing uncertainty over time. Finally, the model doesn’t appear to be making unreasonable optimistic or pessimistic predictions, it shows a balanced view of potential future price movements, based on the historical patterns it learned from.