

2. take derivative<sup>to</sup>  $w$ , it should be same as the separable case, the parameter  $\epsilon$  and  $\beta$  have no effect on derivative of  $w$ .

$$\nabla_w L = w - \sum_{i=1}^m a_i y_i x_i = 0 \Rightarrow w = \sum_{i=1}^m a_i y_i x_i \quad (2)$$

take derivative to  $b$ , same as separable case.

$$\nabla_b L = -\sum_{i=1}^m a_i y_i = 0 \Rightarrow \sum_{i=1}^m a_i y_i = 0 \quad (3)$$

take derivative to  $\epsilon$ , new parameter, but we can just simply take derivative to it

$$\nabla_{\epsilon} L = \sum_{i=1}^m (1 - y_i (w^T x_i + b)) - \sum_{i=1}^m \beta_i = 0 \Rightarrow C = a_i + \beta_i \quad (4)$$

For  $a$ :

$$\forall_i a_i (y_i (w^T x_i + b) - 1 + \epsilon_i) = 0 \Rightarrow a_i = 0 \text{ or } y_i (w^T x_i + b) = 1 - \epsilon_i \quad (5)$$

For  $\beta$ :

$$\forall_i \beta_i \epsilon_i = 0 \Rightarrow \beta_i = 0 \text{ or } \epsilon_i = 0 \quad (6)$$



3. when  $d=3$  with  $x, x' \in \mathbb{R}^3$

$$K(x, x') = (\langle x, x' \rangle + c)^3$$

$$= (x_1 x'_1 + x_2 x'_2 + c)^2 (x_1 x'_1 + x_2 x'_2 + c)$$

$$= (x_1^2 x_1'^2 + x_2^2 x_2'^2 + 2x_1 x'_1 x_2 x'_2 + 2c x_2 x'_2 + 2c x_1 x'_1$$

$$+ c^2)(x_1 x'_1 + x_2 x'_2 + c)$$

$$= x_1^3 x_1'^3 + x_2^2 x_1'^2 x_2 x'_2 + c x_1^2 x_1'^2 + x_2^2 x_2'^2 x_1 x'_1 + x_2^3 x_2'^3 +$$

$$c x_2^2 x_2'^2 + 2x_1^2 x_1'^2 x_2 x'_2 + 2x_2^2 x_2'^2 x_1 x'_1 + 2c x_1 x'_1 x_2 x'_2 +$$

$$2c x_1^2 x_1'^2 + 2c x_1 x'_1 x_2 x'_2 + 2c^2 x_1 x'_1 + 2c x_1 x'_1 x_2 x'_2 +$$

$$2c x_2^2 x_2'^2 + 2c^2 x_2 x'_2 + c^2 x_1 x'_1 + c^2 x_2 x'_2 + c^3$$

combine these polynomial function

$$= x_1^3 x_1'^3 + x_2^3 x_2'^3 + 3c x_1^2 x_1'^2 + 3c x_2^2 x_2'^2 + 3x_1^2 x_1'^2 x_2 x'_2 + 3x_2^2 x_2'^2 x_1 x'_1 +$$

$$6c x_1 x'_1 x_2 x'_2 + 3c^2 x_1 x'_1 + 3c^2 x_2 x'_2 + c^3$$

splitting...

$$= [x_1^3, x_2^3, \sqrt{3}c x_1^2, \sqrt{3}c x_2^2, \sqrt{3}x_2 x_1, \sqrt{3}x_1 x_2, \sqrt{6}c x_1 x_2, \sqrt{3}x_1, \sqrt{3}x_2, c^{\frac{3}{2}}]$$

$$So, \phi(x) = [x_1^3, x_2^3, \sqrt{3}c x_1^2, \sqrt{3}c x_2^2, \sqrt{3}x_2 x_1, \sqrt{3}x_1 x_2, \sqrt{6}c x_1 x_2, \sqrt{3}x_1, \sqrt{3}x_2, c^{\frac{3}{2}}]^T$$

maps a 2-D data point  $x$  into 10-D space as  $\phi(x)$

$$\begin{bmatrix} x_1^3 \\ x_2^3 \\ \sqrt{3}c x_1^2 \\ \sqrt{3}c x_2^2 \\ \sqrt{3}x_2 x_1 \\ \sqrt{3}x_1 x_2 \\ \sqrt{6}c x_1 x_2 \\ \sqrt{3}x_1 \\ \sqrt{3}x_2 \\ c^{\frac{3}{2}} \end{bmatrix}$$