Knowing Bernoulli polynomials can be written in the form:

$$\mathcal{B}_{n-1}(t) = \sum_{i=0}^{n-1} \alpha_i t^i$$

where the alphas are constants and given:

$$\mathcal{B}_n(x) = \int_0^1 tn\mathcal{B}_{n-1}(t)dt + \int_0^x n\mathcal{B}_{n-1}(t)dt$$

then:

$$\mathcal{B}_n(x) = \int_0^1 tn\sum_{i=0}^{n-1} \alpha_i t^i dt + \int_0^x n\sum_{i=0}^{n-1} \alpha_i t^i dt$$

$$= n\int_0^1 \sum_{i=0}^{n-1} \alpha_i t^{i+1} dt + n\int_0^x \sum_{i=0}^{n-1} \alpha_i t^i dt$$

$$= n\sum_{i=0}^{n-1} \frac{\alpha_i}{i+2} + n\sum_{i=0}^{n-1} \frac{\alpha_i x^{i+1}}{i+1}$$