NANYANG POLYTECHNIC

School of Information Technology

Module	:	Business Statistics
Topic	:	Probability Concepts

Learning Outcomes:

By the end of this lesson, you should be able to

1. Describe a probability event by listing all the possible outcomes in that event
2. Calculate probabilities using probability concepts such as rules of addition and multiplication
3. Apply the number of outcomes using principles of counting.

Topic: Probability Concepts

Part I: Probability Concepts

QUESTION 1

Suppose you draw a card from a standard deck of poker cards, which consists of 52 cards. Find the probability that the card is a "4" or an ace.

QUESTION 2

The probability that a family owns a car is 0.48, that it owns a 5-room flat is 0.35, and that it owns both a car and a 5-room flat is 0.21. What is the probability that a randomly selected family owns a car or a 5-room flat?

QUESTION 3

The probability of a successful appendicitis operation is 98%. Find the probability that

(a)   out of three operations, all are successful.
(b)   out of two operations at least one is unsuccessful

Assume that the outcomes of the operations are independent of each other.

QUESTION 4

The table below shows the gender and blood pressure categories of 300 participants

Blood Pressure	Female	Male	Row Total
Normal	39	25	64
Pre-hypertension	61	50	111
High Stage 1	42	47	89
High Stage 2	20	16	36
Column Total	162	138	300

A participant is randomly chosen. Calculate the probability

(a)   participant is male,
(b)   participant has high stage 2 blood pressure,

QUESTION 5

A group of files in a medical clinic classifies the patients by gender and by type of diabetes (I or II). The groupings may be shown as follows. The table gives the number in each classification.

		Type of Diabetes	
		I	II
Gender	Male	25	20
	Female	35	20

If one file is selected at random, find the probability that the individual is a

(a) female.
(b) Type II
(c) Type II, given that the patient is a male.
(d) Are the events "Type II" and "a male" independent?
(e) Are the events "Type I" and "a female" mutually exclusive?

QUESTION 6

1000 people were randomly selected and they were asked whether they are right-handed or left-handed. The following table shows the result of the survey:

	Men	Women
Left-handed	63	50
Right-handed	462	425

(a) A person is selected at random from the sample. Find the probability that the person is
  (i)   left-handed or a woman;
  (ii) right-handed or a man;
  (iii) not right handed given the person is a man;
  (iv) right-handed woman.

(b) Are the events "being right-handed" and "being a woman" mutually exclusive? Explain.

QUESTION 7

The table below shows the results of a survey of the 120 cars in a carpark, in which the colour of each car and the gender of the driver were recorded.

	Male	Female
Green	18	12
Blue	48	22
Red	6	14

One of the cars is selected at random.
  M is the event that the car selected has a male owner.
  G is the event that the car selected is green.
  B is the event that the car selected is blue.
  R is the event that the car selected is red.

Find the following probabilities:

(a) $P(M \cup B)$,
(b) $P(M|R')$.
(c)   Determine whether the events M and G are independent, justifying your answer.

QUESTION 8

Two thousand randomly selected adults were asked if they think they are better off financially than their parents. The following table gives the two-way classification of the responses based on the education levels of the adults and whether they are financially better off, the same, or worse off than their parents.

	Primary	Secondary	Tertiary
Better off	140	450	420
Same	60	250	110
Worse off	200	300	70

Suppose one adult is selected at random from these 2000 adults. Find the probability that the adult is

(a)   better off and has secondary education,
(b)   not the same financially,
(c)   worse off or has primary education,
(d)   not better off given secondary education.

QUESTION 9

A shipment of two boxes, each containing 6 calculators is received by a store. Box 1 contains one defective calculator and box 2 contains two defective calculators. After the boxes are unpacked, a calculator is selected and found to be defective.

(a) Draw a tree diagram to represent the possible outcomes.

(b) Find the probability that the defective calculator came from box 2.

QUESTION 10

A certain virus infects 0.5 % of the population. A test will be positive 80% of the time if the person has the virus and 5 % of the time if the person does not have the virus. Suppose A is the event "the person is infected" and B is the event "the person tests positive".

(a)   Draw a tree diagram to show the outcomes of the tests.

(b)   Find the probability that
      (i)   the person is infected and is tested positive,
      (ii)   the person is tested positive.

QUESTION 11

Two children, Tan and Mui, are each to be given a pen from a box containing 3 red pens and 5 blue pens. One pen is chosen at random and given to Tan. A green pen is then put in the box. A second pen is chosen at random from the box and given to Mui.

(a) Draw a tree diagram to represent the possible outcomes.
(b) Find the conditional probability that Mui's pen is blue, given that Tan's pen is red.
(c) Find the probability that Mui's pen is red.
(d) Find the conditional probability that Tan's pen is red, given that Mui's pen is blue.

Part 2: Principles of Counting - Permutation & Combination

QUESTION 12

Using the letters from the word *COMPUTER*, find

(a) the number of words that can be formed using all the letters,
(b) the number of 4–letter words that can be formed.

QUESTION 13

Find the number of 3-digit PIN codes that can be formed using the digits 1, 2, 3, 4 ,5 ,6 if

(a) no repetitions are allowed
(b) repetitions are allowed.

QUESTION 14

In how many distinguishable ways can the letters in the following words be arranged?

(a) PAPAYA   (b) PERMUTATIONS

QUESTION 15

(a) How many different ways can three of the letters of the word *BYTES* be chosen and written in a row?

(b) How many different ways can this be done if the first letter must be "*B*"?

QUESTION 16

In how many ways can three distinct letters and two distinct digits be arranged if

(a) there is no restriction,
(b) the letters must come first,
(c) the digits must always be together.

QUESTION 17

Find the number of distinguishable ways the word *STATISTICS* can be arranged

(a) without conditions,
(b) if the letter "T"s must be together,
(c) if no two "T"s are together.

QUESTION 18

A meal consists of a main dish, a vegetable dish and two different desserts. Given 10 main dishes, 8 vegetable dishes, and 13 desserts, how many different meals are possible?

QUESTION 19

In a class of 20 people there are 13 girls and 7 boys. Find the number of ways to form a committee of 8 members if

(a)   there are no restrictions,
(b)   the committee is made up of all girls,
(c)   there is exactly 1 boy in the committee,
(d)   there are less than 2 boys in the committee.

QUESTION 20

There are 10 students who are going to spend the evenings in 2 groups; one group goes to the Library and the other plays football. In how many ways can the group for football be selected if there must be at least 4 people in each group?

QUESTION 21

A sample of 5 mice is to be chosen from 7 male and 6 female mice. In how many ways can the sample be selected if it must have at least 2 male and 1 female mice?

QUESTION 22

A shipment of 10 microwave ovens contains two defective units. In how many ways can a restaurant buy three of these units and receive

(a)   no defective units?
(b)   one defective unit?
(c)   at least two non-defective units?

QUESTION 23

Janet has 10 different books that she is going to put on her bookshelf. Of these, 4 are Chemistry books, 3 are Biology books, 2 are Statistics books, and 1 Physics book. Janet wants to arrange her books so that all the books dealing with the same subject are together on the shelf. How many different arrangements are possible?

SUPPLEMENTARY QUESTIONS

QUESTION 24

In a road show, the compere holds a bag containing 4 movie tickets and 6 concert tickets. 4 tickets are to be drawn at random and given away to 4 lucky winners on stage. Find the probability that

(a)   all 4 drawn are concert tickets.
(b)   4 tickets are not of the same type.
(c)   at least 2 movie tickets are drawn.

QUESTION 25

In a group of 10 persons, 4 have a type *A* personality and 6 have a type *B* personality. If two persons are selected at random from this group, what is the probability that the two will have different personality type?

QUESTION 26

If 3 books are picked at random from shelf containing 6 novels, 5 cook books and 1 computer book, what is the probability that

(a) the computer book is selected?
(b) 2 novels and 1 cook book are selected?

QUESTION 27

Four sales representatives for a company are to be chosen to participate in a training program. The company has eight sales representatives, two in each of four regions. In how many ways can the four representatives be chosen if

(a) there are no restrictions?
(b) the selection must include a sales representative from each region?
(c) the selection must be from only two of the four regions?

Answers:

Q1   2/13
Q2   0.62
Q3   (a) 0.941   (b) 0.0396
Q4   (a) 23/50   (b) 3/25
Q5   (a) 11/20   (b) 2/5   (c) 4/9   (d) No   (e) No
Q6   (a) (i) 269/500 (ii) 19/20   (iii) 3/25   (iv) 17/40   (b) No
Q7   (a) 47/60   (b) 33/50   (c) Independent
Q8   (a) 9/40   (b) 79/100   (c) 77/200   (d) 11/20
Q9   (b) 2/3
Q10   (b)(i) 0.004   (ii) 0.05375
Q11   (b) 5/8   (c) 21/64   (d) 3/7
Q12   (a) 40320   (b) 1680
Q13   (i) 120   (ii) 216
Q14   (a) 60   (b) 239,500,800
Q15   (a) 60   (b) 12
Q16   (a) 120   (b) 12   (c) 48
Q17   (a) 50400   (b) 3360   (c) 23520
Q18   6240
Q19   (a) 125970   (b) 1287   (c) 12012   (d) 13299
Q20   672
Q21   1155
Q22   (a) 56   (b) 56   (c) 112
Q23   6912

Q24   (a) 1/14   (b) 97/105   (c) 23/42
Q25   8/15
Q26   (a) 1/4   (b) 15/44
Q27   (a) 70   (b) 16   (c) 6