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LETTER TO THE EDITOR

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LETTER TO THE EDITOR

The thermonuclear burn-up in deuterium–tritium mixtures and hydrides of light elements

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Abstract. The thermonuclear burn-up in deuterium–tritium (DT) mixtures with various D : T ratios is considered. It is shown that the use of small amounts of tritium simplifies the thermonuclear burn-up in the dense deuterium significantly. The general dependence of the critical burn-up parameter $x_c = \rho r_c$ upon the D : T ratio, temperature T and initial density ρ_0 is studied. In particular, we found that when the D : T ratio decreases the critical burn-up parameter $x_c(T, \rho_0)$ diminishes very quickly (at fixed T and ρ_0), i.e. the thermonuclear burn-up simplifies when the relative tritium concentration in the DT-mixture grows. The thermonuclear burn-up in the highly compressed deuterides and DT-hydrides of light elements is discussed briefly in the appendix.

As is well known, the thermonuclear burn-up in the dense ($\rho_0 \geqslant 0.213$ g cm^{-3}) 1 : 1 DT-mixture is quite simple for temperatures $T \geqslant 4$ keV. In particular, this does not require any preliminary additional compression [1–3]. On the other hand, the same process for temperatures $T \geqslant 4$ keV in the pure deuterium D_2 may proceed only after its preliminary compression to the densities $\rho_0 \approx 100$ g cm^{-3} [4]. Briefly, this shows that the thermonuclear ignition and thermonuclear detonation in the pure deuterium are significantly more complicated phenomena than in the dense 1 : 1 DT-mixture. Moreover, it is quite obvious, that the use of the relatively small amounts of tritium may simplify the thermonuclear burn-up in the dense deuterium and, also, in various deuterides of light elements, e.g. in the ^{6}LiD deuteride [4]. In the present study we consider the thermonuclear burn-up for various DT-mixtures with different tritium concentrations. Our main goal is to study the dependence of the critical burn-up parameter $x_c = \rho_0 r_c$ upon the relative tritium concentration in binary DT-mixtures‡. Actually, the thermonuclear burn-up characteristics for the highly compressed deuterium and DT-mixtures with low tritium concentrations are determined and openly discussed for the first time. The dense 1 : 1 DT-mixture and pure deuterium D_2 (or in other words the 1 : 0 DT-mixture) are of specific interest below, since: (i) they are the two limiting cases for our present study, and (ii) the appropriate results for the equimolar (1 : 1) DT-mixture are well known from previous considerations (see e.g. [1–4, 6]).

Let us consider the thermonuclear burn-up in the homogeneous, immovable DT-mixture. The relative amount of tritium we shall designate by y ($0 \leqslant y \leqslant 0.5$). Hence, the relative

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‡ The DT-mixtures with the low tritium concentrations are of specific interest for many practical purposes. In particular, they can be applied as a thermonuclear rocket fuel for interstellar ships. Indeed, the deuterium is stable and significantly lighter than the ^{6}LiD deuteride or even DT-mixture. For a ship with a standard (20 000 tonnes) mass the minimal mass of the thermonuclear fuel can be evaluated as $\approx$ 100 000–400 000 tonnes [5].

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amount of deuterium equals $1 - y$. The burn-up problem is considered in its classical form, i.e. we assume that some part of the mixture is instantaneously heated to a very high temperature. Since the rest of the DT-mixture is cold, the thermal wave begins to propagate in the rest of the mixture. After some time this wave will transform into the shock wave, which propagates and disappears, in the general case, in the cold mixture. However, if the temperature in the central, heated region exceeds the ignition temperature and the space radius of such a hot spot is sufficiently large, then the thermonuclear burning wave may start to propagate out, igniting the rest of the cold DT-mixture. In the general case, such a burning wave propagates initially by the thermal wave, and later by the shock wave, which can be actually considered as the detonation wave, i.e. the shock wave which supports itself.

For the present, we restrict ourselves to the spherical geometry cases only. This means that: (i) the initially heated zone has a spherical form, and (ii) the expanding hot zone will retain such a spherical form at all later times. The initial density ρ_0 of the DT-mixture is a constant, i.e. it does not depend on the spatial coordinates. The temperature behind the shock (or thermal) wave $T = T_f$ is significantly greater than the initial temperature before such a wave, where $T_0 \approx 0$. Actually, without loss of generality, we may also assume that the high temperature T_f behind the shock (or thermal) wave does not depend on the spatial coordinates. The radius of the hot, spherical spot created by the shock (thermal) wave is designated below by r_f. Hence, the velocity of the hot-zone expansion is given by $\mathrm{d}r_f/\mathrm{d}t$. Since the hot spot expands, in principle, either by the detonation or thermal waves [1], the actual (or observed) velocity $V_{\max}$ is the larger of the two corresponding velocities, i.e. $V_{\max} = \max\{(\mathrm{d}r_f/\mathrm{d}t)_d, (\mathrm{d}r_f/\mathrm{d}t)_t\}$. In this notation, the burn-up equation [1] takes the form [1–4]:

$$\mathcal{C}\frac{\mathrm{d}T}{\mathrm{d}t} = -\mathcal{C}\frac{3}{r_f}\frac{\mathrm{d}r_f}{\mathrm{d}t}T + \tilde{q}(r_f, T) = -\mathcal{C}\frac{3}{r_f}V_{\max}T + \tilde{q}(r_f, T) \tag{1}$$

key eqn!

where T is the temperature behind the detonation wave and $\mathcal{C}$ is the specific heat per unit mass, which is also a constant on the spatial coordinates and time. $\tilde{q}(r_f, T)$ is the energy release function (per unit mass). This equation can be rewritten in the form which contains only the burn-up parameter $x = \rho_0 r_f$ [1, 3]:

$$\frac{\mathrm{d}T}{\mathrm{d}x} = -\frac{3}{x}T + \frac{q(x, T)}{\mathcal{C}V_{\max}} \tag{2}$$

where $q(x, T) = \tilde{q}(r_f, T)/\rho_0$ is the normalized energy release function. In the general case, all functions $q(x, T)$, $\mathcal{C}$ and $V_{\max}$ mentioned in (2) depend not only upon x and T, but also upon the initial density ρ_0 and the D : T ratio (i.e. $(1 - y) : y$) in the considered DT-mixture. Now, we want to discuss such dependences in detail.

The specific heat $\mathcal{C}$ (in MJ g^{-1} keV^{-1}) per unit mass for the $(1 - y) : y$ DT-mixture is of the form [4, 6]:

$$\mathcal{C} = \frac{144.716\,4346}{\overline{A}}(1 + \overline{Z}) = \frac{144.716\,4346}{1 + (y/2)} = \frac{144.716\,4346}{1 + (y_1/4)} \tag{3}$$

where the value $y_1 = 2y$ is used. The advantage of using y_1 instead of y is explained below. Note that the last equation is based on the assumption that the electron and ion temperatures are equal to each other and coincide with T [4]. The $\overline{A}$ and $\overline{Z}$ values are the mean atomic mass number and the mean atomic number, respectively. They must be expressed in units in which the proton charge and proton mass equal 1†. Therefore, for the considered DT-mixtures $\overline{Z} = 1$ always, while $\overline{A}$ is a linear function of y (or y_1).

† Here we assume that the difference between the proton and neutron masses is negligible and also that the mass defect for all considered nuclei equals zero exactly. Both these approximations are traditional in thermonuclear problems, and they have been widely used in earlier works (see e.g. [6] or [10]).

To find the velocity of the shock wave $(\mathrm{d}r_f/\mathrm{d}t)_d = V_\mathrm{D}$ the strong-explosion approximation [7] can be applied. This approximation has a very good accuracy for the considered problem, and has been successfully used in many previous works (see e.g. [1–4, 8]) related to the thermonuclear burn-up in the DT-mixtures and dense deuterium. In this approximation one finds (for details see e.g. [3]):

— Detonation shock wave

$$V_\mathrm{D} = \left(\frac{\mathrm{d}r_f}{\mathrm{d}t}\right)_d = a\sqrt{T} = \sqrt{\frac{3\pi(\gamma+1)^2(\gamma-1)\mathcal{C}}{8(3\gamma-1)}}\sqrt{T} = 5.284\,436\,399 \times 10^{-3} \times \sqrt{\mathcal{C}T} \quad (4)$$

where $\gamma = 5/3$ and $\mathcal{C}$ is in MJ g^{-1} keV^{-1} (1 MJ $= 1 \times 10^6$ J), T is in keV and V_D is in cm ns^{-1}. Note that in equation (4) the parameter a is a constant over T, which actually follows from the strong-explosion approximation. From these two formulae given above one easily finds: $\mathcal{C} = 115.773\,1477$ MJ g^{-1} keV^{-1}, $V_\mathrm{D} = 5.685\,944\,233 \times 10^{-2} \times \sqrt{T}$ cm ns^{-1} for the equimolar DT-mixture, and $\mathcal{C} = 144.716\,4346$ MJ g^{-1} keV^{-1}, $V_\mathrm{D} = 6.357\,078\,911 \times 10^{-2} \times \sqrt{T}$ cm ns^{-1} for dense deuterium.

The competing high-temperature thermal wave has the velocity [1]:

$$V_\mathrm{T} = \left(\frac{\mathrm{d}r_f}{\mathrm{d}t}\right)_t = 0.392\,24 \times 10^{-4} \times \frac{\rho_0^{0.08}T^{2.34}}{x} \quad (5)$$

— thermal wave

where x is the burn-up parameter in cm, the temperature T is in keV and V_T is also in cm ns^{-1}. It is obvious that if the temperature T increases to very high values ($\geqslant$ 30–50 keV), then V_T becomes larger than V_D, and therefore, the high-temperature thermal conductivity is a more effective channel to propagate the thermonuclear burning wave at very high temperatures. However, in a large number of actual cases, the thermonuclear burning wave begins to move ahead in the cold compressed mixture already at relatively low temperatures ($T \approx$ 4–10 keV), by the usual detonation expansion. Moreover, in many substances such a low-temperature detonation ($T \leqslant$ 12.5–15 keV) is the only way in which thermonuclear burning can start and propagate, e.g. in the highly compressed ($\rho_0 \geqslant 10^4$ g cm^{-3}) ^{7}LiD, ^{9}BeD$_2$ deuterides, and even, in the highly compressed ^{6}LiD deuteride if the latter burns up without intense neutron fluxes (see the appendix and the discussion in [4]).

Key Point

but

But x-ray bursts are v/hot.

Note, in addition, that the thermonuclear burn wave can also propagate by the α-particles and other fusion products. But, this process becomes important if (and only if) the intensive thermonuclear burning has started already (see discussion and figure 1 in [1]). Obviously, this is not the case in the present study, since our goal is to determine the minimal values of the burn-up parameter $x_\mathrm{c}(T)$. Nevertheless, in our analysis the burn propagation caused by α-particles from the (d,t)-reaction was not ignored completely. The analytical expression for the appropriate velocity is taken to be [1, 9]:

— α heating is important for continued burn not initial ??

$$V_\alpha(T) = \left(\frac{\mathrm{d}r_f}{\mathrm{d}t}\right)_\alpha = \frac{xq(1-k_\alpha)}{3\rho_0\mathcal{C}T} \quad (6)$$

— α heating velocity

where q is the number of α-particles emitted by a unit mass of the DT-mixture per unit time. The factor k_α is the average fraction of the energy with which these particles remains in the reaction zone. The calculation of the k_α factor is discussed in detail in [9] (see also [1]). To solve equation (2) numerically we used in this case $V_\mathrm{max} = \max\{V_\mathrm{D}, V_\mathrm{T} + V_\alpha\}$. The summation of the V_T and V_α velocities follows from the summation of the transported energies (see discussion in [1]). However, our numerical calculations indicate that the appropriate correction (related to $V_\alpha \neq 0$) is really very small. In particular, for all considered temperatures such a correction does not exceed 4–6% of the determined values (x_c). The largest corrections, of $\approx$ 5.5–6%, can be found in the transition area, i.e. when $V_\mathrm{D}(T) \approx V_\mathrm{T}(x, T)$ (in agreement with [1]).

The energy release function $q(x, T)$ for the $(1-y) : y$ DT-mixture can be represented in terms of the two known such functions for the pure deuterium $q_{\mathrm{dd}}(x, T)$ and for the 1 : 1 DT-mixture $q_{\mathrm{dt}}(x, T)$, respectively. This expression takes the form

$$q(x, T) = (1-2y)q_{\mathrm{dd}}(x, T) + 2yq_{\mathrm{dt}}(x, T) = (1-y_1)q_{\mathrm{dd}}(x, T) + y_1 q_{\mathrm{dt}}(x, T). \tag{7}$$

It should be mentioned that this formula is based on the assumption that the (d,t)-reaction is significantly faster in all considered cases than both the competing (d,d)- and (t,t)-reactions. In other words, we apply here the approximation that in the DT-mixture with $y \leqslant 0.5$ all tritium nuclei will react with the equal number of deuterium nuclei, and then the remaining deuterium nuclei will react with each other. Since $y \leqslant 0.5$ in all present cases, the (t,t)-reaction can be ignored. Such an approximation has a good accuracy, since for the considered temperatures (4 keV $\leqslant T \leqslant$ 20 keV) the (d,t)-reaction cross section σ_{dt} is $\approx$ 650–700 times larger than corresponding cross sections for both competing (d,d)- and (t,t)-reactions. This means that if the total number of tritium moles per unit volume is y, then it will consume almost instantaneously an equal number of deuterium moles y, since all tritium burns up very quickly in excess deuterium. Thus, the total number of reacting DT-moles is $2y = y_1$ per unit volume, and hence, the total number of deuterium moles remaining is $1 - 2y = 1 - y_1$. In other words, by using this approximation, we reduce the initial problem for the $(1-y) : y$ DT-mixture to the thermonuclear burn-up in the $(1-y_1) : y_1$ mixture of pure deuterium D_2 and equimolar (1 : 1) DT-mixture. Obviously, both of these components have the same initial density ρ_0.

For the $q_{\mathrm{dt}}(x, T)$ and $q_{\mathrm{dd}}(x, T)$ energy release functions in (7) we shall apply the following general expression:

$$q(x, T) = s(x, T)\left(1 + B\frac{bx}{1+bx}\right) - \frac{C\sqrt{T}}{1 + c\sqrt{\rho_0 x}\,T^{-7/4}} \tag{8}$$

where B, b, C and c are numerical constants and $s(x, T)$ is the 'short' or ionic energy release function, which corresponds to the energy gain produced only by positive high-energy ions arising in the thermonuclear reactions. The last (negative) term in this expression represents the *bremsstrahlung* loss ($\simeq C$) which includes the appropriate correction to its screening in the hot reaction zone ($\simeq c$). The second term ($\simeq B$) is the additional energy gain which neutrons (from thermonuclear reactions) leave in such a hot zone. The short energy release function $s(x, T)$ does not contain any *bremsstrahlung* loss or additional energy gain from the slowing-down neutrons. This function is directly related to the respective $\overline{\sigma v}$ value, e.g. for the 1 : 1 DT-mixture [4]:

$$\begin{aligned} s_{\mathrm{dt}}(x, T) &= \frac{Q_{\mathrm{dt}}(x, T)}{\rho_0^2} = n_{\mathrm{D}} n_{\mathrm{T}} \overline{\sigma v}_{\mathrm{dt}} \frac{E_{\mathrm{dt}}}{5} \times 10^6 \times 1.6021 \times 10^{-19}\ \frac{\mathrm{J\ cm^3}}{\mathrm{g^2\ s}} \\ &= \left(\frac{N_{\mathrm{A}}}{5}\right)^2 \overline{\sigma v}_{\mathrm{dt}} \frac{17.590}{5} \times 10^6 \times 1.6021 \times 10^{-34}\ \frac{\mathrm{MJ\ cm^3}}{\mathrm{g^2\ ns}} \\ &= 8.176\,11 \times 10^{18} \times \overline{\sigma v}_{\mathrm{dt}}\ \frac{\mathrm{MJ\ cm^3}}{\mathrm{g^2\ ns}} \end{aligned} \tag{9}$$

where N_{A} is the Avogadro number, σ_{dt} is the (d,t)-reaction cross section, v is the relative velocity of the deuterium and tritium nuclei and $\overline{\sigma v}_{\mathrm{dt}}$ denotes the averaged (dimensionless) value over the whole range of relative velocities. Note, also that the definition of the $Q_{\mathrm{dt}}(x, T)$ and $Q_{\mathrm{dd}}(x, T)$ functions in this and next two equations coincides exactly with that given in [10]. For the $s_{\mathrm{dd}}(x, T)$ function the appropriate expression is [4]:

$$s_{\mathrm{dd}}(x, T) = \frac{Q_{\mathrm{dd}}(x, T)}{\rho_0^2} = \frac{n_{\mathrm{D}}^2}{2} \overline{\sigma v}_{\mathrm{dd}} \frac{8.3685}{2} \times 10^6 \times 1.6021 \times 10^{-34}\ \frac{\mathrm{MJ\ cm^3}}{\mathrm{g^2\ ns}}$$

$$= 3.039\,79 \times 10^{19} \times \overline{\sigma v}_{\mathrm{dd}} \ \frac{\mathrm{MJ\ cm^3}}{\mathrm{g^2\ ns}}. \tag{10}$$

This formula is written in terms of the two following assumptions: (i) the ^{3}He nuclei from the (d,d)-reactions do not react with neutrons or deuterium nuclei, and (ii) the two channels in the (d,d)-reaction have equal probabilities.

An alternative approach is based on the assumption that all ^{3}He nuclei produced in the (d,d)-reactions are consumed instantaneously in reactions with neutrons. This problem becomes actual if the thermonuclear burn-up takes place in the very intense neutron fluxes. For instance, if the dense deuterium is compressed to some piece of fissionable material, e.g. ^{233}U, ^{239}Pu or ^{247}Cm, then after a quite strong compression the fission chain reaction may begin in the fissionable material, which produces a huge number of neutrons. Obviously, the competing reaction between ^{3}He and deuterium nuclei can be ignored in this case at all temperatures, and the short energy release function $\tilde{s}_{\mathrm{dd}}(x, T)$ takes a different form † [4]:

$$\tilde{s}_{\mathrm{dd}}(x, T) = Y\overline{\sigma v}_{\mathrm{dd}} \ \frac{\mathrm{MJ\ cm^3}}{\mathrm{g^2\ ns}} = 4.6319 \times 10^{19} \times \overline{\sigma v}_{\mathrm{dd}} \ \frac{\mathrm{MJ\ cm^3}}{\mathrm{g^2\ ns}}. \tag{11}$$

The function $s_{\mathrm{dd}}(x, T)$ in (10) can be called the channel closed function, i.e. all reactions for ^{3}He nuclei are ignored. The analogous $\tilde{s}_{\mathrm{dd}}(x, T)$ function in (11) corresponds to the open channel case, since the reactions of ^{3}He nuclei with neutrons as well as the subsequent (d,t)-reactions are included in the consideration.

The both $\overline{\sigma v}_{\mathrm{dt}}$ and $\overline{\sigma v}_{\mathrm{dd}}$ values have been approximated by the following seven-parameter interpolation formula:

$$\begin{aligned}\overline{\sigma v} = {} & \left(\frac{a_1}{T^{2/3}} + \frac{a_2}{T^{1/3}}\right)\exp(-18T^{-1/3}) + \left(\frac{a_3}{T^{2/3}} + \frac{a_4}{T^{1/3}} + a_5\right)\exp(-19T^{-1/3}) \\ & + \left(\frac{a_6}{T^{2/3}} + \frac{a_7}{T^{1/3}}\right)\exp(-20T^{-1/3}).\end{aligned} \tag{12}$$

This formula has quite good accuracy for the temperatures $T \leqslant 30$ keV. Using the known experimental data one can determine all seven coefficients in this formula. The values used in the present calculations are:

$$\begin{aligned} a_1 &= 0.105\,273\,862\,4290 \times 10^{-6} & a_2 &= -0.355\,434\,731\,1801 \times 10^{-6} \\ a_3 &= 0.612\,272\,521\,0748 \times 10^{-6} & a_4 &= -0.228\,092\,194\,2086 \times 10^{-6} \\ a_5 &= 0.227\,747\,682\,3480 \times 10^{-11} & a_6 &= 0.221\,876\,868\,9879 \times 10^{-6} \\ a_7 &= 0.583\,473\,371\,0655 \times 10^{-6} \end{aligned}$$

for $\overline{\sigma v}_{\mathrm{dt}}$, and

$$\begin{aligned} a_1 &= 0.092\,223\,532\,695\,29 \times 10^{-9} & a_2 &= -0.335\,997\,245\,1124 \times 10^{-9} \\ a_3 &= 0.699\,261\,131\,0986 \times 10^{-9} & a_4 &= -0.444\,900\,833\,2372 \times 10^{-9} \\ a_5 &= 0.615\,516\,961\,4240 \times 10^{-13} & a_6 &= 0.329\,760\,714\,9825 \times 10^{-9} \\ a_7 &= 0.780\,003\,664\,1361 \times 10^{-9} \end{aligned}$$

† If in the dense deuterium ($\rho_0 \geqslant 100$ g cm^{-3}) the temperature increases up to 25–40 keV, then the (^{3}He,d)-reaction cannot be ignored, and the constant Y in (11) is $\approx 9.7039 \times 10^{19}$. This means that the total (d,d)-energy gain becomes extremely large under such conditions. Actually, already at $T \approx 20$ keV the appropriate $\overline{\sigma v}$ values for both (d,d)- and (^{3}He,d)-reactions are quite comparable with each other (their ratio is ≈ 0.89 [10]). Despite this, the (^{3}He,d)-reaction is ignored in the present study, since it cannot be considered in terms of the analytical approach developed above. But this one case corresponds to the hardly attainable temperatures $T \geqslant 20$ keV [1]. For all realistic ignition temperatures $T \leqslant 15$ keV the (^{3}He,d)-reaction does not play any significant role.

Table 1. The critical values of the burn-up parameter x_c (in g cm^{-2}) for various densities ρ_0 (in g cm^{-3}), temperatures T (in keV) and relative tritium concentrations (in molar %) for a number of the DT-mixtures.

ρ_0	T	50%	40%	30%	20%	10%	5%	r_{crit} 50% (cm)
	4	4.5961	6.8383	11.2626	22.9906	91.9424	396.6848	"
1.0	10	0.5245	0.6993	0.9958	1.5933	3.3565	6.7405	"
	20	0.2523	0.3330	0.4675	0.7301	1.4506	2.6419	"
	4	3.6874	5.1110	7.5833	12.8090	30.5596	73.1005	0.36874
10.0	10	0.5212	0.6931	0.9827	1.5592	3.2123	6.2098	0.05212
	20	0.2518	0.3322	0.4657	0.7258	1.4350	2.5967	0.02518
	4	3.4030	4.6213	6.6733	10.8100	23.5411	50.0633	0.17015
20.0	10	0.5194	0.6896	0.9754	1.5406	3.1380	5.595 75	0.02597
	20	0.2515	0.3317	0.4647	0.7233	1.4260	2.5707	0.012575
	4	3.0575	4.0524	5.6727	8.7784	17.4229	32.9687	0.06115
50.0	10	0.5160	0.6833	0.9623	1.5079	3.0124	5.5578	0.01032
	20	0.2510	0.3307	0.4628	0.7187	1.4089	2.5217	0.00502
	4	2.8312	3.6938	5.0700	7.6293	14.3479	25.4316	0.028312
100.0	10	0.5126	0.6770	0.9494	1.4764	2.8976	5.2170	0.005126
	20	0.2505	0.3298	0.4608	0.7138	1.3912	2.4710	0.002503

for $\overline{\sigma v}_{dd}$. These coefficients were determined from a number of experimental works for the (d,t)- [11–16] and (d,d)-reactions [17–23], respectively. All the given significant digits in these coefficients are needed to reproduce the results from tables 1 and 2. The values of the parameters in equation (8) for the 1 : 1 DT-mixture are: $B = 4$, $b = 0.0480$, $C = 31.053$ and $c = 1.1$. For the dense deuterium when $s_{dd}(x, T)$ is used they are: $B = 1.9746$, $b = 0.09$, $C = 48.506$ and $c = 1.1$. In the case of the $\tilde{s}_{dd}(x, T)$ function the only difference in equation (8) is that $B = 2.4186$ (see discussion in [4]).

The data presented above are sufficient to solve the burn-up equation, (2), numerically, i.e. to determine the minimum or critical value of the burn-up parameter x_c ($= \rho_0 r_c$) at which the thermonuclear burn-up is still possible in the considered DT-mixture. The formal solution of the burn-up equation can be written in the following integral form [3] (the spherical symmetrical 3D-case):

$$T(x) = T_0 \left(\frac{x_0}{x}\right)^3 + \frac{1}{\mathcal{C} x_0^3} \int_{x_0}^{x} \mathrm{d}y \frac{q(y, T(y)) y^3}{V_{max}(y, T(y))} \tag{13}$$

where $x_0 = \rho_0 r_0$ is the initial value of the burn-up parameter. All the other notation is the same as in (2). The solution of (13) has the following behaviour, when x grows: it starts at $x = x_0$ ($T(x_0) = T_0$), then decreases, reaches a minimum and later increases to infinity. Note that if the initial conditions (i.e. T_0 and x_0) are given, then the solution $T(x)$ is determined uniformly. In particular, the minimal temperature $T_{min} = \min_x\{T(x)\}$ can also be found uniformly from the same initial conditions. The temperature T_{min} cannot be negative and the appropriate condition $T_{min} = 0$ is used to determine the critical value $x_0 = x_c$ of the burn-up parameter.

The results of our numerical calculations are presented in tables 1 and 2. Table 1 contains the x_c values for the six DT-mixtures with the following tritium concentrations: 50%, 40%, 30%, 20%, 10% and 5%. The first case is obviously the 1 : 1 DT-mixture. Table 2 includes the results for the DT-mixtures with relatively small tritium concentrations: 2%,

Table 2. The critical values of the burn-up parameter x_c (in g cm−2) for various densities ρ_0 (in g cm^{-3}), temperatures T (in keV) and relative tritium concentrations (in molar %) for a number of the DT-mixtures.

ρ_0	T	2%[a]	2%[b]	1%[a]	1%[b]	0%[a]	0%[b]
	4	2585.323	1348.924	5716.924	2457.511	17 917.65	5215.356
1.0	10	22.6131	15.0625	65.9269	27.4226	2545.865	148.6236
	20	6.2905	4.8285	10.4071	6.8131	34.7946	11.8769
	4	307.4934	175.8519	627.4971	289.8165	1862.847	569.7449
10.0	10	17.8825	12.7179	37.5916	20.4786	310.5040	54.3114
	20	6.0935	4.07042	9.9513	6.5953	30.3531	11.3247
	4	177.5690	106.7705	342.6201	166.5237	969.9067	309.9928
20.0	10	16.2005	11.7785	31.3848	18.2307	181.3108	41.9982
	20	5.9828	4.6332	9.7036	6.4729	28.4135	11.0282
	4	95.5263	61.4849	168.3255	88.9861	432.0840	151.4326
50.0	10	13.9780	10.4498	24.6920	15.3925	98.1656	30.6229
	20	5.7807	4.5017	9.2651	6.2491	25.5143	10.5078
	4	65.1213	43.8535	107.1271	60.4271	250.3166	96.0152
100.0	10	12.3850	9.4360	20.6499	13.4366	66.3967	24.5334
	20	5.5780	4.3671	8.4808	6.0241	23.1712	10.0089

[a] In this case all reactions with the ^{3}He nuclei are ignored.
[b] In this case the thermonuclear burn-up proceeds in very intense neutron fluxes.

1% and 0% (i.e. the pure deuterium). In table 2 we consider the two subcases for each DT- mixture. The left-hand subcolumn ([a]) in table 2 represents the results for the $s_{dd}(x, T)$ function (equation (10)), while the right-hand subcolumn ([b]) shows the results found with the $\tilde{s}_{dd}(x, T)$ function (equation (11)). The deviation between these two subcolumns becomes larger when the tritium concentration decreases. For table 1 only the $s_{dd}(x, T)$ function was used. In both tables we restrict our analysis to the maximum temperature of 20 keV. Actually, this means that in all considered mixtures the ignition is caused only by the detonation wave. By performing the calculations at higher temperatures, we have found that the high-temperature thermal wave plays a leading role only in the 1 : 1 DT-mixture and only for quite high temperatures $\geqslant$ 25 keV, i.e. for the hardly attainable temperatures [1].

what?? I don't get where this has come from

In discussing the results from tables 1 and 2, we shall assume that the thermonuclear burn-up becomes practically possible if the critical hot-spot radius $r_c = x_c/\rho_0$ decreases to the values which are less than or equal to 1 cm†. It follows from these two tables that the high compressions are not critically important for the thermonuclear burn-up in the dense 1 : 1 DT-mixture nor for such mixtures with 40% and 30% of tritium. However, to produce the thermonuclear ignition in the DT-mixtures with 20 %, 10 % and 5 % of tritium the initial densities required must be already $\approx$ 15–20 g cm^{-3}. In fact, such values are quite high and comparable with the densities of heavy metals. Nevertheless, in the DT-mixtures with 2% or 1% of tritium, and even more in the pure deuterium, the thermonuclear burn-up may begin only after preliminary compressions to the densities $\rho_0 \approx$ 60–100 g cm^{-3}, which are

† For larger r_c values in the highly compressed fuels ($\rho_0 \geqslant$ 10 g cm^{-3}) the thermonuclear burn-up requires enormous amounts of energy, and seems to be very unlikely in practice. Furthermore, without loss of generality, one may assume that the real thermonuclear burn-up proceeds in the fuel which is highly compressed between two finite coaxial cylinders with radii R_{in} and R_{out}. The choice ($r_c \approx$ 1 cm) made in the present study follows from the fact that in all real situations the difference $R_{out} - R_{in}$ is bounded between $\approx$ 0.2 cm and 5 cm.

well referenced Results

significantly higher than the densities of heavy metals. The required densities coincide very well with the known results [24–26] for thermonuclear detonation in the dense deuterium and DT-mixtures with low tritium concentrations. It would be wrong to conclude that the extremely high compressions mentioned increase the thermonuclear energy gain. Clearly, the energy gain grows gradually with compression, whilst the *bremsstrahlung* loss drops very quickly. Finally, the energy balance becomes positive and the mixture burns up.

In addition, it follows from table 2 that the thermonuclear ignition simplifies significantly in the presence of very intense neutron fluxes. This effect is of paramount importance for the DT-mixtures with low tritium concentrations and for relatively small (or 'realistic') temperatures $T \approx 4$–6 keV. In such cases, the critical burn-up parameter drops by approximately three times. Thus, the essential concluding point is that the increasing of tritium concentrations (y) can be applied to simplify the thermonuclear burn-up in the $(1-y) : y$ DT-mixtures with $y \leqslant 0.5$. This acts in the same direction as the use of additional compressions or the presence of intense neutron fluxes. Note that for all considered DT-mixtures presented in tables 1 and 2 ($T \leqslant 20$ keV and $\rho_0 \leqslant 100$ g cm^{-3}) the propagation of the thermonuclear burn is governed by the usual detonation.

What T or ρ would be relevant for Astro?

To complete the solution let us show that the densities obtained are significantly lower than the values which correspond to the Fermi limit of electron degeneracy. As is well known (see e.g. [27]) if the density of the cold matter is quite high, then all electrons should be considered as fermions. In this case, the internal electronic pressures P_e as well as the speed of sound S_s ($\simeq \sqrt{P_e}$) in the cold thermonuclear fuel increase to extremely large values. The appropriate disassembly time $\tau \approx R/S_s$ becomes very short and thermonuclear burn-up cannot start effectively. Here R denotes the spatial radius of the highly compressed volume in the fuel. Such a correction on the electron degeneracy is obviously needed when the actual temperature T is comparable with the equivalent Fermi temperature T_{ef} [27]. For the considered $(1-y) : y$ DT-mixtures T_{ef} takes the following form:

would it be fermi degenerate?

$$T_{ef}\ [\text{keV}] = \frac{2}{5}\left(\frac{3N_A}{8\pi}\right)^{2/3}\frac{h^2}{2m_e}\left(\frac{\rho_0}{2+y}\right)^{2/3} = 9.301\,993\,8991 \times 10^{-3} \times \left(\frac{\rho_0}{2+\frac{y_1}{2}}\right)^{2/3} \quad (14)$$

where N_A, h and m_e are the Avogadro number, Planck constant and electron mass, respectively [28]. The largest T_{ef} value can be found in pure deuterium ($y_1 = y = 0$), where $\rho_0 \leqslant 200$ g cm^{-3}, i.e. $T_{ef} \leqslant 0.200\,41$ keV. This value is significantly smaller than the temperatures used in the present study $T \geqslant 4$ keV, i.e. the appropriate Fermi correction is negligible.

It should be mentioned in conclusion that the minimal energy $\mathcal{E}_c$ required to produce the thermonuclear ignition in the considered DT-mixtures can be easily evaluated in terms of the known x_c parameter:

$$\mathcal{E}_c = \frac{4\pi}{3}x_c^3\rho_0^{-2}\mathcal{C}T$$

nice eqs for E_{in}.

very important when discussing relevance as ⇒ $E_{in} \propto x_c^3$ ∴ must make x_c^3 small

where ρ_0 is the initial density and T is the burn-up temperature. For instance, for the $1:1$ DT-mixture at $\rho_0 = 10$ g cm^{-3} and $T = 4$ keV, one finds easily, using the results from table 1, that $\mathcal{E}_c = 9.726 \times 10^2$ MJ. The analogous result (under the same conditions) for the $0.9:0.1$ DT-mixture is $\mathcal{E}_c = 6.591 \times 10^5$ MJ, while for the dense deuterium $\mathcal{E}_c = 1.569 \times 10^{11}$ MJ (without neutrons) and $\mathcal{E}_c = 4.490 \times 10^9$ MJ in the presence of intense neutron fluxes. The last two figures indicate that the thermonuclear burn-up is very unlikely in the pure deuterium at these densities and temperatures. However, already for $\rho_0 = 100$ g cm^{-3} and $T = 4$ keV such figures are $\mathcal{E}_c = 9.138 \times 10^7$ MJ (without neutrons) and $\mathcal{E}_c = 2.124 \times 10^5$ MJ (with neutrons), respectively, i.e. the burn-up becomes practically

possible. Such a numerical comparison together with the cubic dependence of $\mathcal{E}_c$ upon x_c (i.e. $\mathcal{E}_c \simeq x_c^3$) illustrate clearly why the decreasing of x_c values is so extremely important for practical applications.

Thus, the thermonuclear burn-up in the DT-mixtures with different tritium concentrations (including low concentrations and dense deuterium) has been considered in the present study. The thermonuclear burn-up characteristics for the highly compressed deuterium and DT-mixtures with low tritium concentrations have been determined and openly discussed for the first time. It is shown that the thermonuclear burn wave in the highly compressed DT-mixtures ($\rho_0 \approx 50$–100 g cm^{-3}) with low tritium concentrations is essentially the high-temperature detonation wave (for $T \leqslant 20$ keV). The agreement between our results and data which can be found from previous works seems to be very good. This indicates that our simple method works quite accurately and can be used to analyse the thermonuclear burn-up in really complicated systems which are of interest for numerous applications (e.g. in the ^{6}LiD, ^{7}LiD and ^{9}BeD$_2$ deuterides and their mixtures; see appendix below).

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Appendix

Let us discuss briefly the thermonuclear burn-up in the DT-hydrides and deuterides of light elements. Without loss of generality, such a consideration can be restricted to the ^{6}Li, ^{7}Li and ^{9}Be hydrides only. The thermonuclear burn-up in the appropriate hydrides of heavier elements with $Z \geqslant 5$ and $A \geqslant 10$ (e.g. in ^{10}BD$_3$, ^{12}CD$_4$ or ^{14}ND_3) is very difficult to attain in practice (see discussion in [4]). Note that the thermonuclear burn-up has never been discussed in the literature for the BeDT hydride, dense deuterium (see above) and all the deuterides mentioned in our work. Among DT-hydrides only the ^{6}LiD + ^{6}LiT mixture was considered in [6]. The main goal of the present analysis is to illustrate various complications which can be neglected in binary DT-mixtures. The source of such complications is quite clear: the energy gain drops by 10–20 times in comparison with the corresponding equimolar DT-mixture or pure deuterium (e.g. in ^{9}BeDT or BeD$_2$, respectively), while the appropriate *bremsstrahlung* loss increases by three to seven times. In general, this means that the DT-hydrides and deuterides must be compressed $\approx$ 30–150 times more strongly than the equimolar DT-mixture or pure deuterium, respectively. In other words, to produce the thermonuclear burn-up in the deuterides of light elements, e.g. in ^{7}LiD or ^{9}BeD$_2$, the actual densities must be $\approx 1 \times 10^4$–2.5×10^4 g cm^{-3} or even higher, which are quite close to the respective Fermi limits.

Let us introduce the notation (α, β) to denote (d,d) for the deuterides and (d,t) for the DT-hydrides. Using this notation the corresponding energy release function $q_{\alpha,\beta}(x, T)$ can be written in the following universal form:

$$q_{\alpha,\beta}(x, T) = \mu_{\alpha,\beta}^2 s_{\alpha,\beta}(x, T)\left(1 + B_{\alpha,\beta}\frac{b_{\alpha,\beta}x}{1 + b_{\alpha,\beta}x}\right) - \frac{\kappa_{\alpha,\beta}\mu_{\alpha,\beta}^2 C_{\alpha,\beta}\sqrt{T}}{1 + c_{\alpha,\beta}\sqrt{\rho_0 x}T^{-7/4}} \tag{15}$$

where $\mu_{\alpha,\beta}$ is the ratio of the (α, β) fragment mass to the total atomic (quasi-molecular) mass and the factor $\kappa_{\alpha,\beta}$ takes the well known form [10]:

$$\kappa_{\alpha,\beta} = \frac{(\sum_i n_i Z_i)(\sum_i n_i Z_i^2)}{(\sum_\alpha n_\alpha Z_\alpha)(\sum_\alpha n_\alpha Z_\alpha^2)} = \frac{(\sum_i n_i Z_i)}{2}\frac{(\sum_i n_i Z_i^2)}{2} \tag{16}$$

Table 3. The critical values of the burn-up parameter x_c (in g cm^{-2}) for various densities ρ_0 (in g cm^{-3}), temperatures T (in keV) for a number of the DT-hydrides and deuterides.

T	ρ_0	^{6}LiD + ^{6}LiT	^{6}LiD + ^{7}LiT	^{9}BeDT
4	100	132.8309	133.1406	69.5109
4	200	82.0303	82.3212	45.7513
10	100	15.7774	15.8594	8.7032
10	200	12.8714	12.9484	7.4017
20	100	5.5914	5.6511	2.9993
20	200	5.2120	5.2680	2.8514

T	ρ_0	^{6}LiD	^{7}LiD	^{9}BeD$_2$
4	1×10^3	33 655.92^a /11 107.48^b	33 695.70^a /11 130.04^b	15 399.29^a / 5100.442^b
4	1×10^4	3942.081^a/ 1434.605^b	3979.020^a/ 1454.743^b	1876.883^a / 699.7533^b
10	1×10^3	16 114.78^a / 5218.6325^b	16 128.99^a / 5226.940^b	7289.887^a / 2351.742^b
10	1×10^4	1824.342^a/ 645.1190^b	1837.714^a/ 652.4691^b	856.1467^a/ 309.1134^b
$\bar{x}$	1×10^3	15 996.40^a / 5147.452^b	16 009.46^a / 5154.715^b	7210.59^a / 2297.900^b
$\bar{x}$	1×10^4	1792.03^a / 623.187^b	1803.865^a/ 629.433^b	834.325^a / 294.373^b
T_{cr}	1×10^3	11.26^a / 11.81^b	11.26^a / 11.81^b	11.58^a / 12.47^b
T_{cr}	1×10^4	12.01^a / 12.97^b	12.06^a / 13.03^b	12.50^a / 13.83^b

T	ρ_0	DT + ^{4}He + 2 ^{6}LiD^b	2DT + 2 ^4He + ^{6}LiD^b	DT + ^{4}He^b
4	100	1555.215	322.8455	12.2237
4	500	355.6950	88.9859	8.1258
10	100	173.1460	32.9790	1.9682
10	500	57.1477	16.3843	1.7560
20	100	63.2244	10.3168	0.8771
20	500	31.6085	7.6120	0.8427

[a] In this case all reactions with the ^{3}He nuclei are ignored.
[b] In this case the thermonuclear burn-up proceeds in very intense neutron fluxes.

where the summation is taken over all nuclei present in the initial molecule. For instance, in the ^{6}LiD + ^{6}LiT mixture one easily finds that

$$\mu_{\alpha,\beta} = \mu_{\mathrm{d,t}} = \frac{2+3}{6+2+6+3} = \frac{5}{17} \qquad \kappa_{\alpha,\beta} = \kappa_{\mathrm{d,t}} = \frac{1+3+1+3}{2}\,\frac{1^2+3^2+1^2+3^2}{2} = 40.$$

Finally, $\kappa_{d,t}\mu_{d,t}^2 = 3.460\,2076$ which exactly coincides with the value used in [6]. The use of these two factors, $\mu_{\alpha,\beta}$ and $\kappa_{\alpha,\beta}$, allows us to apply the same short energy release functions $q_{\alpha,\beta}(x, T)$ and *bremsstrahlung* constants $C_{\alpha,\beta}$ as for the corresponding equimolar DT-mixture or pure deuterium. At least in the first order, this approximation seems to be very good. The parameters $B_{\alpha,\beta}$, $b_{\alpha,\beta}$ and $c_{\alpha,\beta}$ depend significantly upon the ionic contents of the considered mixture. But for our present approximate purposes we shall assume that they do not change unless the D : T ratio remains constant. This means that the same parameters $B_{\mathrm{d,t}}$, $b_{\mathrm{d,t}}$ and $c_{\mathrm{d,t}}$ can be used, e.g. in the equimolar DT-mixture, ^{6}LiD + ^{7}LiT and ^{9}BeDT hydrides.

The specific heat per unit mass, $\mathcal{C}$, and other values which are needed to solve the burn-up equation (equation (2) or equation (13)) can be determined using the equations given above. For instance, for the ^{6}LiD + ^{7}LiT mixture we find from (3) that $\mathcal{C}$ =

96.477 623 07 MJ g^{-1} keV^{-1}. The results for the critical burn-up parameters x_c as functions of the temperature T and density ρ_0 for a number of DT-hydrides and deuterides can be found in table 3. As expected, at comparable temperatures and densities the critical values of x_c required for the thermonuclear burn-up in such DT-hydrides are significantly larger than those values for the dense equimolar DT-mixture. The dependence $x_c(\rho_0)$ has a very sharp form for small ρ_0 ($\rho_0 \approx 1$–10 g cm^{-3}), where the ratio of the equimolar DT-results to, e.g. $^6LiD + {}^6LiT$ results, is approximately 200–300. When the density ρ_0 increases the dependence $x_c(\rho_0)$ takes a more smooth form. In addition, when the temperature grows the ratio of the burn-up parameters for the dense equimolar DT-mixture and the appropriate DT-hydrides becomes smaller. But even for $T \geqslant 20$ keV this ratio is a factor of ten (compare tables 1 and 3).

The numerical values of the burn-up parameters for the ^6LiD, ^7LiD and $^9BeD_2$ deuterides are also given in table 3. The results for deuterides in table 3 are presented in the form of a fraction A/B, where A corresponds to the first $q_{dd}(x, T)$ function (equation (10)), while the denominator B represents the x_c values for the second $\tilde{q}_{dd}(x, T)$ function (equation (11)). It follows from numerical calculations that the extremely uneven energy balance in the deuterides produces another complication which cannot be observed for the DT-hydrides or dense deuterium. To explain this, note that the velocity of the detonation wave V_D increases with the temperature as $V_D \simeq a\sqrt{T}$ (equation (4)), while the thermonuclear ignition in any deuteride of light elements obviously takes a longer time than in the dense deuterium. Therefore, we would expect that at some critical temperature T_{cr} ($T_{cr} \approx 11$–14 keV and depends on the initial density ρ_0) the high-temperature thermal wave may transform, in principle, into a usual shock wave. This shock wave breaks away from the surface of the hot sphere and propagates into the rest of the fuel without producing any ignition in it. Later, however, when the temperature behind such a shock wave decreases to $T = T_{cr}$ the burn-up begins, i.e. the high-temperature shock wave transforms (i.e. slows down) to the detonation wave with a lower temperature. But at this moment $x > x_c$, since the hot zone expands continuously. In fact, this means that the critical burn-up parameters $x_c(T, \rho_0)$ have the finite, non-zero limit value $\overline{x}(T_{cr}, \rho_0)$ at $T \to +\infty$. In other words, in all such deuterides (LiD and BeD_2) the minimal burn-up parameter x_c decreases (when the temperature increases) to a specific 'critical' value $\overline{x}(T_{cr}, \rho_0)$, and then does not change. Obviously, this can be reformulated to the form that in all such substances and their mixtures the finite, critical ignition temperature T_{cr} exists, and its value depends only on the initial density ρ_0. This means that at given ρ_0 the critical burn-up radius r_{cr} ($r_{cr} = \overline{x}(T_{cr}, \rho_0)/\rho_0$) can be determined uniformly for each deuteride, and moreover, its value (i) does not vanish when temperature grows, and (ii) does not depend on the temperature T, if $T \geqslant T_{cr}$. The $\overline{x}$ and T_{cr} values are also presented (for each density) in table 3. Since $T_{cr} \leqslant 17$ keV in all cases, the temperature $T = 20$ keV is not considered for the deuterides in table 3.

In conclusion, consider the thermonuclear burn-up in the ^6LiD deuteride in the presence of highly intense neutron fluxes. In this case, as mentioned above, all ^3He nuclei produced in the (d,d;p, ^3He)-reactions will react with neutrons (^3He,n;p,t) producing the tritium nuclei. However, the (^6Li,n;α,t)-reaction has approximately equal cross sections. This means that such additional tritium formation must be taken into account from the very beginning, when the thermonuclear burn-up in ^6LiD is studied in the presence of highly intense neutron fluxes. Now, to avoid having to solve a very complicated problem related to the propagation of the neutrons, we assume that some part z (where $0 \leqslant z \leqslant 1$) of the initial ^6Li nuclei will react with neutrons. In other words, we consider the partially ($z < 1$) or completely ($z = 1$) converted ^6LiD. The conversion means the transformation of the ^6LiD (by neutrons) to the zDT + $z\,^4He + (1 - z)\,^6LiD$ mixture. In fact, this means, that the neutrons move ahead of

the thermonuclear burning (or detonation) wave. Obviously, such a view gives only a quite approximate picture of the real process since in general, the neutrons and burning waves move together and interact with each other. But using this approximation we can easily illustrate the principal difference between ^{6}LiD and ^{7}LiD or BeD_2.

In the present study we used the following three z values in the calculations: $z = 1/3$, 2/3 and 1. The values of the critical burn-up parameter x_c determined are given in table 3. Note that, only the $\tilde{q}_{dd}(x, T)$ function is used in such calculations for table 3. As can be seen from this table, the thermonuclear burn-up of the ^{6}LiD deuteride proceeds at significantly reduced compressions ($\approx$ 70–100 times) in the presence of highly intense neutron fluxes. In fact, the values x_c for the converted ^{6}LiD deuteride are closer to the results for the DT-hydrides than to the values found for the corresponding deuterides. In addition, it should be mentioned that in all such substances (in deuterides as well as in DT-hydrides) the reaction hot zone propagates in the rest of the thermonuclear fuel as a conventional (high-temperature) detonation wave. Only in the dense equimolar DT-mixture at temperatures $\geqslant$ 25 keV may the high-temperature thermal wave play a leading role in such a propagation.

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