IMAGE COMPRESSION USING DFT THROUGH FAST FOURIER TRANSFORM TECHNIQUE

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Abstract ***Images speak more than words. The developments in the field of Teleconferencing, HDTV, and Social media have raised the need of fast and high quality of images. Many web pages use images at very large scale to represent the information. The effective transmission of these images at a very low bit rate requires the compression of images. The authors in this paper discuss an image compression method based on Fast Fourier Transformation. This method provides lossy compression of images both in grayscale and color. The authors analyze the amount of compression by compression ratio with the analysis of quality of image using Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR).***

Keywords: about four key words separated by commas

1. INTRODUCTION

In current era the use of digital images in various applications has increased very rapidly. The reason behind the increased use of digital images is the storage, transmission, modification of these images. The video and television transmission has also become digital and this arises the use of more and more digital images in multimedia applications.

Digital image may be seen as the rectangular array of pixels called Bitmap. Each pixel is a small dot which represents the color (or the gray level for grayscale images) at the particular point on the image. For the transmission of any image the value of color at each pixel is measured and digitally approximated is done. From this approximation, a copy of original image is reconstructed at the receiver side.

The digital image so obtained after the approximation of the color value of each pixel in the image requires a large space for storage purpose. For example, a moderate image size of 512×512 pixels requires 0.75 MB of disk space. One second of digital PAL signal requires 27 MB of disk space. To store and transmit these images compression techniques are needed. The basic aim of image compression is to achieve low bit rate representation, requiring less space and achieving high visual quality of decompressed image.

2. TYPES OF IMAGE COMPRESSION

There are two types of image compression techniques, namely, Lossless Image Compression and Lossy Image compression.

Lossless image compression techniques are used where the reconstructed image must be the exactly same as the original image. These kinds of techniques encode all the information available with the original image, so that at the time of reconstruction of image the exact replica of original image may be obtained. Examples of lossless image compression techniques are Run length encoding, Huffman encoding, Arithmetic encoding, Entropy coding and Area coding etc.

In case of lossy compression techniques the reconstructed image so obtained may differ in quality compared to the original image. Lossy compression gives higher compression ratio compared with the lossless compression techniques. Lossy compression techniques encode only that information which is required for the reconstruction of the image and omits all other unnecessary information. Though lossy compression provides higher compression ration but the reconstructed image somewhere differ from the original image. Lossy compression techniques are more suitable for still images. Examples of lossy compression techniques are Predictive coding, Transform coding (FT/DCT/Wavelets), Chroma sub sampling, Fractal compression and vector quantization etc.

3. PERFORMANCE INDICATORS:

Lossy compression techniques leave the reconstructed image with some distortions, as the reconstructed image is only the approximation to the original image. In order to measure and quantify the performance of compression technique, some performance indicators are used as follows:

- **Compression Ratio**: Compression ratio defines the ratio of number of bits required to represent original image to the number of bits required to represent compressed image. As the compression ratio increases, the quality is compromised. Lossy compression techniques have higher compression ratio than lossless compression techniques.

- **Mean Square Error (MSE):** MSE is the measure of error between the original image and the compressed image. Mean Square Error is the cumulative squared error between the compressed image and the original image. For the lesser distortion and high output quality, the MSE must be as low as possible. Mean Square Error may be calculated using following expression:

$$MSE = \frac{1}{MN}\sum_{y=1}^{M}\sum_{x=1}^{N}[I(x,y) - I'(x,y)]^2$$

- **Peak Signal to Noise Ratio (PSNR):** PSNR is the ratio of maximum power of the signal and the power of unnecessary distorting noise. Here the signal is the original image and the noise is the error in reconstruction. For a better compression the PSNR must be high. The increase in the peak signal to noise ratio results in the decrease in compression ratio.
- Therefore a balance must be obtained between the compression ratio and peak signal to noise ratio for the effective compression. The peak signal to noise ratio may be calculated as:

$$PSNR = 20 \times \log_{10}\left[\frac{255}{\sqrt{MSE}}\right]$$

4. FAST FOURIER TRANSFORM

Fast Fourier Transform (FFT) is the faster and effective method of Discrete Fourier Transform (DFT). Discrete Fourier Transform is the transform which takes the discrete signal in time domain and transforms that signal in its discrete frequency domain representation. This property of DFT signifies the importance of DFT in the area of spectrum analysis. FFT being the high speed and discrete nature equivalent of DFT is suitable for the signal's spectrum analysis in MATLAB in real time.

Discrete Fourier Transform allows us to compute an approximation of the Fourier Transform on a discrete set of frequencies from a discrete set of time samples.

Continuous Signal:

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}\,dt$$

DF Transform:

$$X[k] = \sum_{n=0}^{N-1} x[n]\,e^{-j2\pi\frac{k}{N}n} \; for\; k = 0,1,\ldots,N-1$$

Where, *k* is the index of the discrete frequencies and *n* the index of the time samples.

The inverse discrete fourier transform formula is:

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\,e^{j2\pi\frac{k}{N}n} \; for\; k = 0,1,\ldots,N-1$$

We may also express DFT as:

$$X[k] = \sum_{n=0}^{N-1} x[n]\,W_N^{-kn} \; for\; k = 0,1,\ldots,N-1 \; with\; W_N = e^{j\frac{2\pi}{N}}$$

4.1Cooley-Tukey algorithm for Fast Fourier Transformation of Radix 2 Decimation:

Cooley-Tukey algorithm based on the decimation leads to a factorization of the computations. This algorithm splits the overall computation between the sub computations of odd and even samples. This particular case of the algorithm requires the time sequence length to be a power of 2.

$$X[k] = \sum_{n=0}^{\frac{N}{2}-1} x[2n]W_N^{-k2n} + \sum_{n=0}^{\frac{N}{2}-1} x[2n+1]\,W_N^{-k(2n+1)}$$

Using the property,

$$W_N^2 = W_{\frac{N}{2}}$$

The FFT may be written as:

$$X[k] = \sum_{n=0}^{\frac{N}{2}-1} x[2n]W_{\frac{N}{2}}^{-kn} + W_N^{-k}\sum_{n=0}^{\frac{N}{2}-1} x[2n+1]W_{\frac{N}{2}}^{-kn}$$

Using the property that

$$W_N^{k+\frac{N}{2}} = -W_N^k$$

The entire DFT may be computed with only $k = 0,1,\ldots,\frac{N}{2}-1$ as follows:

$$X[k] = \sum_{n=0}^{\frac{N}{2}-1} x[2n]W_{\frac{N}{2}}^{-kn} + W_N^{-k}\sum_{n=0}^{\frac{N}{2}-1} x[2n+1]W_{\frac{N}{2}}^{-kn}$$

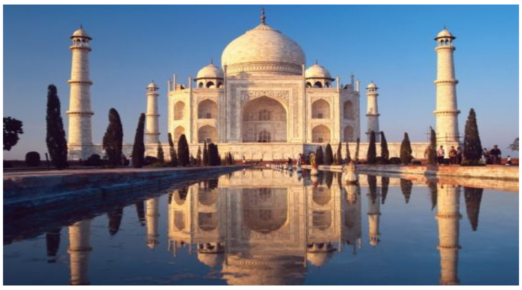
and

$$I)\;\; X\left[k+\frac{N}{2}\right] = \sum_{n=0}^{\frac{N}{2}-1} x[2n]\,W_{\frac{N}{2}}^{-kn} - W_N^{-k}\sum_{n=0}^{\frac{N}{2}-1} x[2n+1]\,W$$

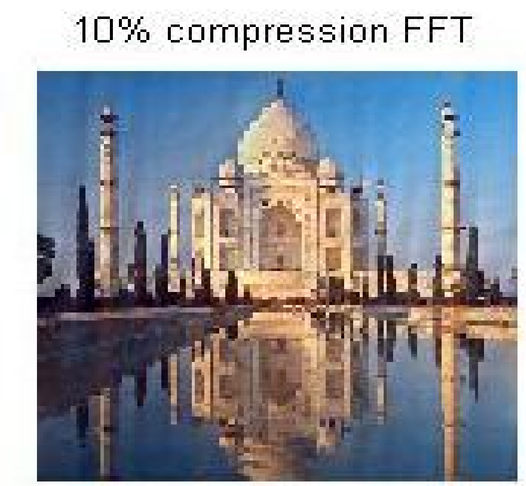
5. RESULT:

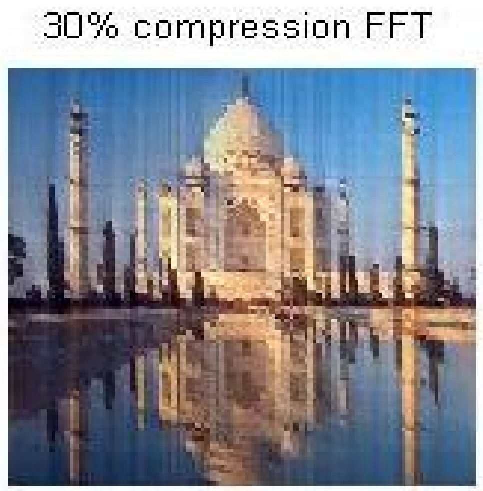
In this paper we have applied some basic MATLAB DCT functions to compress image of almost all types. We have successfully reconstruct all the images with different Compression Ratio(CR), Peak Signal to Noise Ratio and Mean Square Error(MSE).

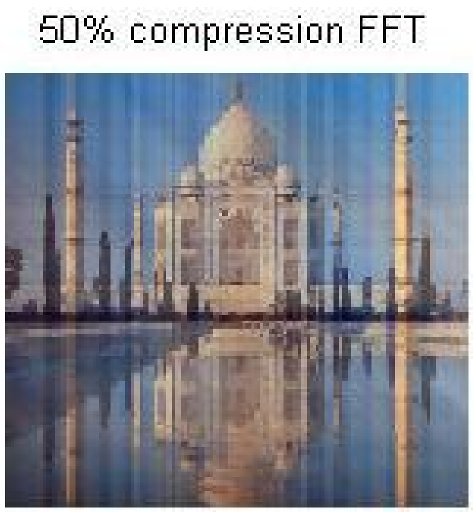
Original Image:



FFT Compression:







Original Image:

FFT Compression:

10% compression FFT

30% compression FFT

50% compression FFT

70% compression FFT

This method gives following results:-

Type of Image	Compression Ratio	Peak Signal to Noise Ratio	Mean Square Error
JPEG	50:1	26.8	105
PNG	27:1	24.6	93
BMP	61:1	28	98

3. Conclusion

In this paper we have applied FFT on almost all kinds of images. In the pictures JPEG images are only shown. This is a lossy compression method so compression ratio is very good but the image quality degrades as the compression ratio is increased.

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