

School of Information Technology

Module	:	Business Statistics
Topic	:	Sampling Distribution and Central Limit Theorem

Learning Outcomes:

By the end of this lesson, you should be able to

1. Define the sampling distribution of the sample means with reference to inferential statistics.
2. Calculate probabilities involving the sample mean by using the standard normal distribution and the central limit theorem.
3. Apply probabilities of the sample mean values in real-life business problems by applying concepts on the sampling distribution of sample means

Topic: Sampling Distribution and Central Limit Theorem

Part 1 Sampling from Normal Populations

QUESTION 1

The price-earnings (P/E) ratio of a stock is usually considered by analysts who put together financial portfolios. Suppose a population of P/E ratios follow an approximately a normal distribution with mean 10.5 and standard deviation of 4.5, find the probability that a random sample of 40 stocks having an average P/E ratio less than 9.

QUESTION 2

General Appliances has microwave ovens that need repair. The mean cost of repair for the microwaves is \$80. The standard deviation of the cost is \$35. The cost can be approximately normally distributed. If a sample of 10 microwaves is selected, what is the probability that the mean cost for the sample is greater than \$100?

QUESTION 3

In a certain population of swordtail fish, the lengths of the individual fish follow approximately a normal distribution with mean 52.0 mm and standard deviation of 6.0 mm. Find the probability that a random sample of 25 swordtail fishes will have an average length of

- a) less than 48.6 mm
- b) between 52.4 and 54.4 mm.

QUESTION 4

The intelligence quotient (IQ) score of a certain population of children is approximately normally distributed with a mean of 102 and a standard deviation of 10. Let X be the random variable 'the IQ score of children.' If a random sample of n children is selected, find the value of n given that $P(\bar{X} > 103) = 0.3446$.

Part 2 Central Limit Theorem

QUESTION 5

The average length of actual running time for television feature films is 1 hour and 40 minutes, with a standard deviation of 15 minutes. If a sample of 49 TV feature films is taken at random, what is the probability that the average running time is 1 hour and 45 minutes or more?

QUESTION 6

The average number of days spent in a particular hospital for a coronary bypass is 9 days and the standard deviation is 4 days. What is the probability that a random sample of 30 patients will have an average stay longer than 9.5 days? State any assumptions required on the distribution on the days spent.

QUESTION 7

In a government hospital, the average medical subsidy for a patient is \$850 with a standard deviation is \$240.

- a) A random sample of 64 patients is selected. What is the probability that the average medical subsidy is between \$820 and \$880?

- b) Suppose that in part (a) the sample size is increased to 100. Without doing any calculations, do you think the probability would increase or decrease?

SUPPLEMENTARY QUESTIONS

QUESTION 8

The monthly electric bill for a small town has a mean of \$120 with a standard deviation of \$25. The electric bill can be approximately normally distributed. If 10 households are selected at random, what is the probability that the sample mean will be between \$110 and \$130?

QUESTION 9

The heartbeat rate of a certain population of babies follows a normal distribution with mean 70 beats/min and standard deviation of 10 beats/min.

- Find the probability that a baby randomly selected from this population has a heartbeat rate of less than 66 beats/min.
- If a sample of 8 babies is randomly selected, find the probability 3 of them will have a heartbeat rate of less than 66 beats/min
- If a random sample of 36 babies is selected, what is the probability that it will have a mean heartbeat rate of more than 68 beats/min.

Answers:

- Q1 0.0174
Q2 0.0351
Q3 (a) 0.0023 (b) 0.3479
Q4 16
Q5 0.0099
Q6 0.2483
Q7 (a) 0.6826 (b) Increase
Q8 0.7924
Q9 (a) 0.3446 (b) 0.2771 (c) 0.8849