Lecture 1

$$\frac{P(Y=1 \mid X=x)}{P(Y=0 \mid X=x)} = \frac{\dfrac{P(x \mid Y=1)\,P(Y=1)}{\cancel{P(x)}}}{\dfrac{P(x \mid Y=0)\,P(Y=0)}{\cancel{P(x)}}}$$

posterior odds

$$= LR \times \text{prior odds}$$

Bayes risk

$$r(\hat{Y}^*(\cdot)) = \mathbb{E}_{X,Y}\left[\ell(Y, \hat{Y}^*(x))\right]$$

$$= \mathbb{E}_{X}\left[\mathbb{E}_{Y|X}\left[\ell(Y, \hat{Y}^*(x))\right]\right]$$

$$= \mathbb{E}_{X}\left[\min_a \mathbb{E}_{Y|X}\left[\ell(Y, a)\right]\right]$$

$$\Downarrow$$

$$r(\hat{Y}(\cdot)) \geq r(\hat{Y}^*(\cdot))$$

$$y \in \mathbb{R}, \quad \ell(y, \hat{y}) = (y - \hat{y})^2$$

$$\underset{a}{\operatorname{argmin}} \ \mathbb{E}_{y|x}\left[(y-a)^2\right] \equiv \hat{y}^*(x)$$

$$\frac{\partial}{\partial a} \mathbb{E}_{y|x}\left[y^2 - 2ya + a^2\right] = 0$$

$$2a - 2\mathbb{E}_{y|x}[y] = 0$$

$$a = \mathbb{E}_{y|x}[y]$$

posterior mean

$$P(y=1 \mid x_1 \cdots x_p) =$$

$$\frac{P(x_1 \cdots x_p \mid y=1)\, P(y=1)}{p(x_1 \cdots x_p)}$$

Naive Bayes assumption:

$$= \frac{\left[\prod_j p(x_j \mid y=1)\right] p(y=1)}{p(x_1 \cdots x_p)}$$

NOTE: The assumption does <u>not</u> imply

$$p(x_1 \cdots x_p) = \prod_j p(x_j)$$

$$= \frac{\left[\prod_j p(x_j \mid y=1)\right] p(y=1)}{\sum_y \left[\prod_j p(x_j \mid y=y)\right] p(y=y)}$$

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