University of California at Santa Cruz

Determination of the Gravitational Constant

PHYS 134 Lab Report

by

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Abstract

Gravity is the attractive force between all objects in the universe with mass. As described by Newton's law of universal gravitation, the force of gravity between two objects is proportional to the product of their two masses and the inverse squared distance between them. However, Newton's law also includes a constant of proportionality, *G*, the universal gravitational constant, which can be used to determine the exact magnitude of the force of gravity between two objects. The experiment detailed in this report uses an apparatus, which was originally designed by English physicist Henry Cavendish, to measure the constant *G*. A boom with two small lead spheres at its ends is suspended from its center by a fine tungsten fiber, and moving a set of larger lead spheres near the smaller spheres causes the boom to oscillate in a damped simple-harmonic fashion. By measuring the period of these oscillations, as well as the new equilibrium angle of the boom after the oscillations had died down, the constant *G* was determined to be $7.24 \times 10^{-11} \pm 1.10 \times 10^{-11}\, m^3/kg \cdot s^2$.

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Introduction

Physics currently describes four different fundamental interactions (or forces) in the universe: the strong nuclear force, the electromagnetic force, the weak nuclear force, and the gravitational force. Of these four, gravity is the weakest, but its effects can still be felt and seen quite strongly. Gravity keeps us tethered to the surface of the Earth, it keeps the Earth in a stable orbit around the sun, and it keeps our solar system orbiting around the center of our galaxy. In fact, neither the Earth, nor the sun, nor our galaxy would even exist without gravity. It is for these reasons, as well as an uncountable number of others, that gravity has long been a topic of intense study among physicists.

The force of gravity between two objects is given by Equation 1.1, Newton's law of universal gravitation (Newton, 1687).

$$\vec{F}_{12} = G\frac{M_1 M_2}{r^2}\hat{e}_{12} \tag{1.1}$$

As the law describes, all objects with mass are attracted to each other via the gravitational force, and the strength of this attraction is proportional to the product of the two objects' masses and the inverse squared distance separating them. To find the exact magnitude of attraction, though, the proportionality constant *G* is also needed. The experiment detailed in this report, which was originally performed by English physicist Henry Cavendish in the late 18th century, seeks to measure this constant G via the torque that arises from the mutual attraction of two sets of lead spheres with known masses attracting each other via the gravitational force.

Two small lead spheres are attached to the end of a boom which is suspended from its center by a very fine tungsten wire, and when a set of larger lead spheres are brought very close to the smaller spheres a torque is exerted on the boom. This torque is countered by the torsion of the twisting tungsten fiber, which causes the boom to oscillate in a damped simple harmonic fashion. Because it is damped, the boom eventually comes to a stop at a new equilibrium position, and by measuring the change between the original equilibrium position and the new equilibrium position, as well as the period of the oscillations, the magnitude of the torque between the spheres, and therefore the constant G, can be calculated.

Once the constant G is known, it can be used in conjunction with Newton's law of universal gravitation to calculate the force of gravity between any two objects provided that their masses and the distance between their respective centers of mass are known. With some manipulation, Newton's law can also be used to determine the mass of an unknown object provided that the force of gravity between the unknown object and a known object is also known.

1.1 Background

Forces are the pushing or pulling between two objects as a result of their mutual interactions. All objects in the universe with mass experience a mutual attraction via the gravitational force, and the strength of this attraction is given by Equation 1.1, Newton's law of universal gravitation, restated below.

$$\vec{F}_{12} = G\frac{M_1 M_2}{r^2}\hat{e}_{12} \tag{1.1}$$

As Newton's law describes, the strength of the gravitational force between two objects (typically measured in newtons) is given by the product of the mass of the first object, M_1, the mass of the second object, M_2, the inverse squared distance separating the two objects, r^2, and the constant of proportionality G, called the *universal gravitational constant*.

For two point masses, the distance r is simply equal to the distance between the two points, but for objects whose mass is distributed over their total volume (i.e. for real objects) the distance r is equal to the distance between the two objects' centers of mass. This is true because the gravitational force is a central force, meaning that the effective force between two objects emanates from their respective "force centers." In this experiment, the important objects between which the gravitational force is measured are uniform lead spheres, meaning that they are spherically symmetric. Because of this, they can be treated like point masses whose total mass is concentrated at their centers (Newton, 1687).

Objects can be made to rotate about a central axis by applying torques to them. In this case, the force exerting the torque is gravity, and thus the torque is given by Equation 1.2

$$\tau = r_l F_G \sin\theta_1 \tag{1.2}$$

where τ is the torque, r_l is the distance from the axis of rotation at which the force is being applied, F_G is the force of gravity, and θ_1 is the angle at which the force is applied with respect to r_l (Giancoli, 2000, p. 248). The net torque being applied to an object can be related to its moment of inertia (its resistance to angular acceleration) by Equation 1.3.

$$\Sigma\tau = I\alpha \tag{1.3}$$

where $\Sigma\tau$ is the net torque, I is the moment of inertia, and α is the angular acceleration resulting from the torques (Giancoli, 2000, 250).

Although described with more detail in the "Methods" section of this report, the apparatus used in this experiment is essentially a torsion pendulum consisting of a boom with two lead spheres at either end which is suspended from its center via a fine tungsten wire.

When more massive lead spheres are brought close to those mounted on the boom, their mutual gravitational attraction exerts a torque, which makes the boom rotate. The larger masses are placed in such a way that the torque exerted on the boom is given by Equation 1.4

$$\tau_g = 2\frac{GmM}{R_s^2}d \tag{1.4}$$

where m is the mass of the smaller lead spheres, M is the mass of the larger lead spheres, R_s is the distance between the centers of the large and small spheres, and d is the distance of the small spheres' centers from the rotation axis (i.e. the tungsten wire). Notice that the $\sin\theta_1$ term from Equation 1.2 is not present. This is because the rotation of the boom is small enough that θ_1 is approximately equal to $\frac{\pi}{2}$, and thus $\sin\theta_1$ is equal to 1.

As the boom rotates it twists the tungsten wire, which provides a restoring torque whose magnitude is given by Equation 1.5

$$\tau_r = -K\theta \tag{1.5}$$

where K is the torsion constant and θ is the angular displacement of the boom from its equilibrium position, when the wire was not twisted (Giancoli, 2000, p. 374). Because the torque supplied by gravity and the restoring torque both depend on displacement, the rotation of the boom will be damped simple harmonic, and therefore the angle θ will change as a function of time. Damped simple harmonic oscillators are governed by Equation 1.6

$$\frac{d^2\theta}{dt^2} + 2b\frac{d\theta}{dt} + \omega_0^2\theta = 0 \tag{1.6}$$

and the solution to Equation 1.6 is given by Equation 1.7

$$\theta(t) = \pm\theta_D + A \cdot exp(-bt) \cdot cos(\omega_1 t + \delta) \tag{1.7}$$

where A is the amplitude of the oscillations, b is the damping constant, ω_1 is the damped angular frequency of oscillation, and δ is the phase angle defined by the origin in time. ω_1 is related to ω_0, the undamped angular frequency of oscillation, by the relation described by Equation 1.8 (Brown, 2022, pp. 4-5):

$$\omega_1 = \sqrt{\omega_0^2 + b^2} \tag{1.8}$$

Because the system is damped, its oscillations will eventually decay to the point where they are effectively negligible, and thus the boom will reach an equilibrium angle θ_D. When this point is reached, the torque arising from the gravitational attraction between the spheres and the restoring torque of the wire will be equal to each other, and thus the relationship between the torque due to gravity and the equilibrium angle θ_D is given by Equation 1.9

$$\theta_D = \frac{\tau_g}{K} \tag{1.9}$$

The moment of inertia of the boom, I, and the torsion constant K are both given by Equations 1.10 and 1.11 respectively.

$$I \approx 2md^2 \tag{1.10}$$

$$K \approx I\omega_1^2 \approx 2md^2\omega_1^2 \tag{1.11}$$

Acknowledging the fact that $\omega_1 = 2\pi/T$, where T is the period of the systems' oscillations, Equations 1.4 and 1.11 can be plugged into their respective positions in Equation 1.9, which can then be solved for G. Doing this yields Equation 1.12 (Brown, 2022, p. 6):

$$G \approx \frac{(2\pi/T)^2 R_s^{\,2} d}{M} \theta_D \tag{1.12}$$

Thus, G can be determined by measuring the equilibrium angle θ_D and the period of the boom's oscillations, T.

2

Methods

A total of three different trials were conducted, each producing its own value for G. For each trial the large lead spheres were placed such that they could exert a gravitational force on the smaller lead spheres of the major boom. The resulting torque caused the major boom to oscillate, and the period of these oscillations was measured. Once the oscillations had died down, the new equilibrium angle was also measured. These two measurements were then used to calculate G, and the three resulting G values were then averaged to obtain the final value for G.

2.1 Description of Apparatus

Figure 2.1, presented below, depicts an overhead view of the apparatus.

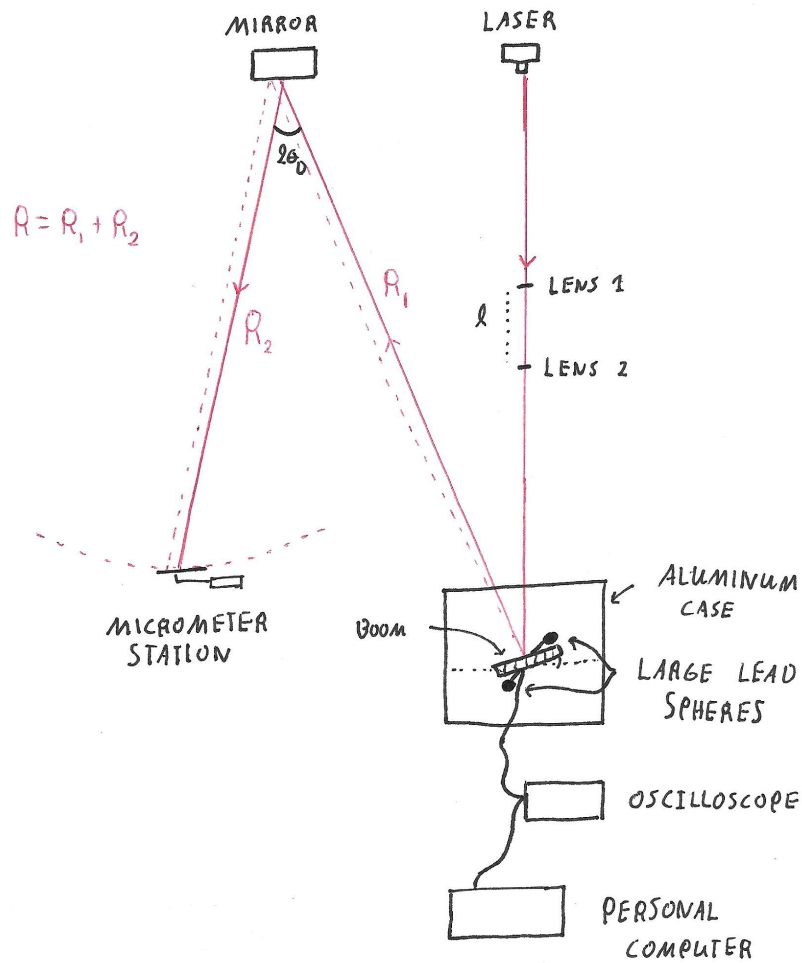

Figure 2.1: A diagram of the experimental apparatus as viewed from above. When the large lead spheres are moved close to the boom it will begin to oscillate simple-harmonically. The angular displacement of the boom is measured electronically and recorded using a computer, allowing the boom's period of oscillation to be determined. Additionally, once the boom's oscillations have settled down, the laser beam, which has been focused by two lenses and reflected off of a mirror on the boom's casing, will move by a distance Δx from its previous position (the dotted red line). This Δx can be used to determine the new equilibrium angle of the boom. R is the travel distance of the beam from the boom, and ℓ is the distance between the two focusing lenses.

The most important part of the apparatus is the major boom, which, as mentioned in the introduction, is essentially a torsion pendulum. Two small lead spheres, each with a radius of 6.72 mm, rest on the ends of the major boom, which itself is suspended from its center via a thin tungsten fiber. The aluminum beam that makes up the major boom is 150 mm long, and the spheres are positioned at its ends such that their centers are 66.56 mm away from the rotation axis at the boom's center. The whole major boom and its components are enclosed by two glass plates, which prevent accidental damage to the fiber or misalignment of the spheres. A more detailed view of this part of the apparatus is shown in Figure 2.2.

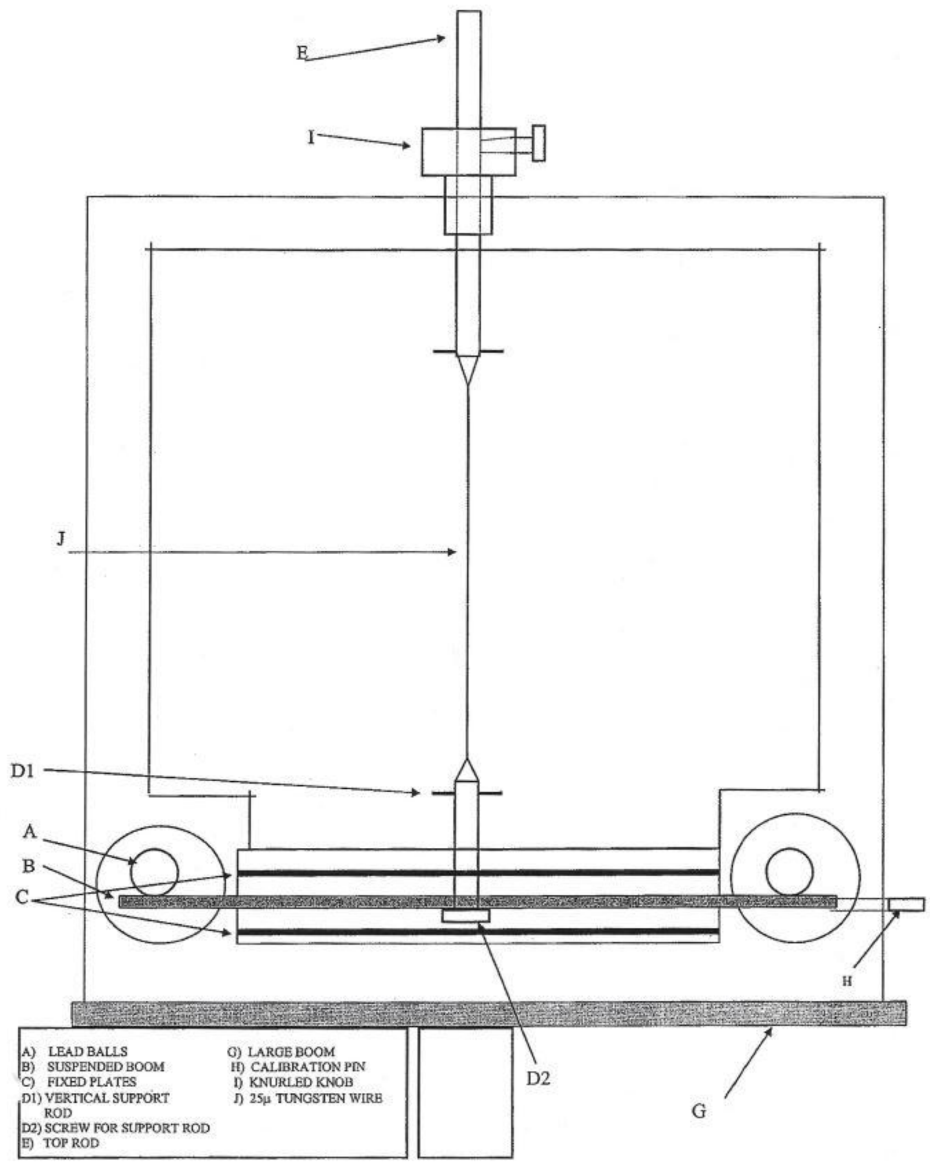

Figure 2.2: A diagram of the major boom part of the apparatus, taken from Tel-Atomic's Computerized Cavendish Balance manual ('Tel-Atomic', p. 3). Two lead spheres (A) are placed on the end of the major boom (B) which is suspended by a fine tungsten fiber (J) and thus is free to rotate. When the large lead spheres (the two larger spheres) are brought close to the smaller lead spheres, the major boom will begin to oscillate.

In addition to the glass plates, both the boom and its mount are enclosed in an aluminum case, which prevents disturbances arising from miscellaneous air currents and electric fields.

Torque is applied to the major boom by two larger spheres (also pictured in Figure 2.2), which are placed at the ends of their own, separate boom for easy adjustment. This minor boom can be rotated such that the larger spheres occupy positions very close to the smaller spheres of the major boom (their centers are separated by 44.9 mm), resulting in a non-negligible gravitational force between each of the large spheres and their corresponding small sphere. As explained in the introduction, the resulting torque causes the major boom to rotate, twisting the tungsten fiber from which it is suspended. This twisting, as a result of the fiber's torsion, provides a counter torque which eventually completely counteracts the gravitationally-induced torque and thus results in the major boom's oscillation.

As the major boom oscillates, its angular displacement from a set zero position is measured via a series of electrical components, and these measurements are output as electrical signals of strengths between 600 and 1000 mV. These signals are first passed through an oscilloscope, which can be used to directly observe the angular displacement of the boom as a function of time, before finally being passed to a personal computer, which is used to record the boom's angular displacement throughout the entire trial. This data can be used to measure the boom's period of oscillation through a procedure which is discussed in more detail later in this section.

After some time, typically about one hour, the boom comes to a stop at a new equilibrium angle, which, as discussed in the introduction, is determined by the magnitude of the torque exerted by the gravitational attraction between the spheres. Measurement of this angle is achieved by first aiming a laser beam, which has been focused by two lenses, through a small hole in the boom’s aluminum enclosure at a small mirror positioned on the major boom’s glass casing. After bouncing off of this mirror, the beam travels back through the hole where, as seen in Figure 2.1, it bounces off of another mirror positioned near the laser emitter. Finally, the beam travels until it hits a measurement card at the micrometer station. The position of the laser beam is then measured two times: once before the adjustment of the large lead spheres, and once after the boom’s oscillations have settled down. Via a procedure which is described with more detail later in this section, these two positions, effectively the two equilibrium positions of the boom, can be compared and used to calculate the desired equilibrium angle.

Once both the period of oscillation and the boom’s equilibrium angle have been measured, they can be used to calculate the constant G.

2.2 Calibration of Apparatus and Measurement of Beam Radius

Before any measurements could be made, the apparatus first needed to be calibrated so that the position of the laser could be measured at the micrometer station. This was done by very subtly adjusting the knurled adjustment knob (Labeled “I” in Figure 2.2) located at the major boom’s attachment point. Doing this altered the position of the boom very slightly such that laser light reflecting off of the boom’s mirror would always be confined to the surface of the larger mirror at the other end of the apparatus, thus ensuring that the beam was also confined to the measurement card of the micrometer station.

It was convenient to measure the beam’s travel distance, R (described in Figure 2.1), at the same time, and this was done using a two meter measuring stick. Two separate measurements were made and their average was taken to obtain the final length of R. The uncertainty in this measurement was determined by calculating the standard deviation of the two measurements.

2.3 Measurement and Calculation of the Boom's Oscillation Period

Determination of the boom's period of oscillation was conducted by first measuring the angular displacement of the boom every two seconds over the course of one hour. As discussed earlier in this section, measurements were taken by the boom electronics and were recorded on a personal computer. A python program was then used to determine the locations of oscillation amplitudes within the data as well as the corresponding times at which they occured.

Using this information, the time difference between the occurrences of 10 oscillation peaks and their corresponding troughs were measured, yielding several half periods. All 10 half periods were then averaged, and this average was used with Equation 2.1 to obtain the full period of oscillation

$$T = 2\bar{t} \tag{2.1}$$

where T is the full period and $\bar{t}$ is the average half-period. The uncertainty in this value was determined by calculating the standard deviation of the half periods according to Equations 2.2 through 2.4

$$\sigma_t^{\,2} = \frac{1}{N-1}\sum_{i=1}^{N}(\bar{t} - t_i)^2 \tag{2.2}$$

$$\sigma_{\bar{t}} = \frac{\sigma_t}{\sqrt{N}} \tag{2.3}$$

$$\sigma_T = 2\sigma_{\bar{t}} \tag{2.4}$$

where N is the number of half-periods, in this case 10.

2.4 Measurement of Δx and θ_D

Whenever the large lead spheres were brought close enough to the major boom such that it began to oscillate before eventually coming to rest at a new equilibrium, the position of the laser beam at the micrometer station changed by a distance Δx. This is depicted in Figure 2.3.

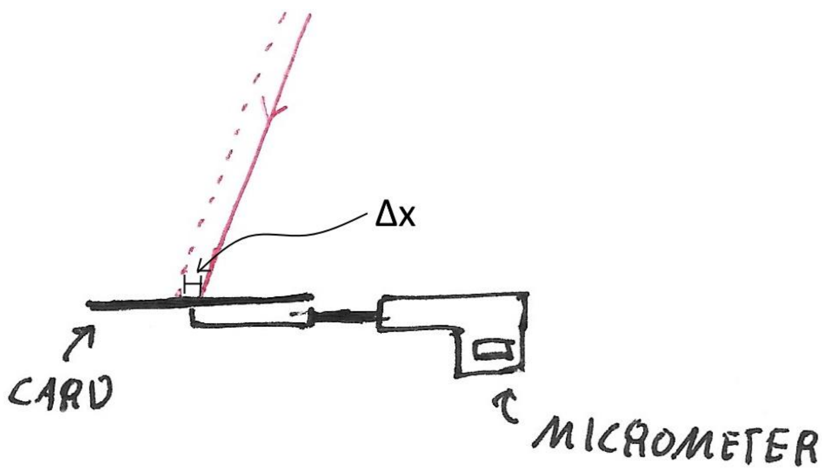

Figure 2.3: The difference between the position of the laser beam before coming to a new equilibrium angle (represented by the dotted red line) and after coming to a new equilibrium angle (represented by the solid red line) is given by Δx. Laser positions are measured both before and after the boom has come to a new equilibrium angle, and the measurements are made by simply rotating the micrometer dial until a crosshair on the card is aligned with the laser's position. The left, center, and right sides of the laser dot are all measured to obtain an accurate position.

As such, two separate measurements of the laser's position at the station were made. The first was taken before the large lead spheres had been adjusted, and the second was taken *after* the adjustment had been made *and* the major boom had been allowed to settle into its new equilibrium position.

The measurements themselves were made by aligning a crosshair, drawn on the card, with the laser's red dot by turning the micrometer dial. The measured distance was simply displayed on the micrometer's digital readout. In order to ensure accuracy, three measurements were made for each laser position: the leftmost side of the dot, the exact center of the dot, and the rightmost side of the dot. The center measurement was taken to be the laser's position, and the uncertainty in this position was obtained by calculating the standard deviation of all three measurements.

This was repeated for both measured positions, yielding two position values and their uncertainties. The final value for Δx was then calculated using Equation 2.5

$$\Delta x = \left| x_1 - x_2 \right| \tag{2.5}$$

Where x_1 and x_2 are the two positions respectively. The uncertainty in Δx was then calculated using Equation 2.6, the error propagation formula for Equation 2.5.

$$\sigma_{\Delta x} = \sqrt{\left(\frac{d\Delta x}{dx_1}\sigma_{x_1}\right)^2 + \left(\frac{d\Delta x}{dx_2}\sigma_{x_2}\right)^2} \tag{2.6}$$

Evaluating the derivatives in Equation 2.6 yielded Equation 2.7, the final uncertainty in Δx.

$$\sigma_{\Delta x} = \sqrt{\sigma_{x_1}^{\ 2} + \sigma_{x_2}^{\ 2}} \tag{2.7}$$

As discussed in the introduction, the oscillation of the boom is described by Equation 1.7, restated here.

$$\theta(t) = \pm \theta_D + A \cdot exp(-bt) \cdot cos(\omega_1 t + \delta) \tag{1.7}$$

After some time, the boom comes to a rest at an equilibrium angle θ_D with respect to the boom's default position (when no torque is present whatsoever).

Because adjusting the minor boom so that it was perpendicular to the major boom (thus producing no torque) was quite difficult to accomplish precisely, the minor boom was instead rotated between each of its two extrema, as depicted in Figure 2.4.

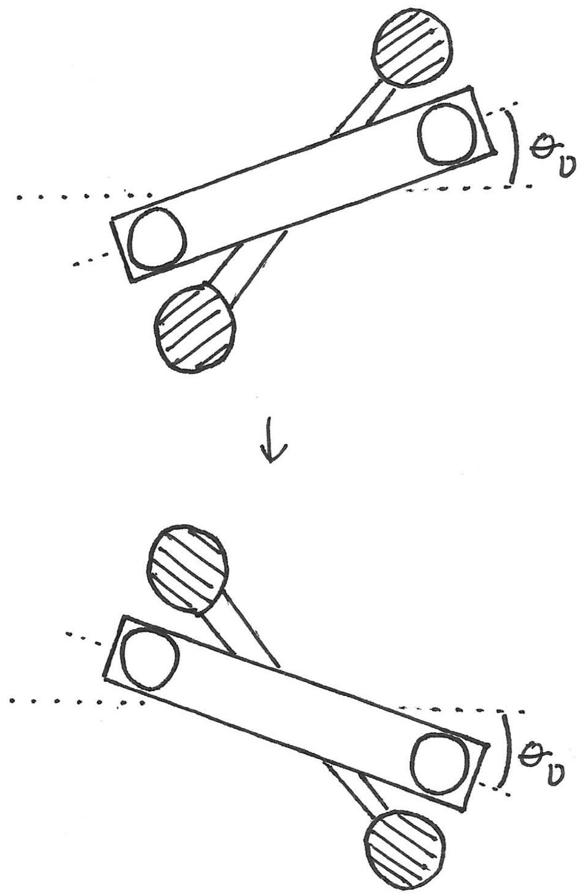

Figure 2.4: A diagram depicting the positions of the minor boom (and therefore the positions of the large lead balls) with respect to the major boom. Each position results in an equilibrium angle of $\pm\ \theta_D$.

As a result of rotating the minor boom between these two extrema rather than between its perpendicular position and an extrema, the difference between the two corresponding major boom equilibrium angles was always $2\theta_D$.

As the major boom rotated to this new equilibrium angle, the position of the laser traversed a part of a circle with radius *R*, the travel distance of the laser, and arc length *s*. However, because the distance Δx was quite small compared to *R*, we approximated *s* as being equal to Δx, thus allowing us to relate θ_D and Δx via the arc length formula, given by Equation 2.8.

$$\Delta\theta = \frac{\Delta x}{R} \tag{2.8}$$

Accounting for the $2\theta_D$ mentioned earlier, and the fact that the laser moves an extra $2\theta_D$ because of its reflection off of the main mirror, as shown in Figure 2.1, we obtain Equation 2.9.

$$4\theta_D = \frac{\Delta x}{R} \tag{2.9}$$

Solving this for θ_D, we obtain the final expression for the deviation angle, given by Equation 2.10.

$$\theta_D = \frac{\Delta x}{4R} \tag{2.10}$$

The uncertainty in this angle is given by error propagation of Equation 2.10, shown in Equation 2.11.

$$\sigma_{\theta_D} = \sqrt{\left(\frac{d\theta_D}{d\Delta x}\sigma_{\Delta x}\right)^2 + \left(\frac{d\theta_D}{dR}\sigma_R\right)^2} \tag{2.11}$$

Evaluating the derivatives in Equation 2.11 and simplifying yields Equation 2.12, the final uncertainty in θ_D:

$$\sigma_{\theta_D} = \sqrt{\left(\frac{\sigma_{\Delta x}}{4R}\right)^2 + \left(\frac{\Delta x\sigma_R}{4R^2}\right)^2} \tag{2.12}$$

2.5 Calculation of G for Each Trial

With the period of oscillation T and the deviation angle θ_D known, the constant G was calculated using Equation 1.12, restated here.

$$G \approx \frac{(2\pi/T)^2 R_s^{\ 2} d}{M} \theta_D \tag{1.12}$$

The uncertainty in this value of *G* is given by Equation 2.13, the error propagation of G.

$$\sigma_G = \sqrt{\left(\frac{dG}{dT}\sigma_T\right)^2 + \left(\frac{dG}{d\theta_D}\sigma_{\theta_D}\right)^2} \tag{2.13}$$

Evaluating the derivatives in Equation 2.13 and simplifying gives Equation 2.14, the final uncertainty in *G*,

$$\sigma_G = \frac{4\pi^2 R_s^{\ 2} d}{T^2 M}\sqrt{\left(\frac{2\theta_D \sigma_T}{T}\right)^2 + \sigma_{\theta_D}^{\ 2}} \tag{2.14}$$

where R_s is the distance between the centers of the large lead spheres and the small lead spheres (44.9 mm), *d* is the distance between the centers of the small lead spheres and the rotation axis (66.56 mm), and *M* is the mass of the large lead spheres (917 g).

2.6 Calculation of the Final G Value

Once a *G* value had been obtained for each of the three trials, the final value of *G* was calculated via a weighted average, given by equation 2.15.

$$\overline{G} = \frac{1}{\Sigma 1/\sigma_{G_i}^{\ 2}} \Sigma \frac{G_i}{\sigma_{G_i}^{\ 2}}$$

(2.15)

The uncertainty in this value is given by Equation 2.16.

$$\sigma_{\overline{G}} = \frac{1}{\Sigma 1/\sigma_{G_i}^{\ 2}} \tag{2.16}$$

Once obtained, the final value of *G* was compared to the commonly accepted value of *G* by computing its Z-score. This was done via Equation 2.17

$$Z = \frac{G_{lit} - \overline{G}}{\sigma_{\overline{G}}} \tag{2.17}$$

where G_{lit} is the commonly accepted value of *G*. Finally, to determine the statistical significance of our final value for *G*, the two-tailed Gaussian p-value was calculated. This was done using Scipy's cumulative distribution function method and the Z-score obtained above. A significance level of 0.05 (5%) was used in this analysis.

3

Results

Measurement of R_s was quite straightforward, and yielded a value of 3625.66667 ± 18.37373 mm. The next three sections present the data from each of the three individual trials, and the final section presents the final value of *G*.

3.1 Results of Trial 1

Table 3.1, presented below, summarizes the position measurements, as well as the calculated value of Δx, for the first trial.

Center (mm)	Left (mm)	Right (mm)	Δx (mm)
-9.052	-7.407	-10.152	
-0.672	-2.213	0.523	
			8.37999 ± 1.58953

Table 3.1: A summary of the laser position measurements and the final Δx value for the first trial.

Additionally, Figure 3.1, presented below, depicts a scatterplot of the major boom's angular displacement throughout the one hour length of the first trial.

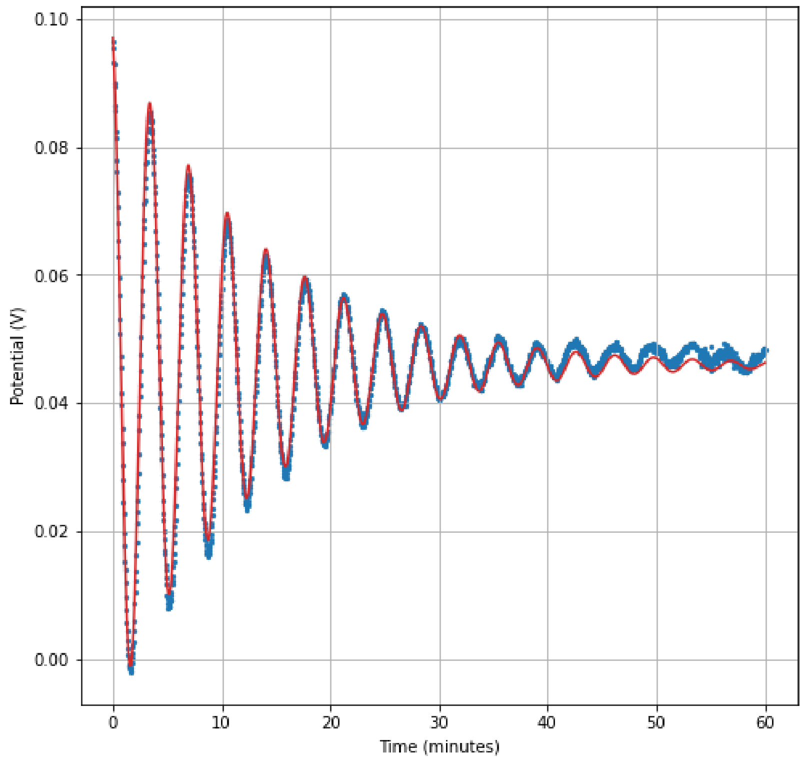

Figure 3.1: A graph of the angular displacement of the major boom as a function of time for the first trial. Half periods of the major boom's oscillation were obtained by measuring the time difference between the occurrence of a peak and its corresponding trough. These half periods were then averaged and multiplied by two to obtain the full period. For display purposes, the raw data (the blue dots) were used to calculate a fit curve. This curve was generated using scipy's *optimize.curve_fit method* and Equation 1.7.

The period T, determined by the method described in Section 2.3, was calculated to be 217.33333 ± 9.42809 seconds. The deviation angle θ_D was determined using the method described in Section 2.4 and the value for Δx presented in Table 3.1, and was calculated to be $5.77825 \times 10^{-4} \pm 1.09642 \times 10^{-4}$ radians. Finally, using these two values in the method described in Section 2.5, the first trial's value for G was calculated.

$$G = 7.067078 \times 10^{-11} \pm 1.47450 \times 10^{-11}\, m^3/kg \cdot s^2$$

3.2 Results of Trial 2

Table 3.2, presented below, summarizes the position measurements, as well as the calculated value of Δx, for the second trial.

Center (mm)	Left (mm)	Right (mm)	Δx (mm)
-0.657	0.525	-2.549	
-9.620	-7.141	-11.523	
			8.963 ± 2.196

Table 3.2: A summary of the laser position measurements and the final Δx value for the second trial.

Additionally, Figure 3.2, presented below, depicts a scatterplot of the major boom's angular displacement throughout the one hour length of the second trial.

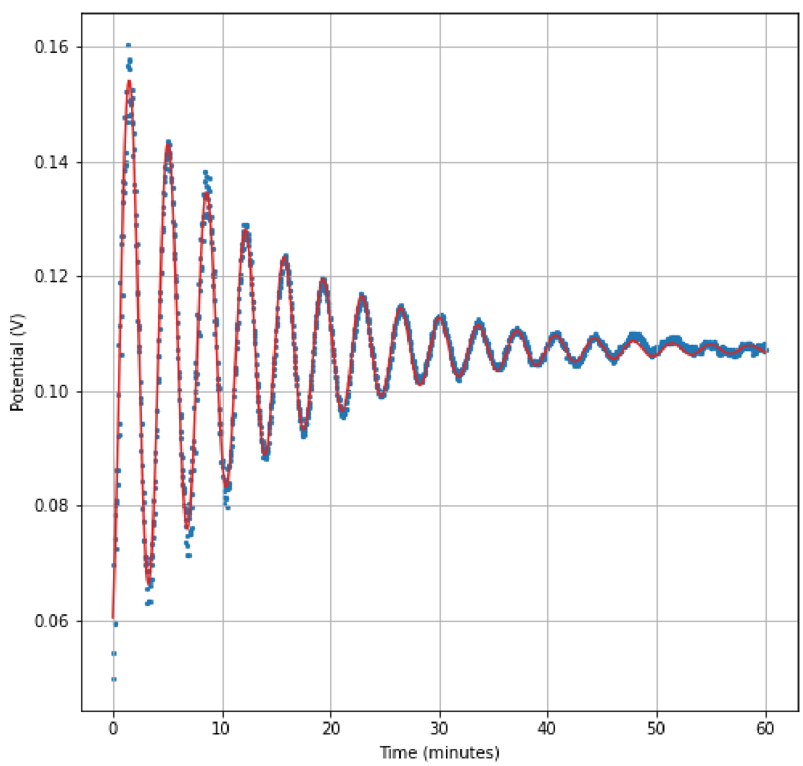

Figure 3.2: A graph of the angular displacement of the major boom as a function of time for the second trial. The full oscillation period of the major boom for this trial was obtained via the same procedure as the previous one. For display purposes, the raw data (the blue dots) were used to calculate a fit curve. This curve was generated using scipy's *optimize.curve_fit method* and Equation 1.7.

Finally, the period *T* was calculated to be 215.55556 ± 18.42167 seconds, the deviation angle θ_D was calculated to be $6.18024 \times 10^{-4} \pm 1.51441 \times 10^{-4}$ radians, and, as before, the second trial's value for *G* was calculated.

$$G = 7.68393 \times 10^{-11} \pm 2.29568 \times 10^{-11}\ m^3/kg \cdot s^2$$

3.3 Results of Trial 3

Table 3.3, presented below, summarizes the position measurements, as well as the calculated value of Δx, for the final trial.

Center (mm)	Left (mm)	Right (mm)	Δx (mm)
-2.007	0.142	-4.848	
-10.289	-8.959	-11.727	
			8.212 ± 2.333

Table 3.2: A summary of the laser position measurements and the final Δx value for the third trial.

Additionally, Figure 3.3, presented below, depicts a scatterplot of the major boom's angular displacement throughout the one hour length of the third trial.

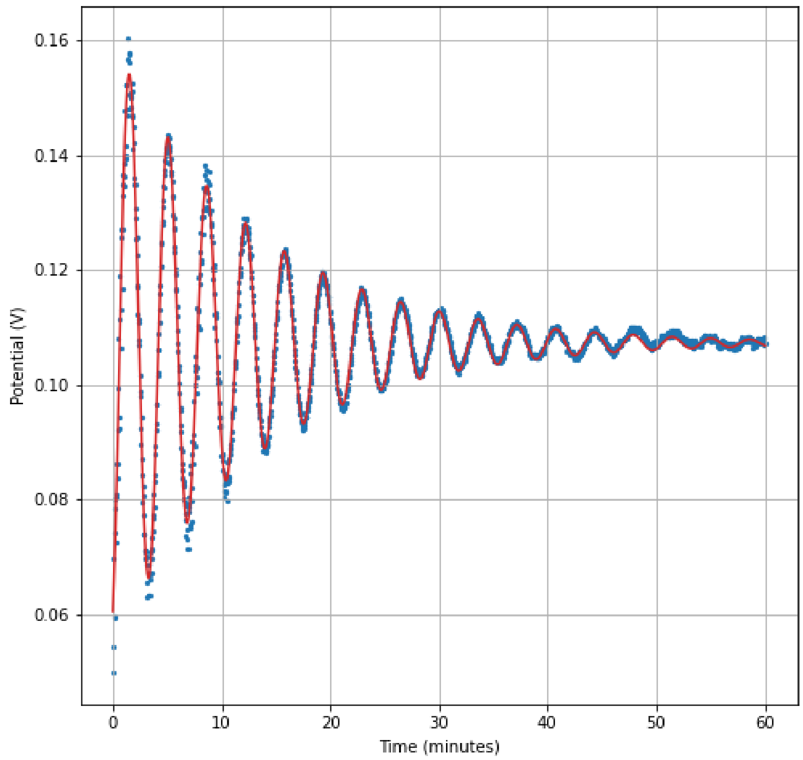

Figure 3.3: A graph of the angular displacement of the major boom as a function of time for the third trial. The full oscillation period of the major boom for this trial was obtained via the same procedure as the previous two trials. For display purposes, the raw data (the blue dots) were used to calculate a fit curve. This curve was generated using scipy's *optimize.curve_fit method* and Equation 1.7.

The period T was calculated to be 215.55556 ± 18.42167 seconds, the deviation angle θ_D was calculated to be $5.66241 \times 10^{-4} \pm 1.60917 \times 10^{-4}$ radians, and, as with the previous two trials, the third trial's value for G was calculated.

$$G = 7.04010 \times 10^{-11} \pm 2.33468 \times 10^{-11}\ m^3/kg \cdot s^2$$

3.4 The Final *G* Value

Figure 3.4, pictured below, presents the three trials' G values, and their associated errors, along with the commonly accepted value for G.

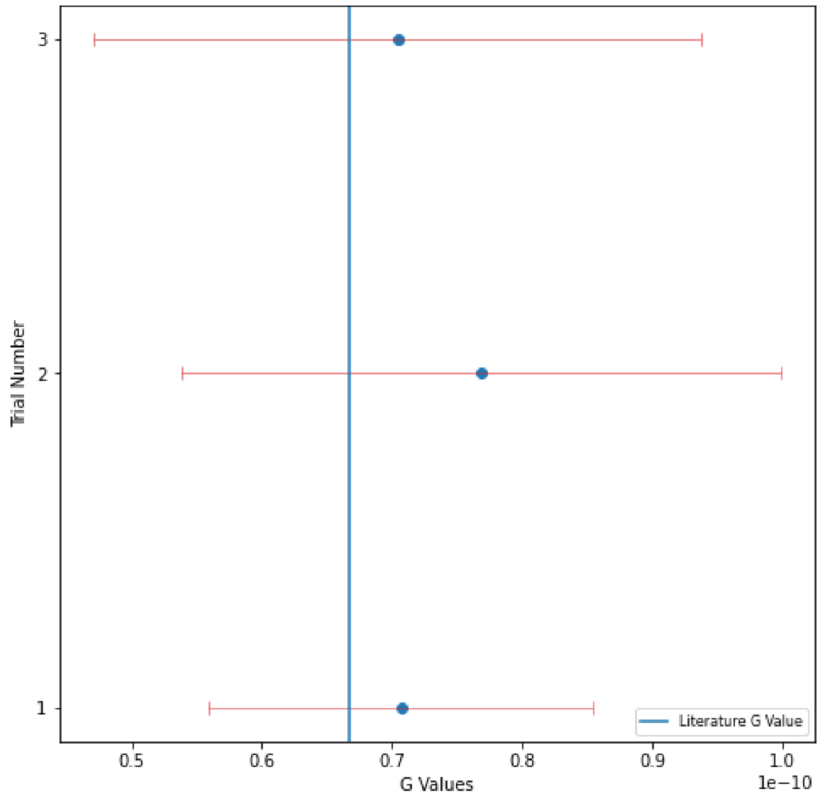

Figure 3.4: A plot depicting the calculated *G* values for each trial along with their associated errors (the orange error bars). Also plotted is the commonly accepted value for *G* (pictured as vertical blue line).

These three values were used in the weighted average described by Section 2.6, and that process yielded the final value for *G*, presented below.

$$\overline{G} = 7.2383 \times 10^{-11} \pm 1.09556 \times 10^{-11} \, m^3/kg \cdot s^2$$

Using this value with Equation 2.14 yielded a Z-score of -0.51072, which essentially means that the final value of *G* is 0.513 standard deviations away from the commonly accepted value of *G*. Finally, using this Z-score, the two-tailed Gaussian p-value was calculated to be 0.60955. This is greater than the significance level of 0.05, indicating that our result is statistically significant.

4

Discussion

Over the course of three separate trials, the universal gravitational constant, *G*, was determined to be $7.68393 \times 10^{-11} \pm 2.29568 \times 10^{-11}\ m^3/kg \cdot s^2$. This final value, as well as its uncertainty, were determined by conducting a weighted average with the *G* values for each of the three trials. As discussed in Section 3.4, this result is about 0.513 standard deviations away from the commonly accepted value for *G*, $6.6743 \times 10^{-11}\ m^3/kg \cdot s^2$ (NIST, 2018).

Our value of *G*, as proven by a two-tailed Gaussian p-value of approximately 0.61, is statistically significant, and the commonly accepted value for *G* is within the error of our *G*, value. However, our result is still somewhat disappointing. The main point of concern for our value is its fairly large error, which is the result of substantial uncertainties in the measurements of θ_D and T (though it was particularly bad for the values of T). As such, there are a couple of improvements that could be made to this experiment to improve its accuracy in determining *G*.

Firstly, the deviation angle θ_D likely could've been measured more accurately if the laser beam had been focused into a smaller point. As mentioned in the Methods section, the laser was focused using two lenses, one concave and one convex, each with a focal length of -1 meter and 1 meter respectively. In combination, these two lenses helped to focus the laser into a smaller point at the micrometer station, and it was found that this was achieved most effectively when the lenses were separated by a distance of 357 mm. However, even with the assistance of the lenses, the laser still arrived at the micrometer station somewhat unfocused, which made it difficult to obtain precise measurements of its position. Considering that the laser travels a distance of at least *R* from the lenses (though in reality the distance is probably closer to 4 or 4.5 meters), future repetitions of this experiment would probably benefit from using lenses of a greater focal length. This would result in better focus of the laser at the micrometer station, and thus more accurate (and less uncertain) measurements of θ_D.

Further, measurements of *T* probably could've been made more precisely if the angular displacement of the major boom had been recorded more frequently. As Section 2.3 describes, measurements of the major boom's angular displacement were made every two seconds by a series of electrical components which were recorded by a computer, and these raw measurements were then later used to make a scatter plot from which peaks and troughs could be identified. Because measurements were only made every two seconds, it is likely that the *exact* times of the real peaks and troughs of oscillation weren't always measured, meaning that the time differences between peaks and troughs of the raw data might not have always corresponded to full half-periods.

Such a situation would lead to quite a bit of variance in the measured half periods, which would explain why there was fairly large uncertainty in the *T* values we obtained. Fortunately, the effects of this situation could be minimized by simply measuring the angular displacement of the major boom more regularly. If, for example, measurements were taken every 0.5 seconds instead of every two seconds, it would be much more likely that the real peaks and troughs of oscillation would be measured, leading to much less variance in the measured half periods and, thus, a more accurate measurement of the full oscillation period.

Although the accuracy of the measured value for G was somewhat disappointing, these two adjustments are simple to make and would likely lead to a value of G which is not only more accurate, but significantly less uncertain.

5

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