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Homework #
Control

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這是一個模板供使用。

I'm writing to demonstrate use of automatically-generated footnote markers¹ and footnotes which use a marker value provided to the command⁴².

Now, I will use another automatically-generated footnote marker².

1 Equation Example

In this paper, the 3–2–1 intrinsic convention is considered. The direction cosine matrix (DCM), ${}^G\mathbf{C}^B(\phi, \theta, \psi)$, is constructed in terms of the Euler angles as follows:

$${}^G\mathbf{C}^B(\phi, \theta, \psi) = \mathbf{R}_z(\psi)\mathbf{R}_y(\theta)\mathbf{R}_x(\phi) \quad (1)$$

where

$$\begin{aligned} \mathbf{R}_x(\phi) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi & c_\phi \end{bmatrix}, \quad \mathbf{R}_y(\theta) = \begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix}, \\ \mathbf{R}_z(\psi) &= \begin{bmatrix} c_\psi & -s_\psi & 0 \\ s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (2)$$

are the rotation matrices and the notations $s_{(\cdot)}$ and $c_{(\cdot)}$ represent $\sin(\cdot)$ and $\cos(\cdot)$, respectively.

$$\begin{aligned} \dot{\mathbf{e}}_{1,\Theta} &= \mathbf{e}_{2,\Theta} \\ \dot{\mathbf{e}}_{2,\Theta} &= -\mathbf{T}^{-1}\mathbf{J}^{-1}\mathbf{J}_0\mathbf{T}\left(\mathbf{K}_{p,\Theta}\mathbf{e}_{1,\Theta} + \mathbf{K}_{i,\Theta}\int_0^t \mathbf{e}_{1,\Theta} d\tau \right. \\ &\quad \left. + \mathbf{K}_{d,\Theta}\mathbf{e}_{2,\Theta}\right) + [\mathbf{J}\mathbf{T}]^{-1}\mathbf{d}_\Theta \\ &\quad + \left([\mathbf{J}\mathbf{T}]^{-1}\mathbf{J}_0\mathbf{T} - \mathbf{I}_3\right)\ddot{\mathbf{x}}_{1d,\Theta} \\ &\quad - [\mathbf{J}\mathbf{T}]^{-1}\left([\mathbf{T}\mathbf{x}_{2,\Theta}]^\times \Delta\mathbf{J}\mathbf{T}\mathbf{x}_{2,\Theta} - \Delta\mathbf{J}\dot{\mathbf{T}}\mathbf{x}_{2,\Theta}\right) \end{aligned} \quad (3)$$

2 Figure Example

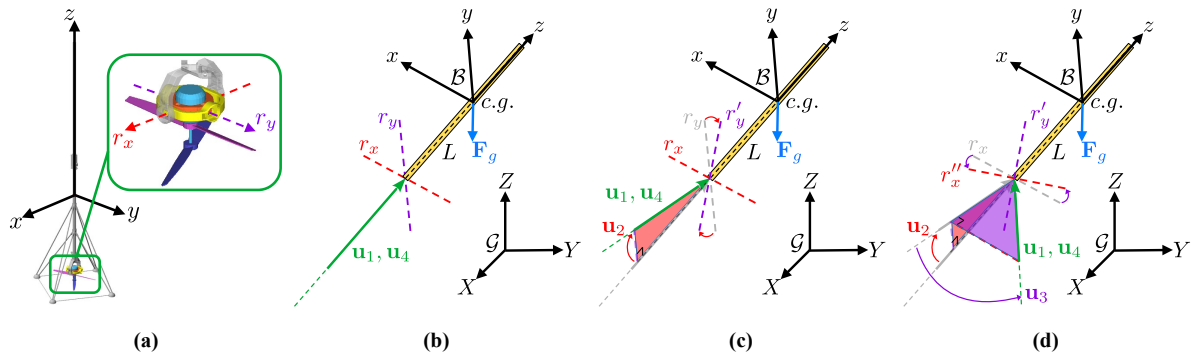


Figure. 1. Illustration of the rocket-type UAV force analysis.

¹Automatically generated footnote markers work fine!

⁴²...is that the answer to everything?

²Now, footnote markers are 1, 42, but then back to 2? That will be confusing if the automatically-generated number also reaches 42!

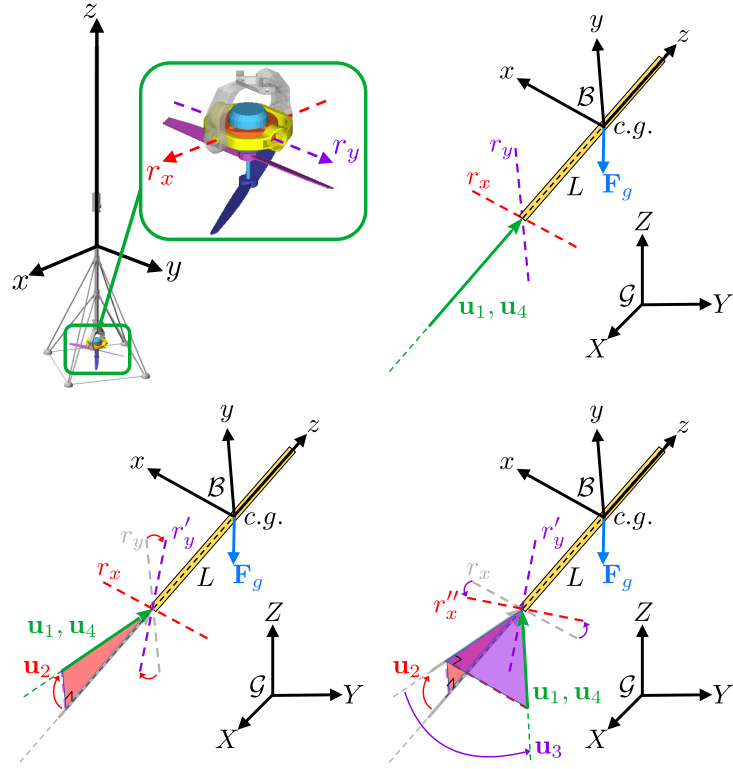


Figure. 2. Illustration of the rocket-type UAV force analysis.

3 Table Example

Table. 1. Simulated system parameters

| m (kg) | g (m/s ²) | \mathbf{J} (kg-m ²) | L (m) | C_d |
|----------|-------------------------|---|---------|-------|
| 0.65 | 9.8 | $\begin{bmatrix} 0.1287 & 0 & 0 \\ 0 & 0.1287 & 0 \\ 0 & 0 & 0.052 \end{bmatrix}$ | 0.45 | 0.05 |

Table. 2. Levenberg-Marquardt Method pseudo code.

Algorithm 1 Levenberg-Marquardt Method

given an initial value $\mathbf{u}^{(0)}$, $\lambda^{(0)} = 1000$, $\epsilon = 10^{-5}$.

repeat

1. Determine a Jacobian matrix $\mathbf{J}_{\mathbf{r}}^{(k)}$.

2. Update the damping parameter $\lambda^{(k)}$.

3. Update the LM step.

$$\mathbf{d}^{(k)} = - \left(\mathbf{J}_{\mathbf{r}}^{(k)T} \mathbf{J}_{\mathbf{r}}^{(k)} + \lambda^{(k)} \mathbf{I}_4 \right)^{-1} \mathbf{J}_{\mathbf{r}}^{(k)T} \mathbf{r}(\mathbf{u}^{(k)}).$$

4. Update the control variables.

$$\mathbf{u}^{(k+1)} = \mathbf{u}^{(k)} + \mathbf{d}^{(k)}.$$

$$k \leftarrow k + 1.$$

5. Compute the residual error vector to evaluate the stopping condition.

$$\mathbf{r}(\mathbf{u}^{(k+1)}) = [r_1, r_2, r_3, r_4]^T$$

until $\|\mathbf{r}\| < \epsilon$ is satisfied, $\mathbf{u}^* = \mathbf{u}^{(k+1)}$.
