The below equations are those used in the modeling of Integrated Concentrating Solar and Heat Losses for a High Temperature Thermal Fluid and Surrounding Air Cavity of a Double Skin Facade.

Definitions:

θ_i = Temperature of water at node "i"

ϕ_i = Temperature of air at node "i"

where:

$T_i = \theta_i$; $T_{i+1} = \phi_i$; $T_{i+2} = theta_{i+1}$; $T_{i+3} = \theta_{i+1}$ and so on

0.1 Residual: Heat Balance Between Thermal Fluid and Surrounding Air

0.1.1 Region 1

Water

Heat balance equation for the water in Region 1:

$$m_{water} C_{p_{water}} (T_{i+2} - T_i) = q_{wa} = h_{pipe} A \frac{\left(\frac{T_{i+3}+T_{i+1}}{2} - \frac{T_{i+2}+T_i}{2}\right)}{R_{Pipe}}$$

$$\boxed{q_{water} = m_{water} C_{p_{water}} (T_{i+2} - T_i) - \frac{\left(\frac{T_{i+3}+T_{i+1}}{2} - \frac{T_{i+2}+T_i}{2}\right)}{R_{Pipe}} = 0} \tag{1}$$

$$R_{Pipe} = \frac{1}{h_{PipeFlow} A_{insideS}} + \frac{\frac{r_{outerTubing}}{r_{innerTubing}}}{k_{siliconTubing} A_{tubingS}} + \frac{\frac{r_{insulation}}{r_{outerTubing}}}{k_{Insulation} A_{insulationS}} + \frac{1}{h_{extPipeAirFlow} A_{outsideS}}$$

Air

Heat balance equation for the air in Region 1:

$$\boxed{q_{air} = m_{air} Cp_{air} (T_{i+3} - T_{i+1}) + \frac{\left(\frac{T_{i+3}+T_{i+1}}{2} - \frac{T_{i+2}+T_i}{2}\right)}{R_{Pipe}} - \frac{T_{int} - \frac{T_{i+3}+T_{i+1}}{2}}{R_{Int}} - \frac{T_{ext} - \frac{T_{i+3}+T_{i+1}}{2}}{R_{Ext}} = 0} \tag{2}$$

$$R_{IntToCavity} = \frac{L_{glass}}{k_{glass}A_{surface}} + \frac{L_{gap}}{k_{argon}A_{surface}} \frac{L_{glass}}{k_{glass}A_{surface}} + \frac{1}{h_{cavity}A_{SurfGlass}}$$

$$R_{ExtToCavity} = \frac{L_{glass}}{k_{glass}A_{surface}} + \frac{1}{h_{cavity}A_{SurfGlass}}$$

0.1.2 Region 2

$$Q_{water} = m_{water}C_{p_{water}}(T_{i+4} - T_{i+2}) + Q_{receiver}$$

$$Q_{air} = m_{air}C_{p_{air}}(T_{i+5} - T_{i+3}) + Q_{receiver}$$

0.2 Temperature Solver

By rearranging the equations to separate temperatures, coefficients can be determined for each temperature. The input (or given) temperatures of the system can then be used to solve for the subsequent unknown temperatures. *This study includes Regions 1 but excludes Region 2 to simplify the problem and exclude heat generation.*

0.2.1 Using Equation 1: Equations for q_{water}, determine A_{12}, A_{22}, and F_1

Equation 1,

$$q_{water} = m_{water}C_{p_{water}}(T_{i+2} - T_i) - \frac{\left(\frac{T_{i+3}+T_{i+1}}{2} - \frac{T_{i+2}+T_i}{2}\right)}{R_{Pipe}} = 0$$

Rearrange into,

$$T_2A_{12} + T_3A_{13} = F_1$$

Rearrange to determine coefficient for T_2:

$$A_{12} = [m_wCp_w + \frac{1}{2R_{pipe}}]$$

Rearrange to determine coefficients for T_3:

$$A_{13} = [-\frac{1}{2R_pipe}]$$

Rearrange all other temperatures (T_0 and T_1) to the right side of the equation and solve for coefficients to each:

$$F_1 = T_0[m_w Cp_w - \frac{1}{2R_{pipe}}] + T_1[\frac{1}{2R_{pipe}}]$$

There is no dependence on T_{int} and T_{ext} in the water heat balance.

0.2.2 Using Equation 2: Equations for q_{air}, determing A_{22}, A_{23}, and F_2

Equation 2,

$$q_{air} = m_{air}Cp_{air}(T_{i+3} - T_{i+1}) + \frac{\left(\frac{T_{i+3}+T_{i+1}}{2} - \frac{T_{i+2}+T_i}{2}\right)}{R_{Pipe}} - \frac{T_{int} - \frac{T_{i+3}+T_{i+1}}{2}}{R_{Int}} - \frac{T_{ext} - \frac{T_{i+3}+T_{i+1}}{2}}{R_{Ext}} = 0$$

Rearrange into,

$$T_2 A_{22} + T_3 A_{23} = F_1$$

Rearrange to determine coefficients for T_2:

$$A_{22} = [-\frac{1}{2R_{pipe}}]$$

Rearrange to determine coefficients for T_3:

$$A_{23} = [m_a Cp_a + \frac{1}{2R_{pipe}} + \frac{1}{2R_{int}} + \frac{1}{2R_{ext}}]$$

Rearrange all other temperatures (T_0, T_1, T_{int} and T_{ext}) to the right side and solve for coefficients to each:

$$F_2 = T_0[\frac{1}{2R_{pipe}}] + T_1[m_a Cp_a - \frac{1}{2R_{pipe}} - \frac{1}{2R_{int}} - \frac{1}{2R_{ext}}] + T_{int}[\frac{1}{R_{int}}] + T_{ext}[\frac{1}{R_{ext}}]$$

0.2.3 Solving Unknowns

The unknowns temperatures at T_2 and T_3. The unknown equations:

$$T_2 A_{12} + T_3 A_{13} = F_1$$

$$T_2 A_{12} + T_3 A_{23} = F_1$$

were plugged into MatLab in the form,

$$\begin{bmatrix} A_{12} & A_{13} \\ A_{22} & A_{23} \end{bmatrix} \begin{bmatrix} T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

and solving in Matlab using,

$$T = A \backslash F$$

where,

$$A = \begin{bmatrix} A_{12} & A_{13} \\ A_{22} & A_{23} \end{bmatrix}$$

and

$$F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

With the given values of,

```
T_0 = 13;
T_1 = 20;
T_int = 22.5;
T_ext = 25;
m_w = 0.00084931862198712224;
m_a = 0.3597862499999999;
Cp_w = 4.188774760737728;
Cp_a = 1.005;
R_pipe = 1472.0223510771341;
R_int = 0.52972312781694775;
R_ext = 0.10670725480107474;
```

the resultant temperatures are:

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T_2 = 14.970373956130462

T_3 = 28.607571102687491
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