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| 1 | 2 | 3 | Σ |
|---|---|---|---|
| | | | |

# Übungsblatt Nr. 8

(Abgabetermin 28.01.2022)

## Aufgabe 1

### a

$$\begin{array}{r} \texttt{F O U R} \\ +\texttt{F I V E} \\ \hline \texttt{N I N E} \end{array} \quad (1)$$

Constraints: AllDiff(F,O,U,R,I,V,E,N)
R + E = E + 10 * $C_1$
$C_1$ + U + V = N + 10 * $C_2$
$C_2$ + O + I = I + 10 * $C_3$
$C_3$ + F + F = N

1. R + E = E + 10 * $C_1$
   Assume R = 0 and E = 1
   0 + 1 = 1 + 10 * 0

2. $C_1$ + U + V = N + 10 * $C_2$
   Assume U = 7 and V = 8
   0 + 7 + 8 = 5 + 10 * 1

3. $C_2$ + O + I = I + 10 * $C_3$
   Assume O = 9 and I = 4
   1 + 9 + 4 = 4 + 10 * 1

4. $C_3$ + F + F = N
   $C_3$ + 2 + 2 = 4

$$\begin{array}{r} \texttt{2 9 7 0} \\ +\texttt{2 4 8 1} \\ \hline \texttt{5 4 5 1} \end{array} \quad (2)$$

**b**

$$\begin{array}{r} \texttt{C A K E} \\ +\texttt{C O O K I E} \\ \hline \texttt{A L I E N S} \end{array} \tag{3}$$

Constraints: AllDiff(C,A,K,E,O,I,L,N,S)
$E + E = S + 10 * C_1$
$C_1 + K + I = N + 10 * C_2$
$C_2 + A + K = E + 10 * C_3$
$C_3 + C + O = I + 10 * C_4$
$C_4 + O = L + 10 * C_5$
$C_5 + C = A$

$O = 9$ because we need a carry at the left most calculation.

$$\begin{array}{r} \texttt{C A K E} \\ +\texttt{C 9 9 K I E} \\ \hline \texttt{A L I E N S} \end{array} \tag{4}$$

$L = 0$ because carry can be max. 1, 9 + 1 = 10.

$$\begin{array}{r} \texttt{C A K E} \\ +\texttt{C 9 9 K I E} \\ \hline \texttt{A 0 I E N S} \end{array} \tag{5}$$

$C = 2$, must be lower 8, then $A = 3$ and $I = 1$.

$$\begin{array}{r} \texttt{2 3 K E} \\ +\texttt{2 9 9 K 1 E} \\ \hline \texttt{3 0 1 E N S} \end{array} \tag{6}$$

$S\%2 = 0$, $3 + K < 10$, thus $K = 5$, thus $E = 8$ thus $N = 7$

$$\begin{array}{r} \texttt{2 3 5 8} \\ +\texttt{2 9 9 5 1 8} \\ \hline \texttt{3 0 1 8 7 6} \end{array} \tag{7}$$

# Aufgabe 2

Original Domains (No Constraints considered):
$D_{1,1} = \{1\}$
$D_{1,2} = \{1, 2, 3, 4\}$
$D_{1,3} = \{1, 2, 3, 4\}$
$D_{1,4} = \{1, 2, 3, 4\}$

$D_{2,1} = \{1, 2, 3, 4\}$
$D_{2,2} = \{1, 2, 3, 4\}$
$D_{2,3} = \{1, 2, 3, 4\}$
$D_{2,4} = \{3\}$

$D_{3,1} = \{1, 2, 3, 4\}$
$D_{3,2} = \{1, 2, 3, 4\}$
$D_{3,3} = \{1, 2, 3, 4\}$
$D_{3,4} = \{1, 2, 3, 4\}$

$D_{4,1} = \{1, 2, 3, 4\}$
$D_{4,2} = \{2\}$
$D_{4,3} = \{4\}$
$D_{4,4} = \{1, 2, 3, 4\}$

Constraints:
Row:
AllDiff($D_{1,1}$,$D_{1,2}$,$D_{1,3}$,$D_{1,4}$)
AllDiff($D_{2,1}$,$D_{2,2}$,$D_{2,3}$,$D_{2,4}$)
AllDiff($D_{3,1}$,$D_{3,2}$,$D_{3,3}$,$D_{3,4}$)
AllDiff($D_{4,1}$,$D_{4,2}$,$D_{4,3}$,$D_{4,4}$)

Column:
AllDiff($D_{1,1}$,$D_{2,1}$,$D_{3,1}$,$D_{4,1}$)
AllDiff($D_{1,2}$,$D_{2,2}$,$D_{3,2}$,$D_{4,2}$)
AllDiff($D_{1,3}$,$D_{2,3}$,$D_{3,3}$,$D_{4,3}$)
AllDiff($D_{1,4}$,$D_{2,4}$,$D_{3,4}$,$D_{4,4}$)

In-box:
AllDiff($D_{1,1}$,$D_{1,2}$,$D_{2,1}$,$D_{2,2}$)
AllDiff($D_{1,3}$,$D_{1,4}$,$D_{2,3}$,$D_{2,4}$)
AllDiff($D_{3,1}$,$D_{3,2}$,$D_{4,1}$,$D_{4,2}$)
AllDiff($D_{3,3}$,$D_{3,4}$,$D_{4,3}$,$D_{4,4}$)

1. Examine $E_{4,1}$
   - Original Domain
     $\{1, 2, 3, 4\}$
   - Row Constraints
     AllDiff($D_{4,1}$,$D_{4,2}$,$D_{4,3}$,$D_{4,4}$)
     Exclude 2,4
   - Column Constraints
     AllDiff($D_{1,1}$,$D_{2,1}$,$D_{3,1}$,$D_{4,1}$)
     Exclude 1
   - In-box constraints
     AllDiff($D_{3,1}$,$D_{3,2}$,$D_{4,1}$,$D_{4,2}$)
   - Domain is now $\{3\}$ and $E_{4,1}$ must be 3
2. Examine $E_{4,4}$
   - Original Domain
     $\{1, 2, 3, 4\}$
   - Row Constraints
     AllDiff($D_{4,1}$,$D_{4,2}$,$D_{4,3}$,$D_{4,4}$)
     Exclude 2,3,4
   - Column Constraints
     AllDiff($D_{1,4}$,$D_{2,4}$,$D_{3,4}$,$D_{4,4}$)
   - In-box constraints
     AllDiff($D_{3,3}$,$D_{3,4}$,$D_{4,3}$,$D_{4,4}$)
   - Domain is now $\{1\}$ and $E_{4,4}$ must be 1
3. Examine $E_{1,3}$
   - Original Domain
     $\{1, 2, 3, 4\}$
   - Row Constraints
     AllDiff($D_{1,1}$,$D_{1,2}$,$D_{1,3}$,$D_{1,4}$)
     Exclude 1
   - Column Constraints
     AllDiff($D_{1,3}$,$D_{2,3}$,$D_{3,3}$,$D_{4,3}$)
     Exclude 4
   - In-box constraints
     AllDiff($D_{1,3}$,$D_{1,4}$,$D_{2,3}$,$D_{2,4}$)
     Exclude 3
   - Domain is now $\{2\}$ and $E_{1,3}$ must be 2

4. $E_{1,4}$ must be 4
5. $E_{1,2}$ must be 3
6. $E_{2,3}$ must be 1
7. $E_{2,2}$ must be 4
8. $E_{2,1}$ must be 2
9. $E_{3,1}$ must be 4
10. $E_{3,2}$ must be 1
11. $E_{3,3}$ must be 3
12. $E_{3,4}$ must be 2

| | | | |
|---|---|---|---|
| 1 | 3 | 2 | 4 |
| 2 | 4 | 1 | 3 |
| 4 | 1 | 3 | 2 |
| 3 | 2 | 4 | 1 |

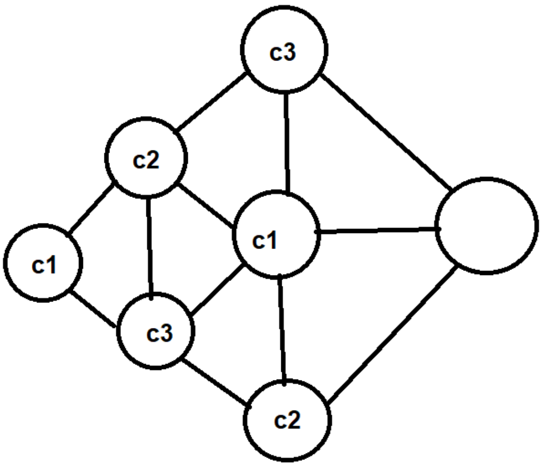
## Aufgabe 3

### a

It is obvious that A,B and D have to have three different colors.
Therefore E must have the same color as A.
It is then also clear to see that C has to be the same color as D and F the same as B.

Since G has to fulfill the Constrain AllDiff(C(C),C(E),C(F),C(G)) and C,E and F have different colors G can not have a color assignet and the puzzle is not solvable.

Doing the algorithm 3 Times but changing around the assigments of color 1, 2 and 3 will not change the outcome of the algorithm.




| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| | r,g,b | r,g,b | r,g,b | r,g,b | r,g,b | r,g,b | r,g,b |
| A-r | r | g,b | r,g,b | g,b | r,g,b | r,g,b | r,g,b |
| B-g | r | g | r,b | b | r,b | r,g,b | r,g,b |
| C- b | r | g | r | b | r | r,g,b | r,g |
| D- b | r | g | r | b | | r,g | g,b |
| Back | | | | | | | |
| C- r | r | g | r | b | b | r,g,b | g,b |
| D- b | r | g | r | b | | r,g | g,b |
| Back | | | | | | | |
| B- b | r | b | r,g | g | r,g | r,g,b | r,g,b |
| C- r | r | b | r | g | g | r,g,b | g,b |
| D- g | r | b | r | g | | r,b | g,b |
| Back | | | | | | | |
| C- g | r | b | g | g | r | r,g,b | r,b |
| D- g | r | b | g | g | r | r,b | r,b |
| E- r | r | b | g | g | r | b | b |
| F- b | r | b | g | g | r | b | |

### b