ICSolar Model

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1 Steady Model

Consider the model of air and water interaction consisting of an initial inlet region (denoted by 0) and a pair of regions, an open region with pipe followed by a module, denoted by (1,2) satisfying

$$\begin{aligned}
W_1: &\quad \dot{m}_w C_{p,w}(T_{w,1} - T_{w,0}) - h_{wa}(T_{a,1} - T_{w,1}) = 0 && (1)\\
A_1: &\quad \dot{m}_a C_{p,a}(T_{a,1} - T_{a,0}) - h_{wa}(T_{w,1} - T_{a,1}) - h_e(T_e - T_{a,1}) - h_i(T_i - T_{a,1}) = 0 && (2)\\
W_2: &\quad \dot{m}_w C_{p,w}(T_{w,2} - T_{w,1}) - Q_w = 0 && (3)\\
A_2: &\quad \dot{m}_a C_{p,a}(T_{a,2} - T_{a,1}) - Q_a = 0 && (4)
\end{aligned}$$

Where i and e are interior and exterior contributions. Each pair of these forms a 'module'. In this work, we use

$$\begin{aligned}
C_{p,w} &= 4.218 kJ/(kgK) && (5)\\
\dot{m}_w &= 0.0008483 kg/s && (6)\\
C_{p,a} &= 1.005 kJ/(kgK) && (7)\\
\dot{m}_a &= 0.384 kg/s && (8)\\
h_{wa} &= 4.823 \times 10^{-5} kW/(Km) && (9)\\
h_i &= 1.572 \times 10^{-4} kW/(Km) && (10)\\
h_e &= 4.837 \times 10^{-4} kW/(Km) && (11)\\
& && (12)
\end{aligned}$$

With Initial and Boundary Conditions of $T_{a,0} = 20C, T_i = 25.0C, T_e = 22.5C$. At this point, we set $Q_a = 0$ as the surrounding air acts like a reservoir and its effect is currently minimal. Our inputs are $T_{w,0}$ and $Q_{w,i}$ from experimental data. We also occasionally have access to $T_{a,0}$, the ambient air temperature.

2 Unsteady Model

Consider the steady model in **??** and introduce the time derivative, $mC_p\frac{\partial T}{\partial t}$ and rearrange to get

$$W_1: \quad m_{w,1}C_{p,w}\frac{\partial T}{\partial t} + \dot{m}_w C_{p,w}(T_{w,1} - T_{w,0}) - h_{wa}(T_{a,1} - T_{w,1}) = 0 \tag{13}$$

$$W_2: \quad m_{w,2}C_{p,w}\frac{\partial T}{\partial t} + \dot{m}_w C_{p,w}(T_{w,2} - T_{w,1}) - Q_w(t) = 0 \tag{14}$$

$$A_1: \quad m_{a,1}C_{p,a}\frac{\partial T}{\partial t} + \dot{m}_a C_{p,a}(T_{a,1} - T_{a,0}) - h_{wa}(T_{w,1} - T_{a,1}) - h_e(T_e - T_{a,1}) - h_i(T_i - T_{a,1}) = 0 \tag{15}$$

$$A_2: \quad m_{a,2}C_{p,a}\frac{\partial T}{\partial t} + \dot{m}_a C_{p,a}(T_{a,2} - T_{a,1}) - Q_a = 0 \tag{16}$$

To handle the mass term, we need the volume. We have a length of the first tube as $L_1 = 0.15m$ and $L_{3,5,...} = 0.3m$. The cross sectional area of the tube is based on inner diameter, $d = 0.003m$ and outer diameter of $d = 0.0142m$. The volume of the surrounding air we are interested in has a cross section of $0.4m \times 0.4m$. Using the density and the specific heat, we get that $m_a = 0.0576kg$, $m_w = 2.12 \times 10^{-3}kg$.

For regions with modules, we can create a small volume, $m_{a,2,4,6,...} \approx Cm_a$ and $m_{w,2,4,6,...}Cm_w$.

3 Steady Model - Matrix Form

Lets write the equations in the form $\mathbf{Ax} = \mathbf{b}$

$$W_1: \quad (\dot{m}_w C_{p,w} + h_{wa})T_{w,1} - h_{wa}T_{a,1} = \dot{m}_w C_{p,w} T_{w,0} \tag{17}$$

$$A_1: \quad (\dot{m}_a C_{p,a} + h_{wa} + h_e + h_i)T_{a,1} - h_{wa}T_{w,1} = \dot{m}_a C_{p,a} T_{a,0} + h_e T_e + h_i T_i \tag{18}$$

$$W_2: \quad \dot{m}_w C_{p,w} T_{w,2} - \dot{m}_w C_{p,w} T_{w,1} = Q_w \tag{19}$$

$$A_2: \quad \dot{m}_a C_{p,a} T_{a,2} - \dot{m}_a C_{p,a} T_{a,1} = Q_a \tag{20}$$

For a two module system, the system is as follows. The extension to additional modules is obvious

$$
\begin{bmatrix}
C_w + h_{wa} & -h_{wa} & 0 & 0 & 0 & 0 & 0 & 0 \\
-h_{wa} & C_a + h_{win} & 0 & 0 & 0 & 0 & 0 & 0 \\
-C_w & 0 & C_w & 0 & 0 & 0 & 0 & 0 \\
0 & -C_a & 0 & C_a & 0 & 0 & 0 & 0 \\
0 & 0 & -C_w & 0 & C_w + h_{wa} & -h_{wa} & 0 & 0 \\
0 & 0 & 0 & -C_a & -h_{wa} & C_a + h_{win} & 0 & 0 \\
0 & 0 & 0 & 0 & -C_w & 0 & C_w & 0 \\
0 & 0 & 0 & 0 & 0 & -C_a & 0 & C_a
\end{bmatrix}
\begin{bmatrix}
T_{w,1} \\ T_{a,1} \\ T_{w,2} \\ T_{a,2} \\ T_{w,3} \\ T_{a,3} \\ T_{w,4} \\ T_{a,4}
\end{bmatrix}
=
\begin{bmatrix}
C_w T_{w,0} \\ C_a T_{a,0} + Q_{win} \\ Q_w \\ Q_a \\ 0 \\ Q_{win} \\ Q_w \\ Q_a
\end{bmatrix}
\tag{21}
$$

$$
\begin{aligned}
C_w &= \dot{m}_w C_{p,w} && (22) \\
C_a &= \dot{m}_a C_{p,a} && (23) \\
h_{win} &= h_{wa} + h_e + h_i && (24) \\
Q_{win} &= h_e T_e + h_i T_i && (25)
\end{aligned}
$$