Week 3 assignment

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JT393

Problem 1

- **Question:**
- Vary $\lambda \in (0, 1)$. Use PCA and plot the cumulative variance explained by each eigenvalue for each λ chosen.
- **What does this tell us about values of λ and the effect it has on the covariance matrix?**

Problem 1

- In this problem, first we need to build up some functions to calculate weighted covariance matrix. Below, are the based formulas.

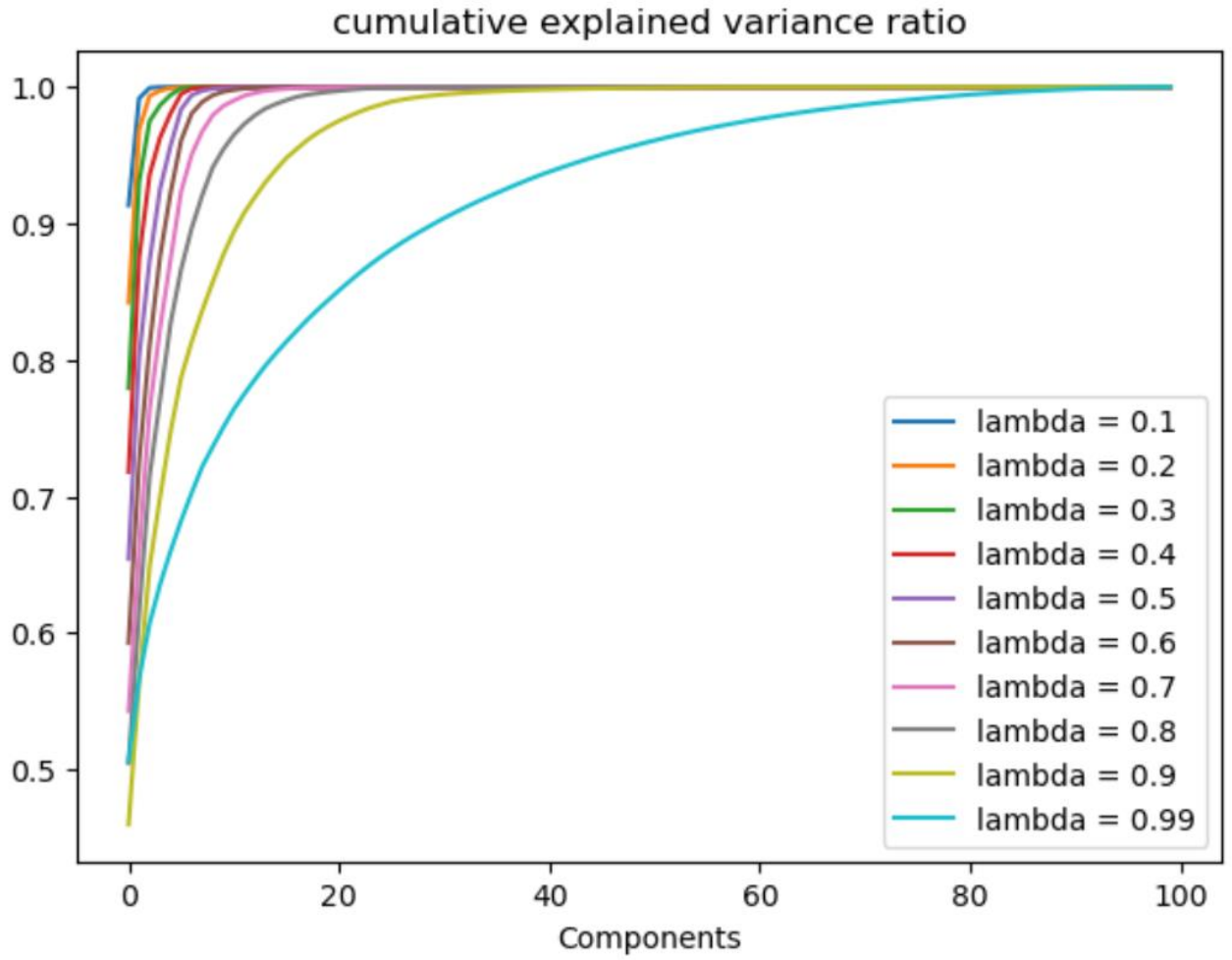
$$w_{t-i} = (1 - \lambda)\lambda^{i-1} \qquad \widehat{w_{t-i}} = \frac{w_{t-i}}{\sum_{j=1}^{n} w_{t-j}}$$

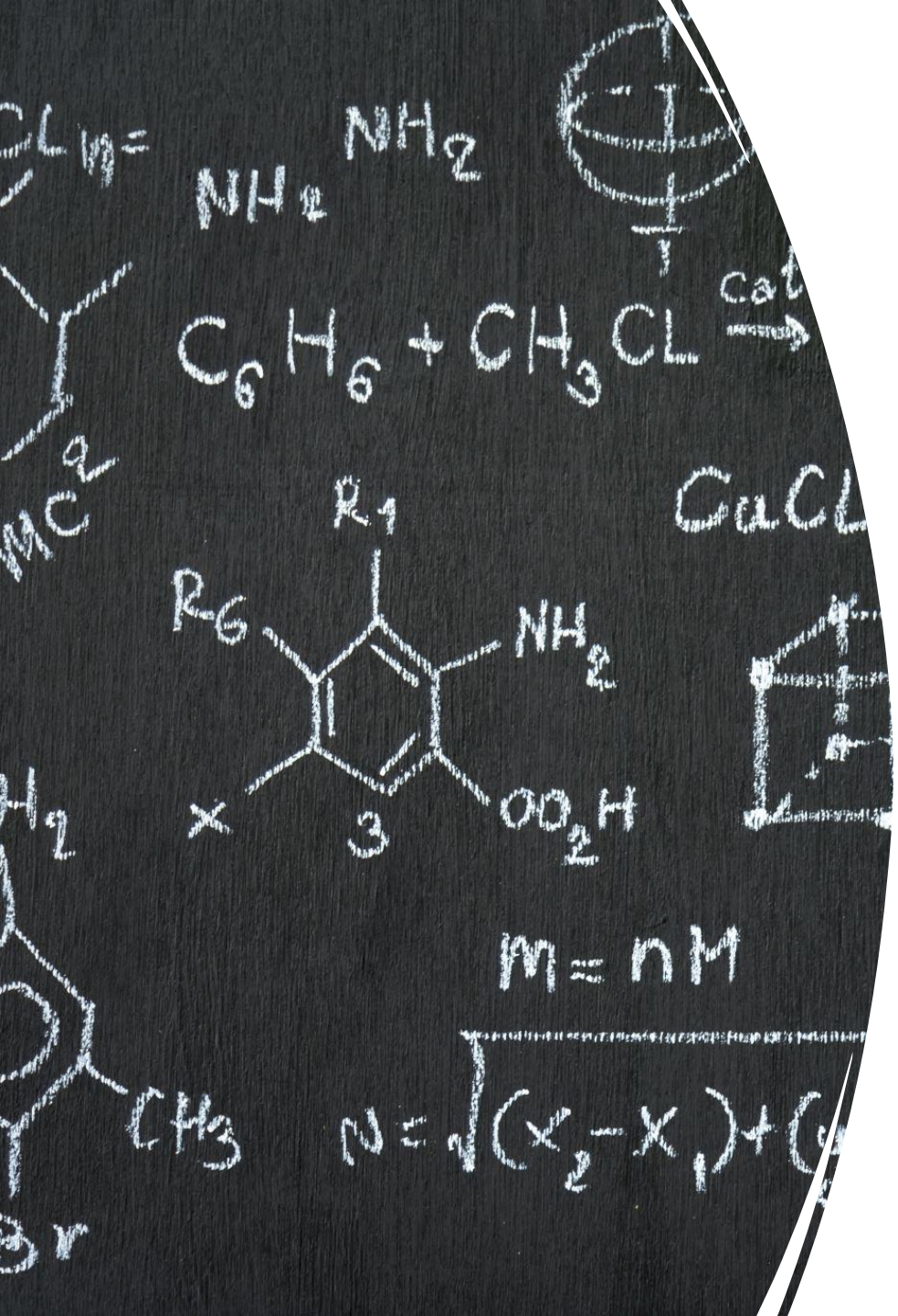
$$\widehat{\sigma_t^2} = \sum_{=1}^{n} w_{t-i}\left(x_{t-i} - \bar{x}\right)^2$$

$$\widehat{cov(x,y)} = \sum_{i=1}^{n} w_{t-i}\left(x_{t-i} - \bar{x}\right)\left(y_{t-i} - \bar{y}\right)$$

Problem 1

- **Ans:** Based on the graph, we can see that different values of λ will have different impacts on the eigenvalues and the cumulative variance explained.
- When lambda becomes larger, the covariance matrix is more evenly distributed throughout the time and went slowly towards the larger components.
- Larger eigenvalues (λ) indicate that more variance is captured by the corresponding principal component.





Problem 2

Question:

- Copy the chol_psd(), and near_psd() functions from the course repository – implement in your programming language of choice. These are core functions you will need throughout the remainder of the class.
- Implement Higham's 2002 nearest psd correlation function.
- Generate a non-psd correlation matrix that is 500x500.
- Use near_psd() and Higham's method to fix the matrix. Confirm the matrix is now PSD.
- Compare the results of both using the Frobenius Norm.
- Compare the run time between the two. How does the run time of each function compare as N increases?
- Based on the above, discuss the pros and cons of each method and when you would use each. There is no wrong answer here, I want you to think through this and tell me what you think.

Problem 2

- **Ans: First, we checked two functions returned matrix are now psd matrix.**

```python
#check if the matrix is now PSD
def is_psd(matrix, tol=1e-7):
    return np.all(np.linalg.eigvals(matrix) >= -tol)

#Generate a non-psd correlation matrix that is 500x500
n = 500
sigma = np.matrix(np.full((n, n), 0.9))
np.fill_diagonal(sigma, 1)
sigma[0, 1] = 0.7357
sigma[1, 0] = 0.7357

#print comfirm result
near_psd_matrix = near_psd(sigma)
print(is_psd(near_psd_matrix))
higham_psd_matrix = higham_psd(sigma)
print(is_psd(higham_psd_matrix))
```

```
True
True
```

Problem 2

- This shows that the result from higham_psd_matrix function is more accurate than the one from near_psd _matrix function.

```
#Compare the results of both using the Frobenius Norm
print(frobenius_norm(near_psd_matrix - sigma))
print(frobenius_norm(higham_psd_matrix - sigma))
```

✓ 0.0s

```
0.6275226557650357
0.08964799629214762
```

Problem 2

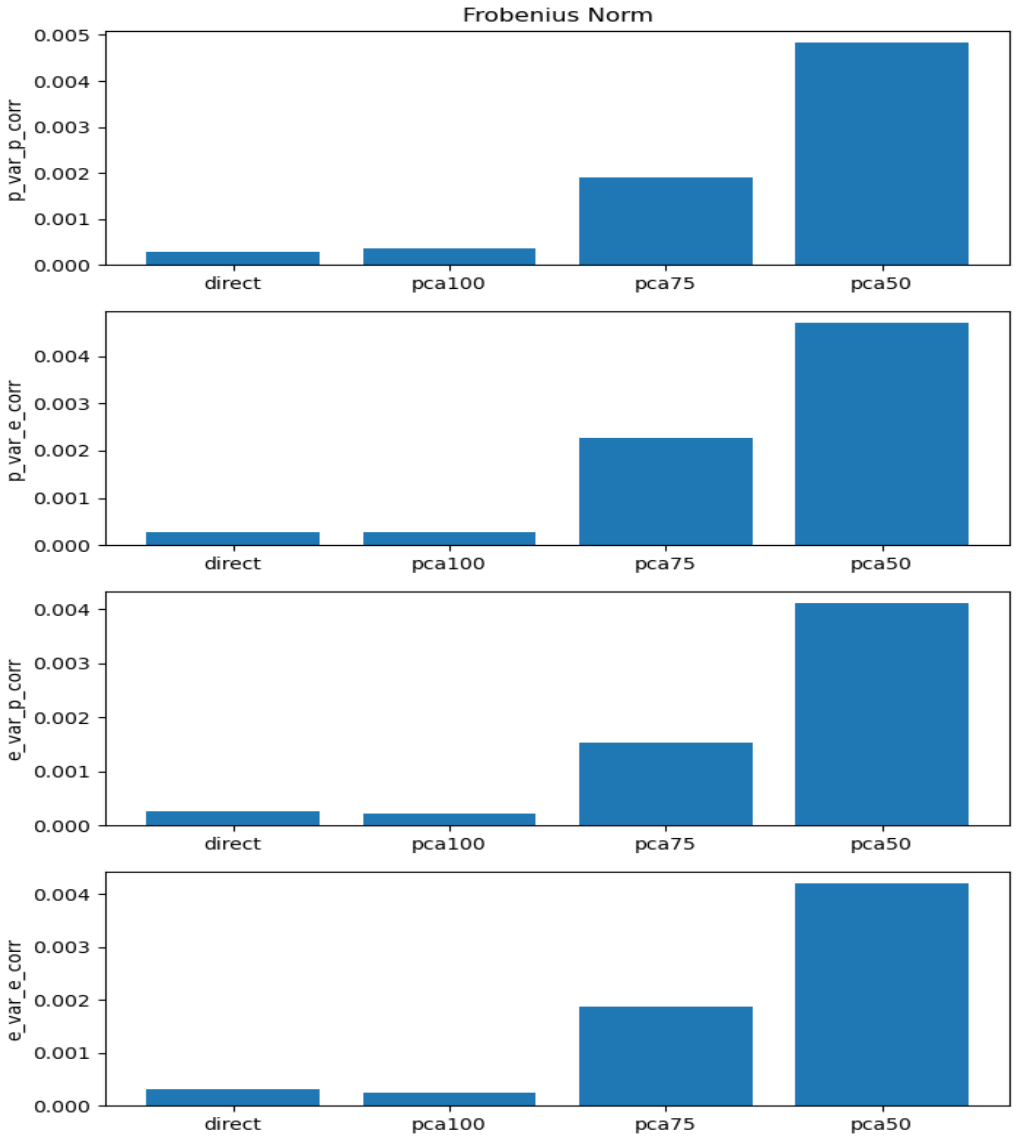
- **Ans:**
- Based on the graph, we can see that near_psd method requires much less time to run than higham_psd method. As the run time increase, the gap between these two methods also increase.
- We know that the near_psd function is less accurate than higham_psd, but it runs very fast. And higham_psd runs slower but more accurate.
- If we have to run it in a large number of times and we do have a time constraint, we should use near_psd. Otherwise, we should choose higham, which is because it is more accurate.

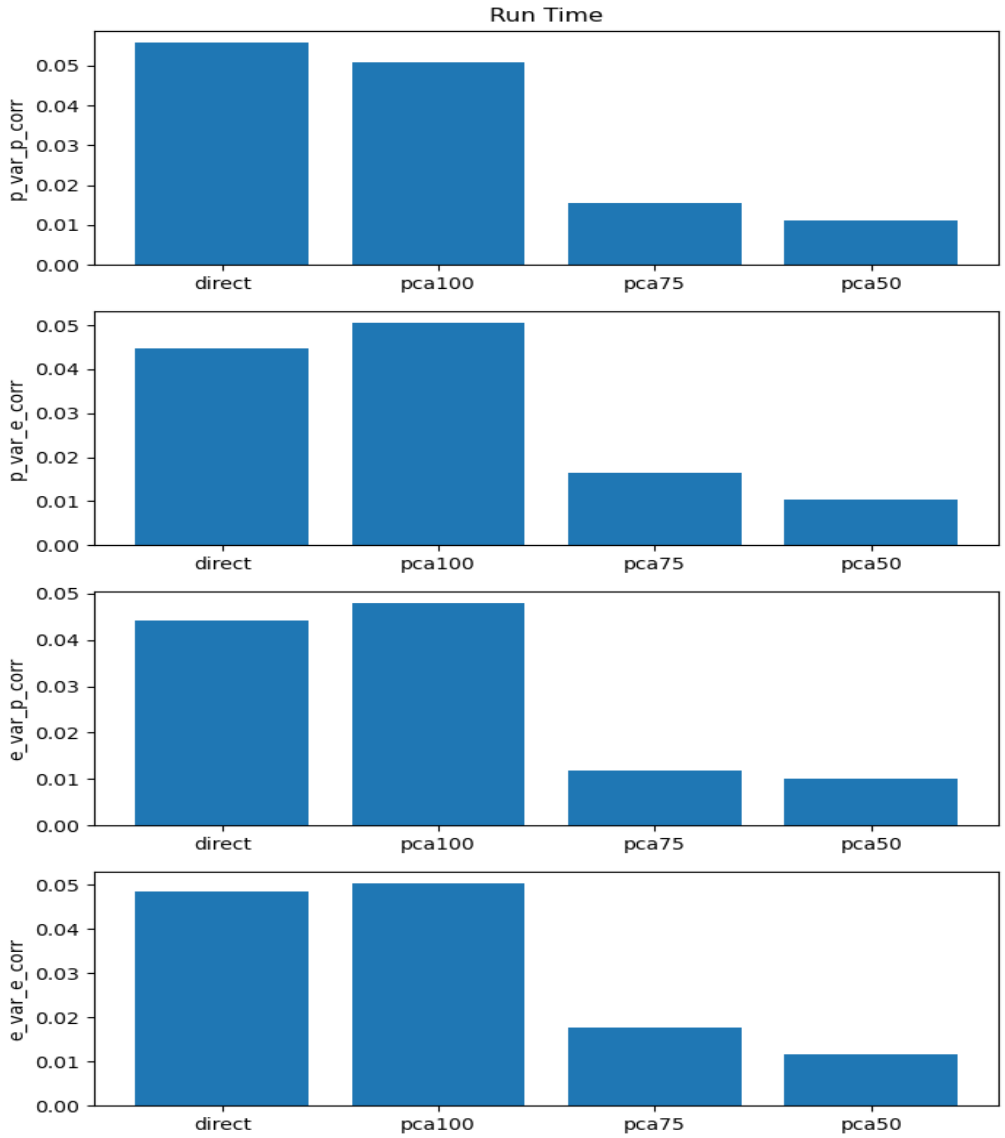
RUN TIME	Near_psd	Higham_psd	Ratio
300	0.0504258	0.60389328	11.97589
500	0.0668557	1.316893101	19.69755
700	0.0800281	2.366162777	29.56667

Problem 3

- **Questions:**
- Implement a multivariate normal simulation that allows for simulation directly from a covariance matrix or using PCA with an optional parameter for % variance explained.
- Generate a correlation matrix and variance vector 2 ways: 1. Standard Pearson correlation/variance (you do not need to reimplement the cor() and var() functions). 2. Exponentially weighted $\lambda = 0.97$
- Combine these to form 4 different covariance matrices. (Pearson correlation + var()), Pearson correlation + EW variance, etc.)
- Simulate 25,000 draws from each covariance matrix using:
    1. Direct Simulation
    2. PCA with 100% explained.
    3. PCA with 75% explained.
    4. PCA with 50% explained.
- Calculate the covariance of the simulated values. Compare the simulated covariance to it's input matrix using the Frobenius Norm (L2 norm, sum of the square of the difference between the matrices).
- Compare the run times for each simulation. What can we say about the trade offs between time to run and accuracy
- What can we say about the trade offs between time to run and accuracy

Problem 3


Frobenius Norm
p_var_p_corr
0.005
0.004
0.003
0.002
0.001
0.000
direct
pca100
pca75
pca50
p_var_e_corr
e_var_p_corr
e_var_e_corr


Run Time
p_var_p_corr
0.05
0.04
0.03
0.02
0.01
0.00
direct
pca100
pca75
pca50
p_var_e_corr
e_var_p_corr
e_var_e_corr

Problem 3

- **Ans:** Based on these graphs, we can see that as explained percentage decreases on PCA, the accuracy decreases while the runtime also decreases.
- I think the trade off is not fair. The advantage that the faster runtime brings cannot compensate the loss of the accuracy. This means that the overall gain from decreasing PCA explained percentage is not good.
- However, if we do have a time constraint that forces us cannot use more accurate method, we should choose to use PCA with less explained percentage.