Artificial Intelligence

Assignment 12

This Assignment will not be graded.

Question 1 Unification

Unify the following sentences. Please specify the most general unifier (MGU), and in case the unification is not possible, justify your answer. Take into account that only small letters are variables. Standardizing apart is not allowed in this question.

(a) $P(A, A, B), \quad P(x, y, z)$.

Solution:
$MGU = \{x/A, y/A, z/B\}$ Unified: $P(A, A, B)$

(b) $H(y, z, G(A, B)), \quad H(G(x, x), C, y)$.

Solution:
$MGU = \{y/G(x, x), z/C, x/A, ...$ Failure!
The unification is not possible, because x cannot be substituted by both A and B.

(c) $P(x, Q(x, A), H(B, y, z)), \quad P(B, Q(B, z), H(x, C, z))$.

Solution:
$MGU = \{x/B, z/A, y/C\}$ Unified: $P(B, Q(B, A), H(B, C, A))$

(d) $Q(P(y, z), x, H(x)), \quad Q(x, P(A, B), H(x))$.

Solution:
$MGU = \{x/P(y, z), y/A, z/B\}$ Unified: $Q(P(A, B), P(A, B), H(P(A, B)))$

(e) $H(A, v, P(l)), \quad H(m, B, l), \quad H(A, u, l)$.

Solution:
$MGU = \{m/A, v/u, ...$ Failure!
The unification is not possible, because the occur-check does not allow the unification of l with $P(l)$.

(f) $Older(Father(y), y), \quad Older(Father(x), John)$.

Solution:
$MGU = \{y/x, y/John\}$
Unified: $Older(Father(John), John)$

Question 2 Substitution and Skolemization

In this question Sk_0 is a Skolemization constant.

(a) Given the premise $\forall x\ \exists y\ P(x, y)$, it is not valid to conclude that $\exists q\ P(q, q)$. Give an example of a predicate P where the first is true but the second is false.

(b) Show that from $\forall q\ P(q, q)$, you can conclude $\forall x\ \exists y\ P(x, y)$.

(c) Show that from $\forall x\ \exists y\ P(x, y)$, you can not conclude $\forall x\ P(x, Sk_0)$.

(d) Show that from $\forall x\ \exists y\ P(x, y)$, you can conclude $P(Sk_0, F(Sk_0))$.

Solution:

(a) Let $P(x, y) = Father(x, y)$ where $Father(x, y)$ denotes the father/child relationship saying x has y as father. The statement $\exists q\ Father(q, q)$ is of course not possible.

(b) We know that

$$\forall q\ P(q, q) \tag{1}$$

but

$$\neg\ \forall x\ \exists y\ P(x, y) \equiv \exists x\ \forall y\ \neg P(x, y)$$

Let's say X_o is this x such that the above holds: $\forall y\ \neg P(X_o, y)$
if $y = X_o$ then

$$\neg P(X_o, X_o) \tag{2}$$

From 1 we know that $P(X_o, X_o)$ that contradicts 2.

(c) Let $P(x, y) = Loves(x, y)$ where $Loves(x, y)$ denotes that a person x loves the person y. Then statement $\forall x\ P(x, Sk_0)$ is not possible because otherwise everybody needs to love the person indicated by Sk_0 (Sk_0 is constant).

(d) As in $\forall x\ \exists y\ P(x, y)$, existential quantifier is embedded in a universally quantified statement when we use a Skolem function we have $\forall x\ P(x, F(x))$. If $x = Sk_0$ then we have $P(Sk_0, F(Sk_0))$.

Question 3 Resolution in First-Order-Logic

Prove the statement $A(x) \wedge B(y, f(x))$ by using resolution from the following knowledge base. For each step, also give the numbers of the clauses used and the unifier (the substitution) θ.

1. $B(x_1, f(x_2)) \vee \neg A(x_1) \vee \neg C(x_2)$
2. $\neg D(y_1, y_2) \vee A(f(y_2))$
3. $C(f(G))$
4. $D(z_1, G)$

Solution:

1. $B(x_1, f(x_2)) \vee \neg A(x_1) \vee \neg C(x_2)$
2. $\neg D(y_1, y_2) \vee A(f(y_2))$

3. $C(f(G))$
4. $D(z_1, G)$
5. $\neg A(x) \vee \neg B(y, f(x))$ (negation)
6. $A(f(G))$, $\theta = \{y_1/z_1, y_2/G\}$, (2 and 4)
7. $B(x_1, f(f(G))) \vee \neg A(x_1)$, $\theta = \{x_2/f(G)\}$, (1 and 3)
8. $B(f(G), f(f(G)))$, $\theta = \{x_1/f(G)\}$, (6 and 7)
9. $\neg B(y, f(f(G)))$, $\theta = \{x/f(G)\}$, (5 and 6)
10. empty clause, $\theta = \{y/f(G)\}$, (8 and 9)

Question 4 First order logic

(a) Consider a vocabulary with the following predicates:

- $Student(x)$: x is a student
- $Takes(x, c)$: student x takes course c
- $Passes(x, c)$: student x passes course c

Constants denoting courses are: $English$, $German$, $French$. Constants denoting students are: $Alex$ and $James$. Use these symbols to write the following assertions in first order logic:

- Students who take French cannot take German.
- If there is a student who took English and failed it, then it must be $James$ who failed it.
- If and only if Alex fails German, he can take French.

Solution:

- $\forall\ x\ Student(x) \wedge Takes(x, French) \Rightarrow \neg Takes(x, German)$
- $\exists\, x\ Student(x) \wedge Takes(x, English) \wedge \neg Passes(x, English) \wedge \neg Passes(James, English)$
  $\exists\, x\ Student(x) \wedge Takes(x, English) \wedge \neg Passes(x, English) \Rightarrow \neg Passes(James, English)$
  $\exists\ x\ Student(x) \wedge Takes(x, English) \wedge \neg Passes(x, English) \wedge x = James$
  $\exists\ x\ Student(x) \wedge Takes(x, English) \wedge \neg Passes(x, English) \Rightarrow x = James)$
- $\neg Passes(Alex, German) \Leftrightarrow Takes(Alex, French)$

(b) Consider a vocabulary with the following predicates:

- $Person(x)$: x is a person
- $Born(x, c)$: person x is born in country c
- $Parent(x, y)$: person x is a parent of y
- $Citizen(x, c)$: person x is a citizen of country c
- $Resident(x, c)$: person x is a resident of country c

Use these symbols to write the following assertions in first order logic:

- A person born inside the UK, both of whose parents are not UK citizens, is not a UK citizen.

- A person born outside the US, both of whose parents are US citizens but not US residents, is not a US citizen.

<u>Solution:</u>

- $\forall\ x\ Person(x) \land Born(x, UK) \land (\forall\ y\ Parent(y, x) \land \neg Citizen(y, UK))$
  $\Rightarrow \neg Citizen(x, UK)$
- $\forall\ x\ Person(x) \land \neg Born(x, US) \land$
  $(\forall\ y\ Parent(y, x) \land Citizen(y, US) \land \neg Resident(y, US)) \Rightarrow \neg Citizen(x, US)$